Chapter 43
Nuclear Physics

1. Properties of nuclei
2. Nuclear binding and nuclear structure
3. Nuclear stability and radioactivity
4. Activities and half-lives
5. Biological effects of radiation
6. Nuclear reactions
7. Nuclear fission
8. Nuclear fusion

Nuclear energy: good & evil

- Nuclear medicine Proton therapy
- Little Boy Atomic Bomb

Properties of the nuclei: mass, radius & density

Scattering experiments following the Rutherford experiment show that we can model the nucleus as a sphere with radius of R:

$$R = R_0 A^{1/3}$$

Where A is the nucleon number or mass number and it is the nearest whole number to the mass of the nucleous in atomic mass unit (amu or u for short).

$$1u = 1.66053886(28)\times 10^{-27}\,kg$$

R_0 is an experimentally determined constant $R_0 = 1.2\times 10^{-15}\,m = 1.2\,fm$

$m_p =$ mass of proton $\simeq$ $m_n =$ mass of nutron $\simeq 1u$

Calculating nuclear density:

$$\rho = \frac{m}{V} = \frac{Au}{\frac{4}{3}\pi R^3} = \frac{Au}{\frac{4}{3}\pi\left(R_0 A^{1/3}\right)^3} = \frac{3}{4}\frac{u}{\pi R_0^{\,3}} = \text{constant}$$

Nuclear density is independent of the type of material.

Example: Mass of iron nucleus is 56. Calculate its nuclear radius, approximate mass, and density.

Nuclides & isotopes

The basic building blocks of the nucleus:

Z : atomic number $=$ number of protons $\quad m_p = 1.007276u = 1.672622 \times 10^{-27} kg$

N : neutron number $=$ number of neutrons $\quad m_n = 1.008665u = 1.674927 \times 10^{-27} kg$

For a neutral atom number of electrons $= Z \quad m_e = 0.000548580u = 9.10938 \times 10^{-31} kg$

A: the nucleun number of mass number $A = Z + N$

Isotopes: are the atoms with same atomic number Z but different neutron number N

Chemical properties of the matter are determined by its electrons which is determined by Z.

Isotopes are the same material from chemical point of view.

Physical properties of the material such as melting/boiling point, diffusion rate, etc. change with their atomic mass or A

We represent the atoms with their chemical symbol with atomic number and atomic mass as follows: $\,^{Atomic\ mass}_{atomic\ number}Symbol \rightarrow \,^{A}_{Z}C$

A is usually a good approximation for the atomic mass.

Table 43.1 Compositions of Some Common Nuclides

Nucleus	Mass Number (Total Number of Nucleons), A	Atomic Number (Number of Protons), Z	Neutron Number, $N = A - Z$
$^{1}_{1}H$	1	1	0
$^{2}_{1}D$	2	1	1
$^{4}_{2}He$	4	2	2
$^{6}_{3}Li$	6	3	3
$^{7}_{3}Li$	7	3	4
$^{9}_{4}Be$	9	4	5
$^{10}_{5}B$	10	5	5
$^{11}_{5}B$	11	5	6
$^{12}_{6}C$	12	6	6
$^{13}_{6}C$	13	6	7
$^{14}_{7}N$	14	7	7
$^{16}_{8}O$	16	8	8
$^{23}_{11}Na$	23	11	12
$^{65}_{29}Cu$	65	29	36
$^{200}_{80}Hg$	200	80	120
$^{235}_{92}U$	235	92	143
$^{238}_{92}U$	238	92	146

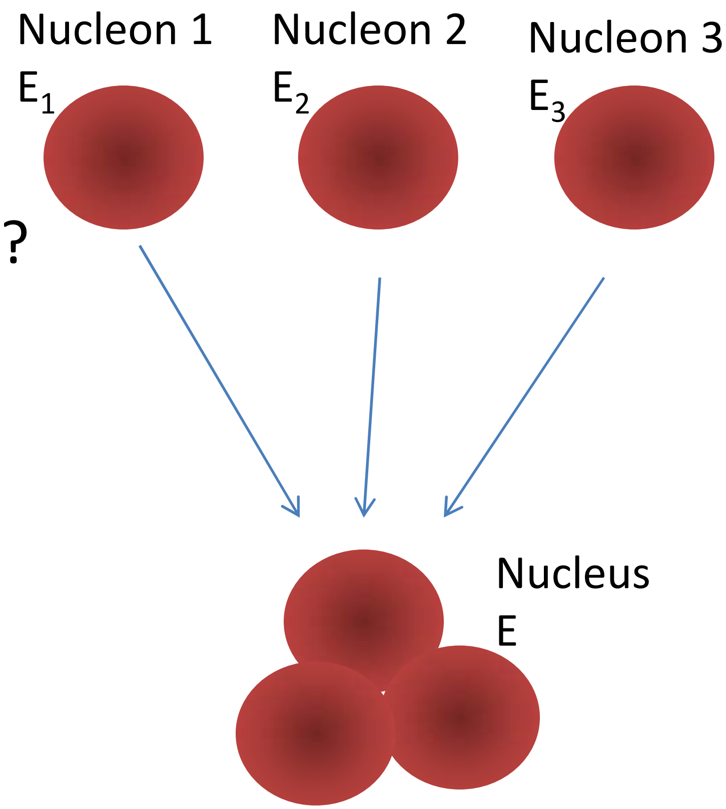
Table 43.2 Neutral Atomic Masses for Some Light Nuclides

Element and Isotope	Atomic Number Z	Neutron Number N	Atomic Mass (u)	Mass Number A
Hydrogen ($^{1}_{1}H$)	1	0	1.007825	1
Deuterium ($^{2}_{1}H$)	1	1	2.014102	2
Tritium ($^{3}_{1}H$)	1	2	3.016049	3
Helium ($^{3}_{2}He$)	2	1	3.016029	3
Helium ($^{4}_{2}He$)	2	2	4.002603	4
Lithium ($^{6}_{3}Li$)	3	3	6.015122	6
Lithium ($^{7}_{3}Li$)	3	4	7.016004	7
Beryllium ($^{9}_{4}Be$)	4	5	9.012182	9
Boron ($^{10}_{5}B$)	5	5	10.012937	10
Boron ($^{11}_{5}B$)	5	6	11.009305	11
Carbon ($^{12}_{6}C$)	6	6	12.000000	12
Carbon ($^{13}_{6}C$)	6	7	13.003355	13
Nitrogen ($^{14}_{7}N$)	7	7	14.003074	14
Nitrogen ($^{15}_{7}N$)	7	8	15.000109	15
Oxygen ($^{16}_{8}O$)	8	8	15.994915	16
Oxygen ($^{17}_{8}O$)	8	9	16.999132	17
Oxygen ($^{18}_{8}O$)	8	10	17.999160	18

Source: A. H. Wapstra and G. Audi, *Nuclear Physics* **A595,** 4 (1995).

Nuclear binding and nuclear force

- What do you think of the relationship between E & $E_1+E_2+E_3$?
- Why the nucleus should exist instead of the individual nucleons?



