Chapter 37
Relativity

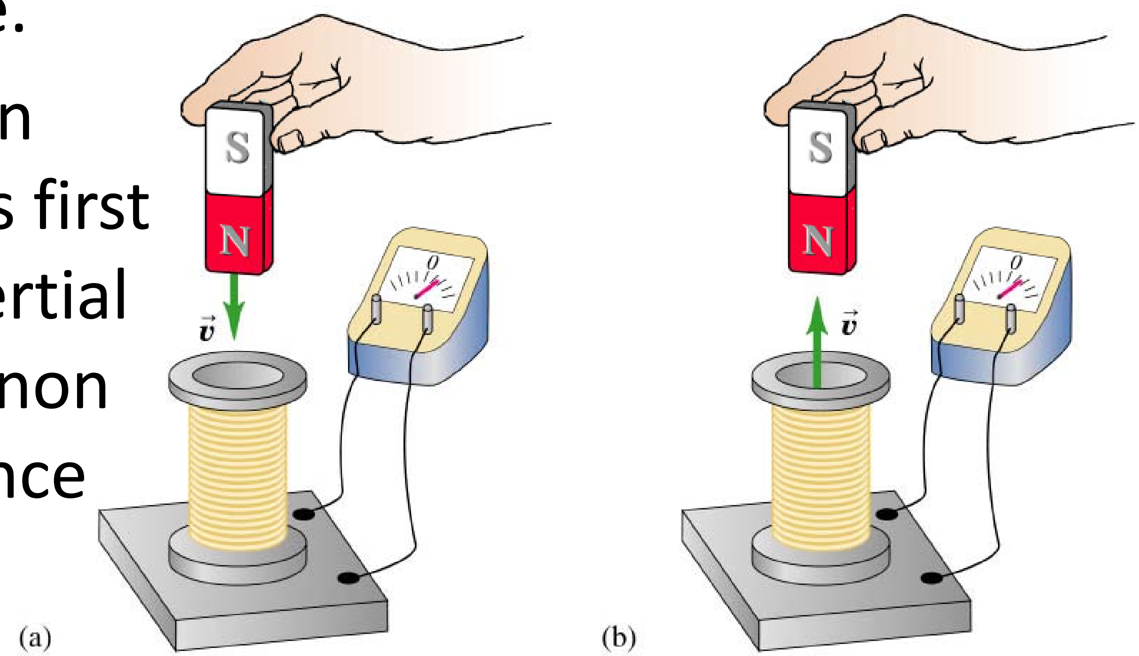
1. Invariance of physical laws
2. Relativity of Simultaneity
3. Relativity of time intervals
4. Relativity of length
5. The Lorentz transformation
6. The Doppler effect for electromagnetic waves
7. Relativistic momentum
8. Relativistic work and energy
9. Newtonian mechanics and relativity

“Special” theory of relativity

- A drastic revision of the Newtonian concepts of space and time, postulated by Albert Einstein in 1905
  1. Laws of physics are the same in all inertial reference frames
  2. Speed of light in vacuum is the same in all inertial reference frames.
- Implications of the Einstein’s postulates
  1. Events that are simultaneous for one observer may not be simultaneous for another one
  2. Two observers that are moving relative to each other may not agree in their time interval and length measurements
  3. In order to keep the conservation of momentum & energy valid, we need to revise the Newton’s second law and equations for momentum and kinetic energy.
- Consequences of relativity are seen in electromagnetism, atomic and nuclear physics, high-energy physics and the theory is in solid agreement with experimental results.

37.1 Invariance of physical laws Einstein's first postulate