$E < E_1 + E_2 + E_3$

Lower energy and more stability is the key for any reaction and nuclear reactions are not exceptional.

We define the nuclear binding energy:

$$E_B = Z\left(m_p c^2 + m_e c^2\right) + N m_n c^2 - {}_Z^AM c^2$$

$$\boxed{E_B = (ZM_H + Nm_n - {}_Z^AM)c^2} > 0 \text{ bonding; } < 0 \text{ no bonding}$$

Where $M_H = m_p + m_e$

Is it OK to use $M_H = m_p + m_e$?

Example: Binding energy/nucleon

- Find the binding energy **per** nucleon of the ^{62}Ni if its neutral atomic mass is 61.928349u

$$E_B = \left(ZM_H + Nm_n - {}^A_Z M \right) c^2$$

We have Ni so $Z = 28$; $A = 62$ & $N = A - Z = 34$; ${}^A_Z M = 61.928349u$;

$M_H = 1.007825u$; $m_n = 1.008665u$; $\underbrace{c = 2.997925}_{\text{Use the exact value}} \times 10^8 m/s$; $1u = 1.660539 \times 10^{-27} kg$

$$uc^2 = 931.5 MeV \rightarrow c^2 = \frac{931.5 MeV}{u}$$

$$E_B = \left(ZM_H + Nm_n - {}^A_Z M \right) c^2$$

$$E_B = \left(28 \times 1.007825u + 34 \times 1.008665u - 61.928349u \right) \frac{931.5 MeV}{u}$$

$\boxed{E_B = 545.3 MeV !!!}$ > 0 so this atom can be stable.

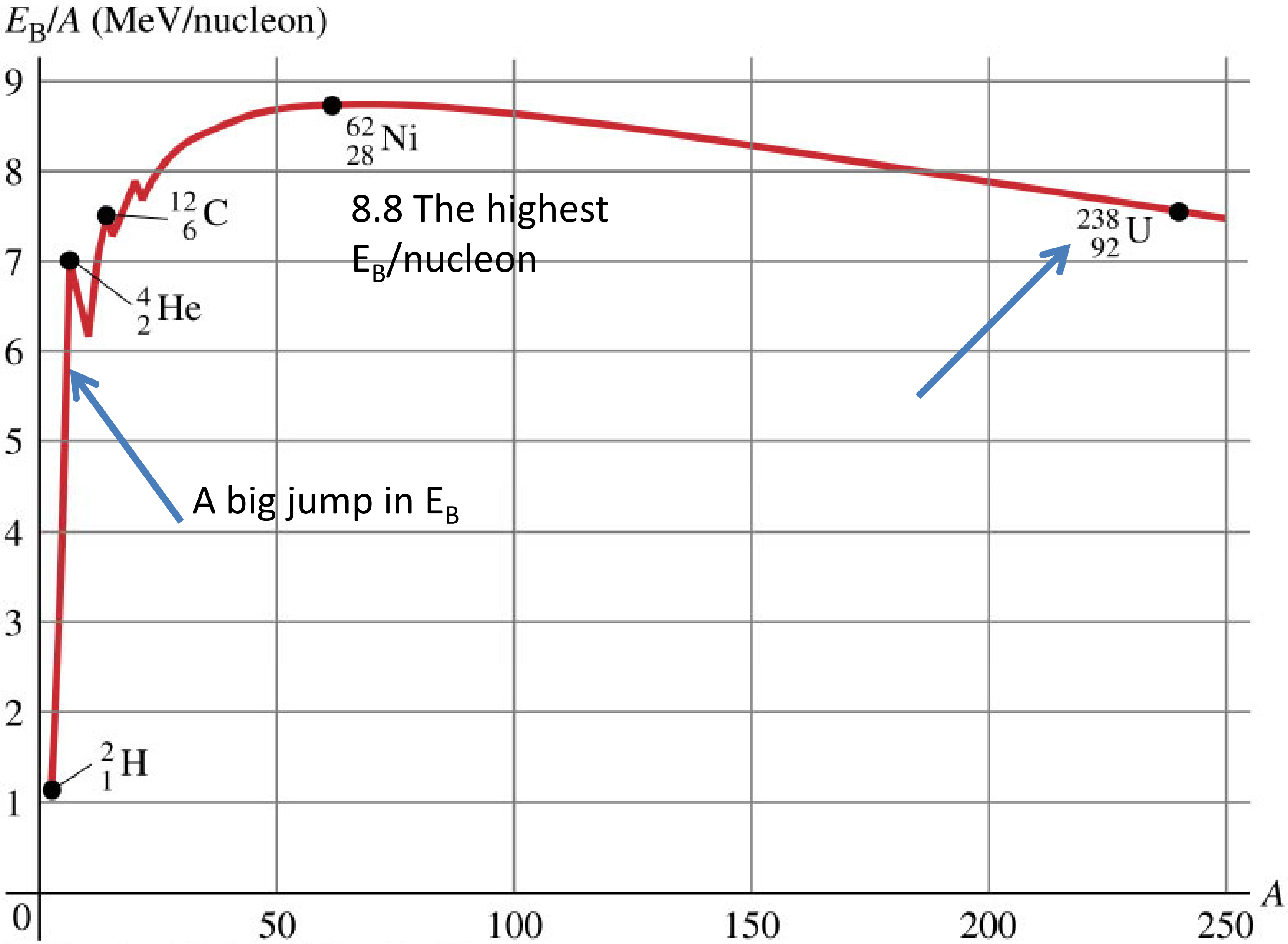
Keep the energies in units of eV for nuclear physics.

$E_B / nucleon = E_B / A = 545.3 MeV / 62$

$E_B / nucleon = \underline{8.795 MeV / nucleoon} ?!!!$

Compare this to the atomic binding energies.

Binding energy/nucleon
A very important graph



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The nuclear force

- The nuclear force binds the protons to protons despite of the Coulomb repulsion. It belongs to the category of the strong forces
  - Does not depend on the charge and both neutrons and protons are bound the same way.
  - It is a very short-range force ~10^{-15} m (otherwise very large nuclei would be possible to form)
  - There is a saturation property due to the very short range similar to covalent bonding.
  - There is pairing between the nucleons of opposite spin and pairing of pairs like alpha particles (protons and neutrons are spin-1/2 particles or they are fermions).
  - Much stronger than the electrical forces (otherwise the nucleus would fall apart).
- We don't have a clear picture of inside of the nuclei
- We gain some insight by studying simple models such as liquid drop or shell model.

Liquid-drop model (1928 George Gamow)

Based on the observation that all the nuclei have almost the same density.

The nuclei are hold together similar to the molecules of a liquid drop with short-range interactions and surface-tension effects.

Consider the following contributions to the binding energy:

1) Saturation: an individual nucleon interacts only with few of its nearest neighbors $E_S = C_1 A$.

2) Surface effect: the nucleons on the surface are less tightly bound than interior ones.

$$E_{surface} \propto \underbrace{-}_{\text{Reduction}} \underbrace{4\pi R^2}_{Surface} = -C4\pi\left(R_0 A^{1/3}\right)^2 = -C_2 A^{2/3}$$

3) Coulomb force: every one of Z pronons repels $(Z-1)$ other protons with a force proportional to

$$1/R^2 \to E_{\substack{Coulomb\\ potential}} = -C\frac{Z\left(Z-1\right)}{4\pi\varepsilon_0 R} = \underbrace{-}_{\text{Reduction}} C_3 \frac{Z\left(Z-1\right)}{A^{1/3}}$$

Liquid-drop model (1928 George Gamow) II

4) The most stable muclei are the ones with balanced number of neutrons and protons. As nuclei depart from this rule a negative energy term is introduced to the E_B.

Based on experimental data

$$E_{balance} \propto -\left(N-Z\right)^2 / A = -C_4\left(A-2Z\right)^2 / A$$

5) Pairing: nuclear force favors pairing of protons and neutrons. Pairing energy is experimentally determined

$$E_{pairing} = \pm C_5 A^{-4/3}.$$

It is $+$ for both N & Z even and $-$ for both N & Z odd and zero otherwise.

Binding energy based on liquid-drop model

Putting all together: $E_B = \underbrace{C_1 A}_{Saturation} \underbrace{-C_2 A^{2/3}}_{Surface} \underbrace{-C_3 \frac{Z(Z-1)}{A^{1/3}}}_{Coulomb} \underbrace{-C_4 \frac{(A-2Z)^2}{A}}_{p-n \text{ balance}} \underbrace{\pm C_5 A^{-4/3}}_{Pairing}$

Experimentally determined constants:

$C_1 = 15.75\ MeV$ $\quad C_2 = 17.80 MeV$ $\quad C_3 = 0.7100 MeV$

$C_4 = 23.69 MeV$ $\quad C_5 = 39 MeV$

We can use E_B formula to estimate mass of any neutral atom:

$${}_Z^M M = ZM_H + Nm_n - \Delta m \rightarrow \boxed{{}_Z^M M = ZM_H + Nm_n - \frac{E_B}{c^2}}$$ A semiempirical formula

To separate all of the nucleons of an atom we need to invest energy as much as E_B which is equivalent to convertion of that energy to mass of $\Delta m = \frac{E_B}{c^2}$

$ZM_H + Nm_n = {}_Z^M M + \frac{E_B}{c^2}$ $\leftarrow$ Example of conservation of mass-energy law.

The model explains the nuclear masses and decay of the unstable nuclei but has no use in explanation of the angular nomentum and excited states.

Example: binding energy and mass

- Calculate the five terms in the binding energy and the total E_B for the ${}^{62}_{28}Ni$ nucleus. compare it to E_B calculates based on measured M=61.928349u
- Find its neutral atomic mass and compare it to the measured value

1) $C_1 A = 976.5 MeV$

2) $-C_2 A^{2/3} = -278.8 MeV$

3) $-C_3 \frac{Z(Z-1)}{A^{1/3}} = -135.6 MeV$

4) $-C_4 \frac{(A-2Z)^2}{A} = -13.8 MeV$

5) $+C_5 A^{-4/3} = 0.2 MeV$

$E_B = 548.5 MeV$

Compared to $545.3 MeV$, it is 6% larger.

$$M = 28(1.007825u) + 34(1.008665u) - \frac{548.5 MeV}{931.5 MeV/u}$$

$$M = 61.925u$$

$$\Delta u\% = \frac{61.928349u - 61.925u}{(61.928349u + 61.925)0.5} \times 100 = 0.005\%$$

Not bad!!

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- in good academic standing, and completed Physics 2a/b or 50 series, and Chem 1a
- US citizens or permanent residents

For more information, contact Dr. Kress at mkress@science.sjsu.edu

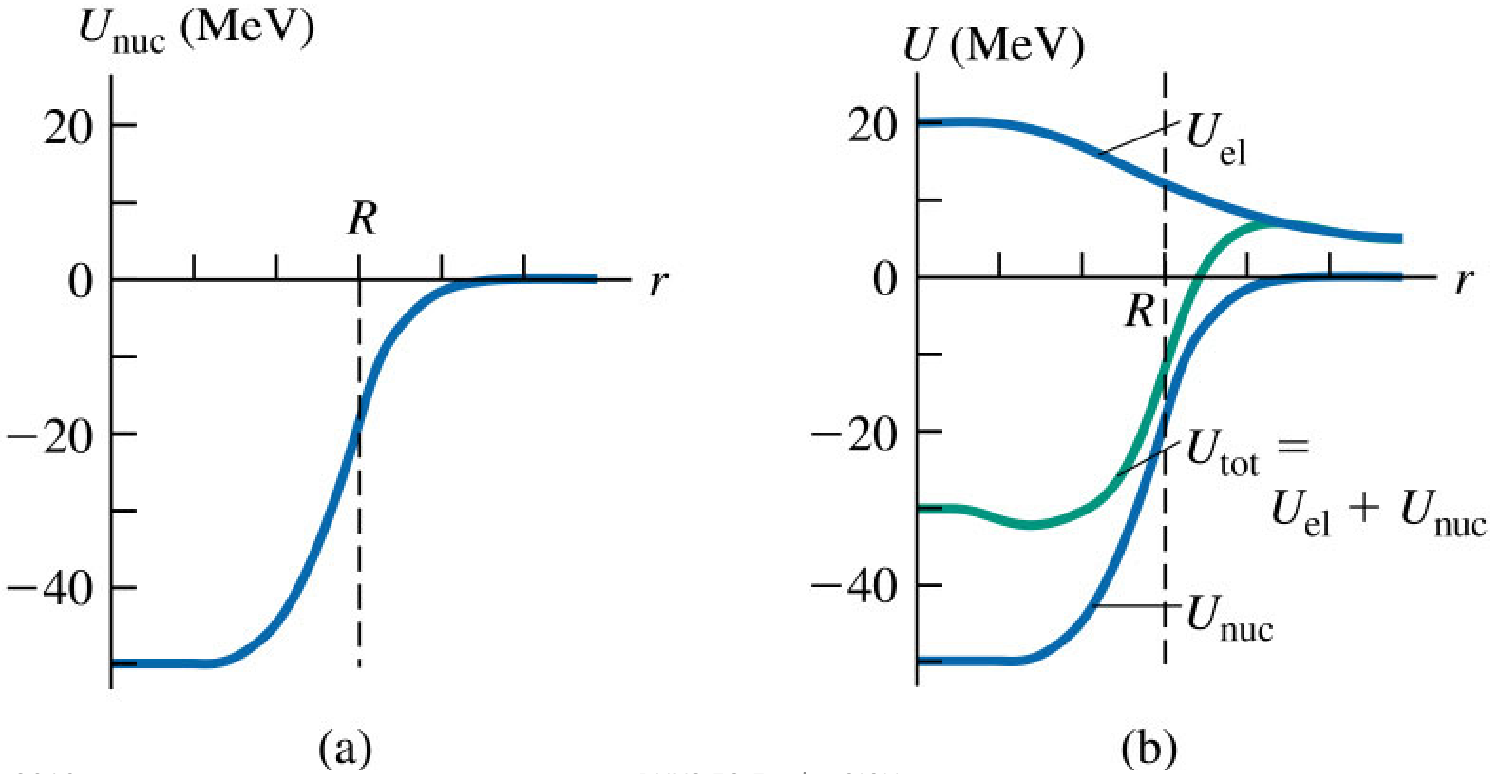
The shell model

Using the central-field approximation, we assume each nucleon moving in a averaged-out potential field of all other nucleons. Due to short-range nature of the nuclear forces this may not work but it explaoins many properties of the nuclei surprisingly well.

The potential field is approximated by a 3D square potential well.

We put that together with the potential for the mutual repulsion of the protons.

We can solve the Schrodinger equation and find the energy levels, shells and subshells.



The shell model and the magic numbers

The central field approximation in atomic model successfully explained the chamical stability of the inert gasses as those atoms with complete shell.

$Z = 2, 10, 16, 18, 36, 54$

Nuclear structure exhibits a comparable effect with different atomic numbers called magic numbers:

$Z \ or \ N = 2, 8, 20, 28, 50, 82, \underline{126}$

The reason for the difference is the difference in the potential-energy function and also a much stronger spin-orbit interaction in the nuclear structure.

Nuclei with both N and Z equal to a magic number are called doubly magic nucleides. Magic and doubly magic nucleides have:

a) unusually high number of stable isotopes,

b) zero nuclear spin,

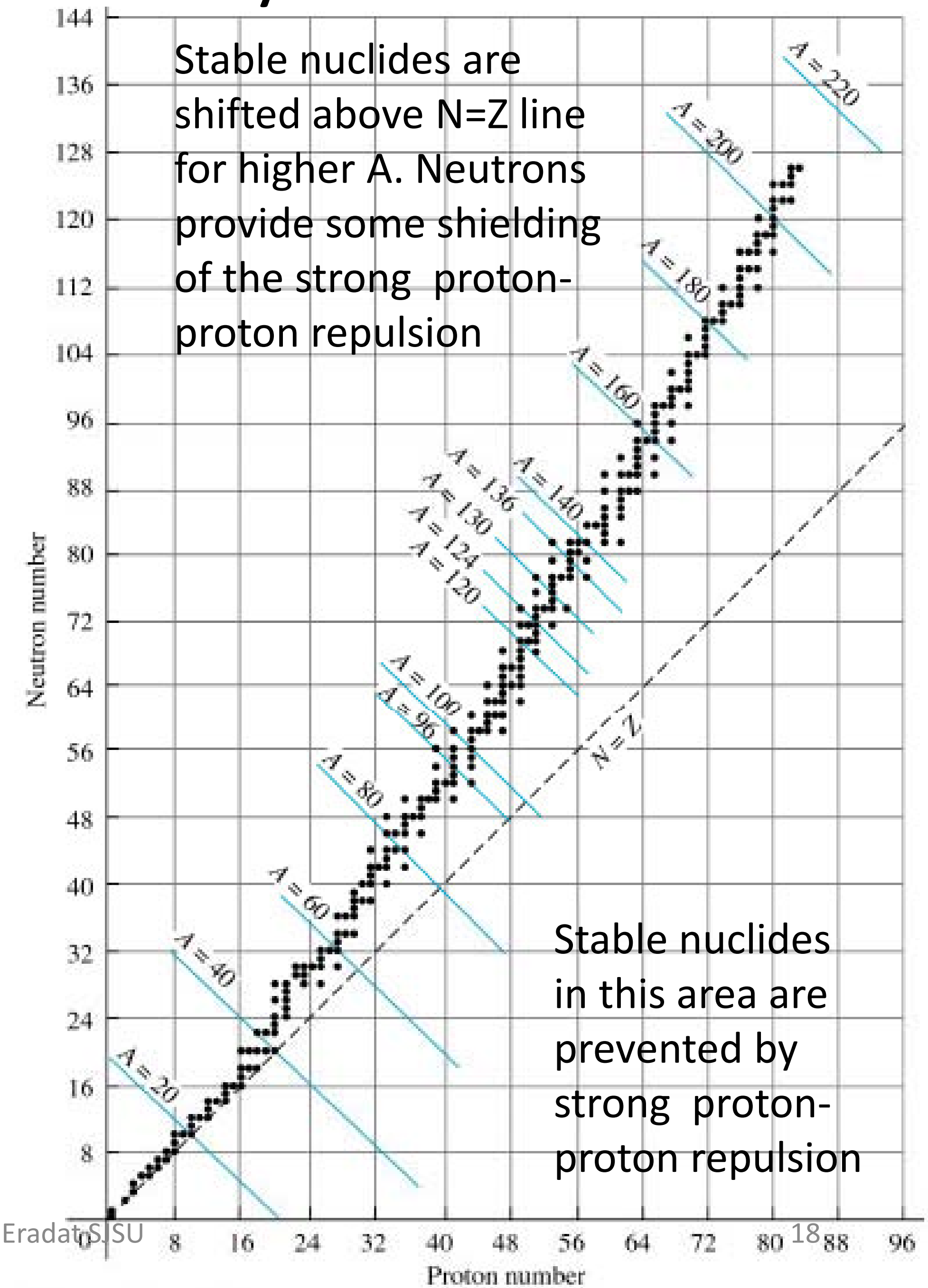
c) filled-shell or sub-shell,

d) large jump in E_B compared to their neighbors.

Nuclear stability and radioactivity

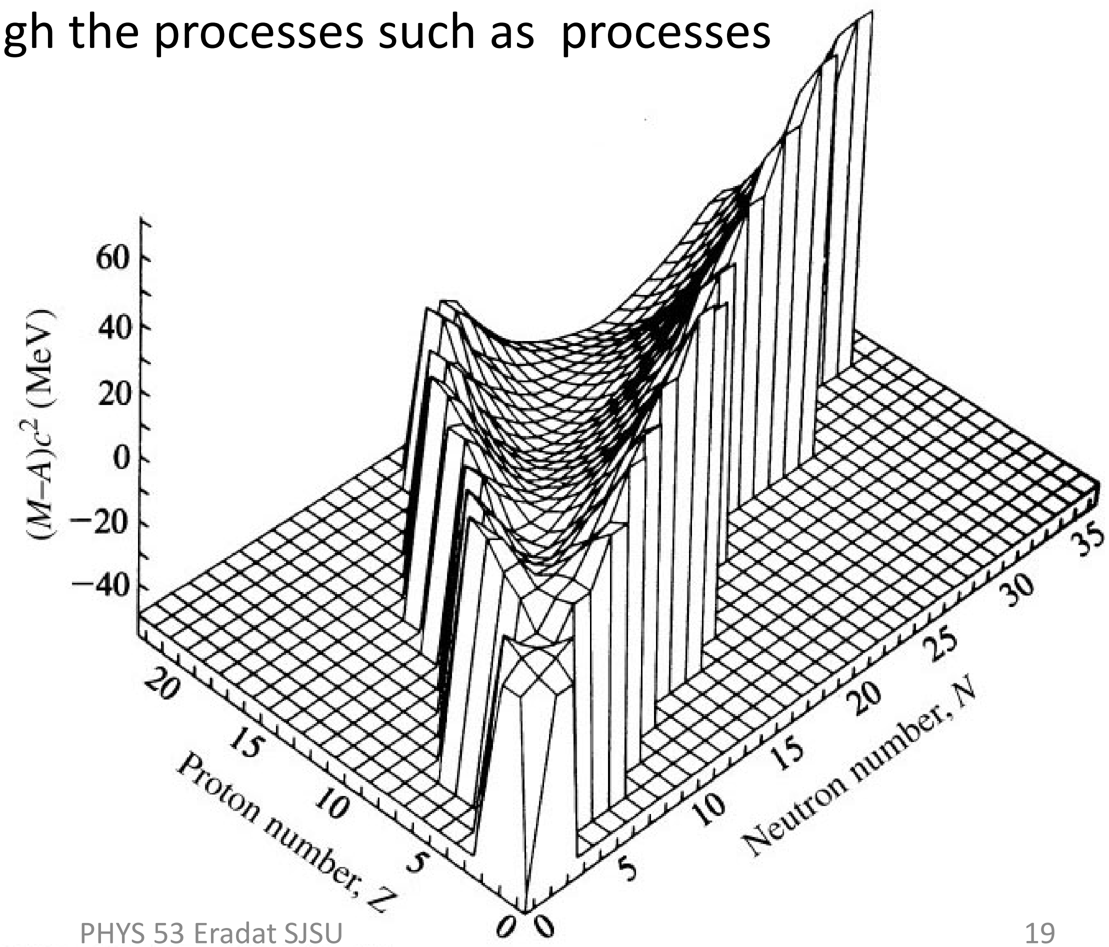
Segre chart (1905-1989)

- 2500 known nuclides
- 300 stable nuclides
- Radioactive nuclides: unstable nuclides emit particles and EM radiation to become stable. The process can take from micro-seconds to billions of years.
- Segre chart shows the stable nuclides as a function of N and Z. The blue lines are the *A=constant* lines.
- Only 4 stable odd-odd nuclides exist. Extreme stability of the doubly magic $^{4}_{2}He$ prevents A=5 and A=8 nuclides from existence.



3-D version of the Segre chart

- Unstable nuclides with higher A and higher N/Z ratio decay to lighter nuclides with balanced N to Z ration closer to one. In the process neutrons convert to protons and …
- 90% of the known nuclides (2500) are radioactive and they decay to lighter and more stable nuclides through the processes such as processes
  - α-decay
  - β-decay
  - γ-decay



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Alpha decay

An α particle is ${}^{4}He$ nucleus with two protons and two neutrons with total spin of zero.

The nuclei that are too large to exist can reduce their A by α emission.

The nuclear reaction has to satisfy several conservation laws (can you name them?).

$${}^{A_A}_{Z_A}A + {}^{A_B}_{Z_B}B \rightarrow {}^{A_C}_{Z_C}C + {}^{A_D}_{Z_D}D + Energy$$

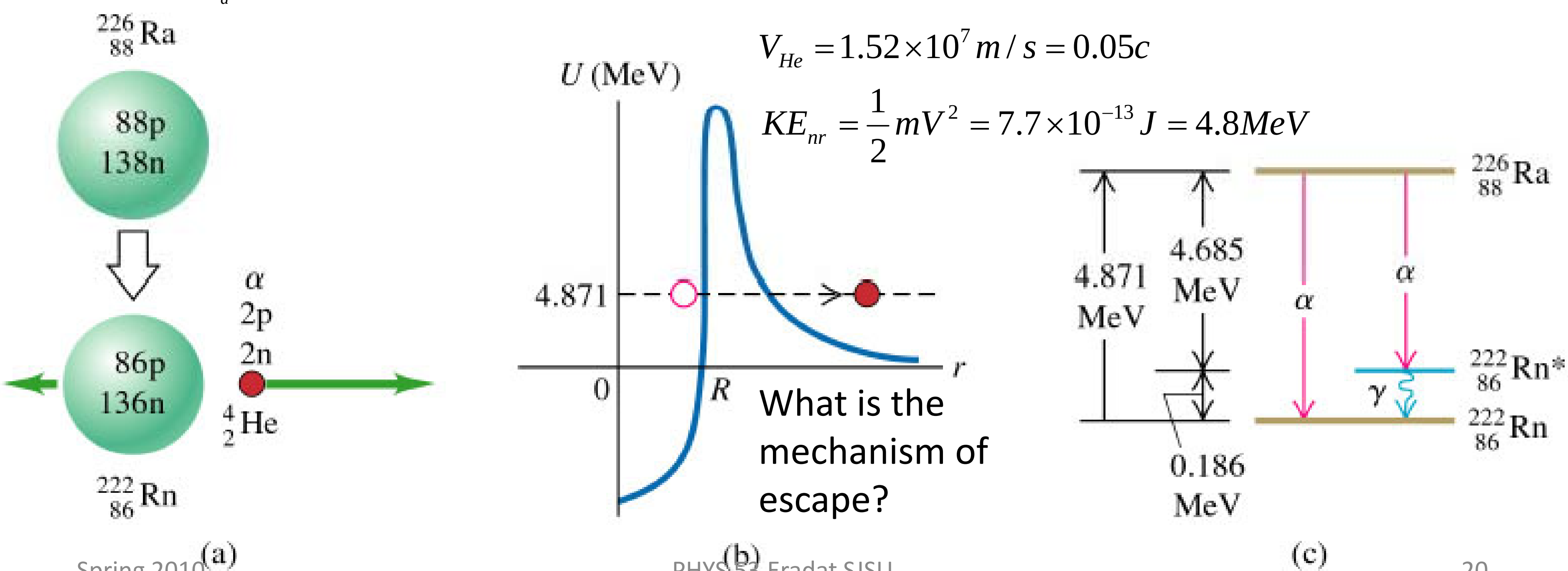
Mass-Energy conservation: $\gamma_A M_A c^2 + \gamma_B M_B c^2 = \gamma_C M_C c^2 + \gamma_D M_D c^2; \quad E_{total} = \gamma mc^2 = K + mc^2$

We need to decide if relativistic or classical treatment is needed for a given situation.

$\underbrace{{}^{226}_{88}Ra}_{Mp} \rightarrow \underbrace{{}^{222}_{86}Rn^*}_{M_d} + K_d + \underbrace{{}^{4}_{2}He}_{M_\alpha} + K_\alpha$ Note: $M_p > M_d + M_\alpha$ to have α-decay possible.

$V_{He} = 1.52 \times 10^7 m/s = 0.05c$

$KE_{nr} = \frac{1}{2}mV^2 = 7.7 \times 10^{-13} J = 4.8 MeV$



What is the mechanism of escape?

Beta decay

There are three types of β decay:

Beta-minus β^- (electron)$: n \rightarrow p + \beta^- + \overline{\nu_e}$ happens whenever N/Z is too large;

$$^{A_A}_{Z_A}A \rightarrow ^{A_A}_{Z_{A+1}}C + \beta^- + \overline{\nu_e} + Energy; \quad M_p > M_d$$

Beta-plus β^+ (positron)$: p \rightarrow n + \beta^+ + \nu_e$ happens whenever N/Z is too small;

$$^{A_A}_{Z_A}A \rightarrow ^{A_A}_{Z_{A-1}}C + \beta^+ + \nu_e + Energy; \quad M_p - M_d > 2m_e$$

Electron capture: $p + \beta^- \rightarrow n + \nu_e$ happens whenever N/Z is too small;

β^+ is energetically not possible

$$^{A_A}_{Z_A}A \rightarrow ^{A_A}_{Z_{A-1}}C + \nu_e + Energy; \qquad M_p > M_d$$

The electron is usually captured from the $K-$shell

Important for β decay: $m_n = 1.008665u > m_p = 1.007276u$

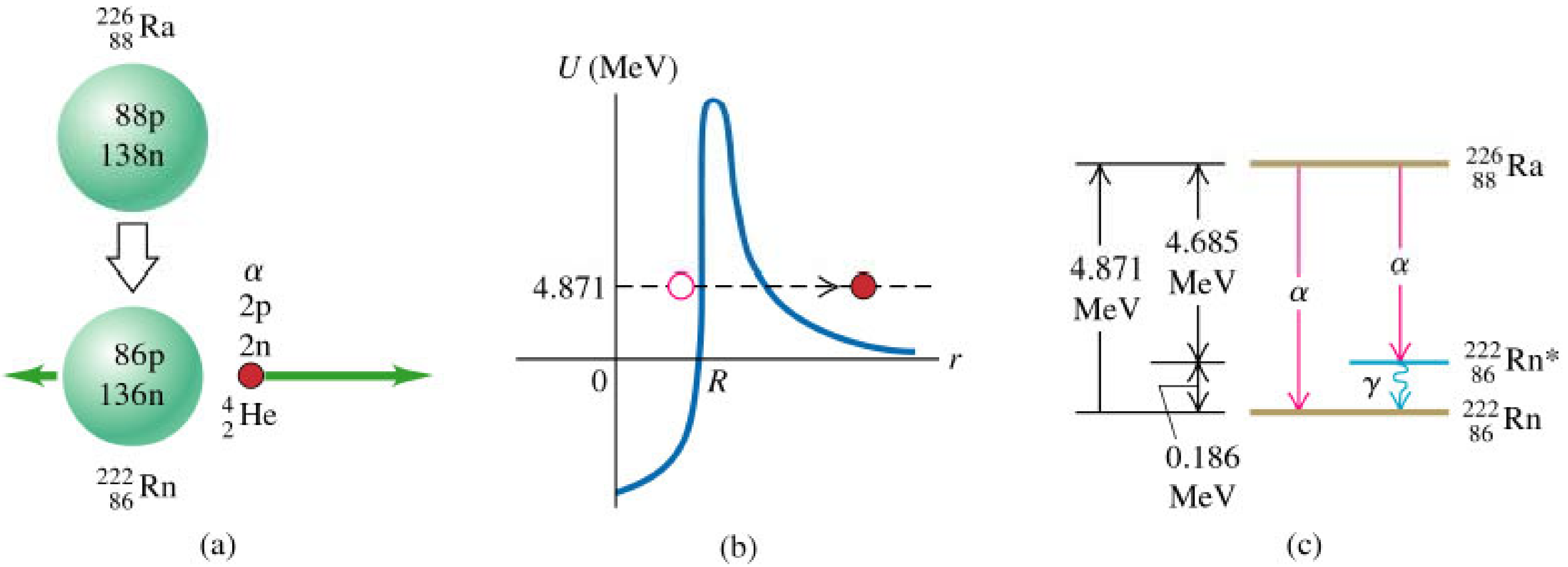
The nutrino ν_e and anti-nutrino $\overline{\nu_e}$ are particle-antiparticle pairs with little mass and no charge. Very difficult to detect (detected by Raines & Cowan in 1953).

Gamma decay

γ-rays are high-energy photons 10KeV-5MeV emitted from the nuclei that are in excited state or undergoing nuclear reactions.

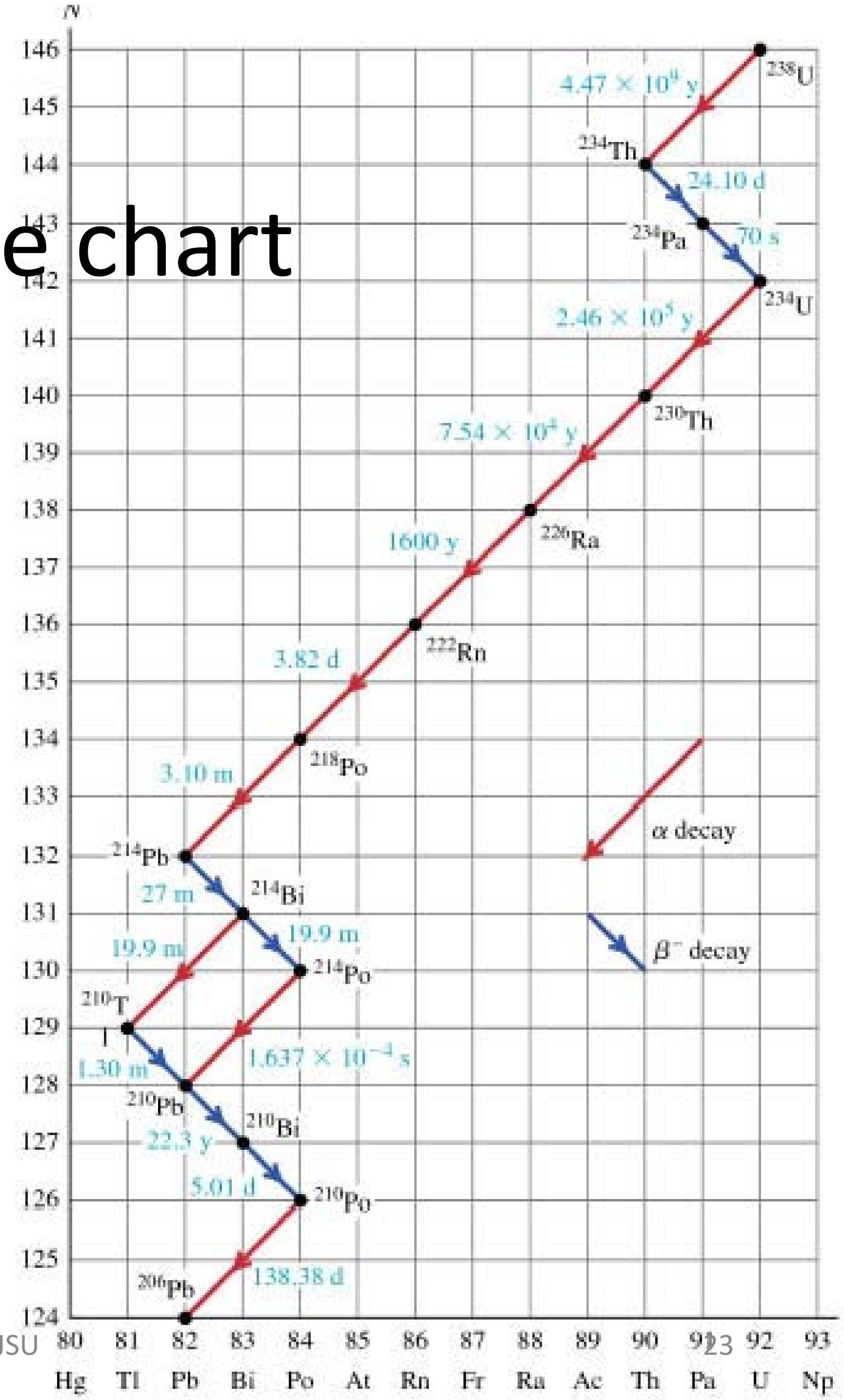
This is simmilar to emission from the atoms when they relax from an excited state to the ground state.

The difference is in magnitude of the energy of each emitted photon which depands on the spacing of the energy levels.



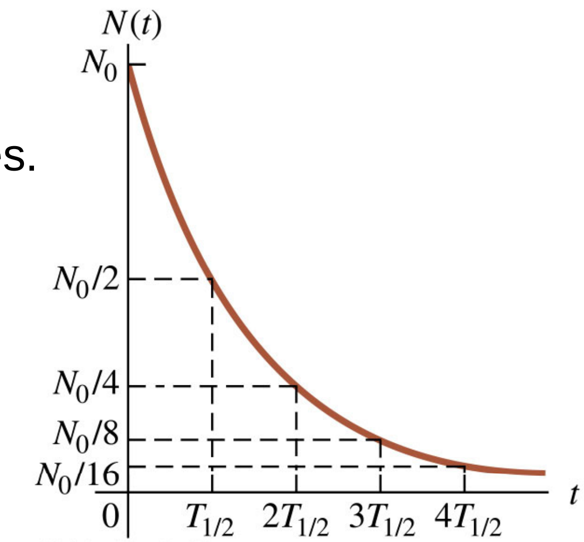
Natural radioactivity Reactions on the Segre chart

- We have all kinds of heavy nuclei from the time of formation of stars and planets.
- All the unstable ones are in the process of decaying into stable ones.
- Some of these processes are very slow.
- Example: Segre chart of ^{238}U decay series terminating with stable nuclide ^{206}Pb. Note the half lives (blue fonts)



Activities & half-lives

Rate of decay of a radioactive nucleon is not affected by any chemical or physical environmental effects. It is a totally random phenomenon. We analyze it with statistical techniques.



$N(t)$: Number (very large) of radioactive nuclei

$dN(t)$: change (negative) in $N(t)$ during a short time dt

$-\frac{dN(t)}{dt}$: activity or decay rate that is proportional to the amount of material present in the sample or $\boxed{-\frac{dN(t)}{dt} = \lambda N(t)}$

λ: decay constant or probability of decay/t $\begin{cases} \text{large for rapidly decaying} \\ \text{small for the slowly decaying} \end{cases}$

$$\frac{dN(t)}{N(t)} = -\lambda dt \rightarrow \ln\left(N(t)\right)\Big|_{N_0}^{N} = -\lambda t\Big|_0^t \rightarrow \boxed{N(t) = N_0 e^{-\lambda t}}$$ # of remaining nuclei

Half-life of the radioactive material: $\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}} \rightarrow \boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$

Lifetime: $\boxed{T_{mean} = \frac{1}{\text{decay constant}} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693}}$ $\underbrace{\boxed{1Ci = 3.70 \times 10^{10} Bq = 3.70 \times 10^{10} decays/s}}_{\text{Unit of activity}}$

Example: Activity of ^{57}Co

Radioactive isotope of ^{57}Co decays by electron capture with a half-life of 272 days.

a) Find the decay constant and the lifetime of the process.

b) If you have a radiation source of ^{57}Co with activity of 2.00μCi, how many radioactive nuclei does it contain?

c) What will be the activity of the source after one year?

$$^{57}_{27}Co \rightarrow ^{57}_{26}Fe + \nu_e$$

$$T_{1/2} = 272 days = (272 days)(86400 s/day) = 2.35\times10^7 s$$

a) Lifetime: $T_{mean} = T_{1/2}/\ln 2 = \dfrac{2.35\times10^7 s}{0.693} = 3.39\times10^7 s$

Decay constant $\lambda = 1/T_{mean} = 2.95\times10^{-8}/s$

b)The activity $\dfrac{-dN(t)}{dt} = 2.00\mu Ci$

$$= (2.00\times10^{-6})(3.70\times10^{10}/s) = 7.40\times10^4 decays/s$$

# of nuclei: $N(t) = \dfrac{-dN(t)/dt}{\lambda} = \dfrac{7.40\times10^4 decays/s}{2.95\times10^{-8}/s}$

$$N(t) = 2.51\times10^{12} nuclei$$

c)activity after 1 year: $N(1y) = N_0 e^{-\lambda t}$

$$= N_0 e^{-(2.95\times10^{-8}/s)365 day/y(86400 s/day)} = 0.394 N_0$$

$$-dN(t)/dt = \lambda N(1y) = \lambda(0.394 N_0) = \underbrace{\lambda N_0}_{\text{Original activity}} (0.394)$$

$$-dN(1y)/dt = (2.00\mu Ci)(0.394) = 0.788\mu Ci$$

Radiocarbon dating

Nuclear spins & magnetic moments

Table 43.3 Relative Biological Effectiveness (RBE) for Several Types of Radiation

Radiation	RBE (Sv/Gy or rem/rad)
X rays and γ rays	1
Electrons	1.0–1.5
Slow neutrons	3–5
Protons	10
α particles	20
Heavy ions	20

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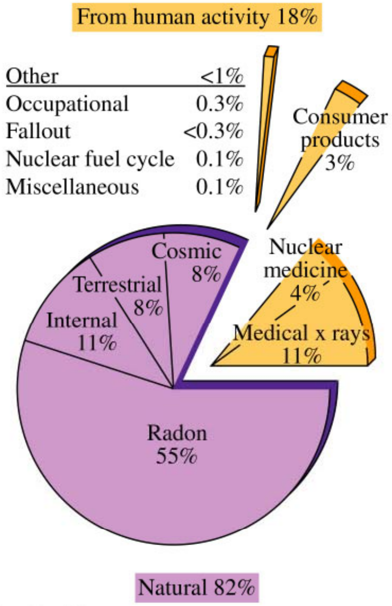
Radioactive dating



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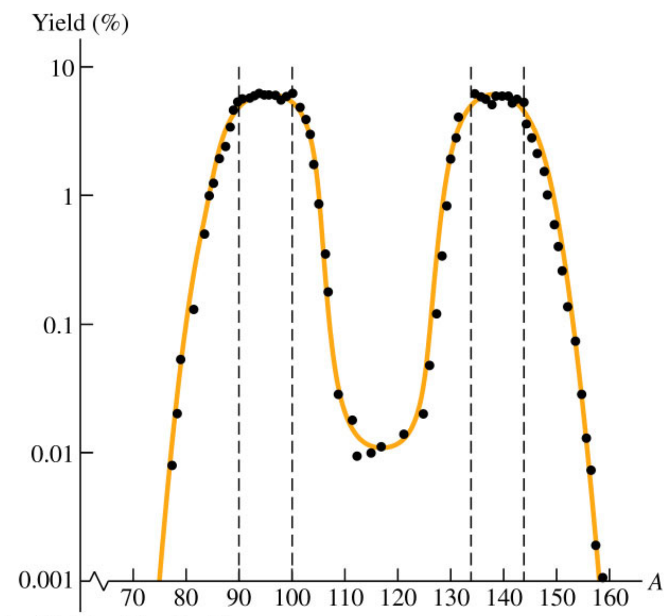

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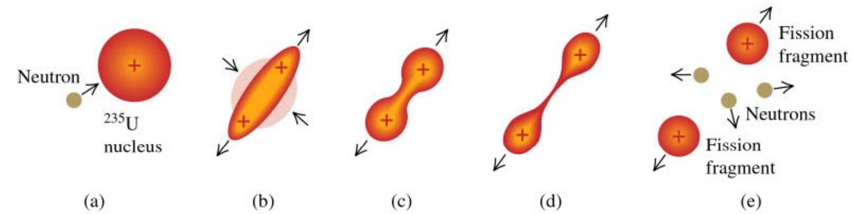

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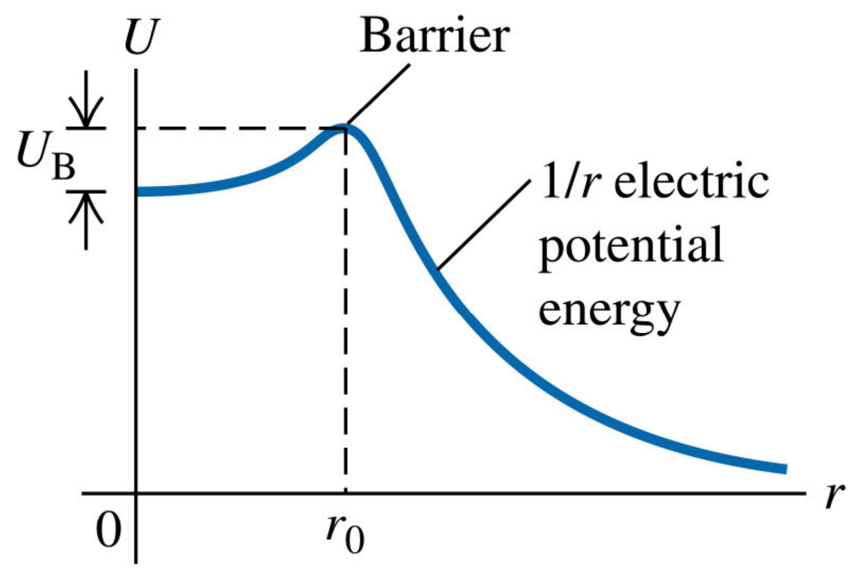

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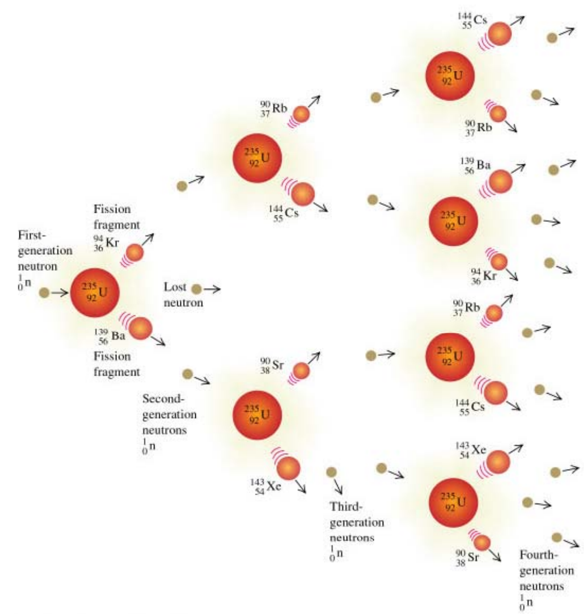

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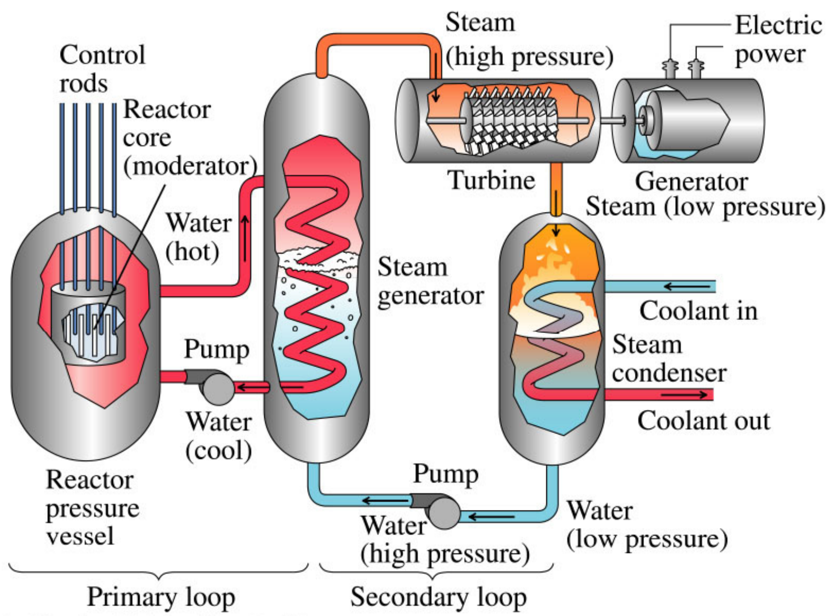

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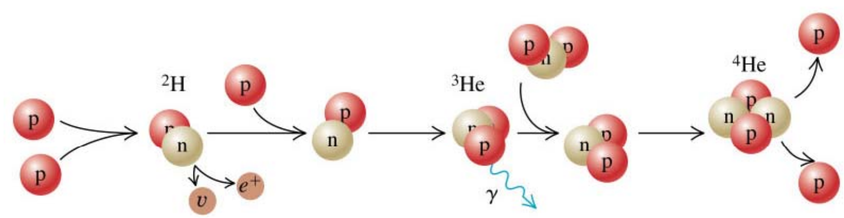

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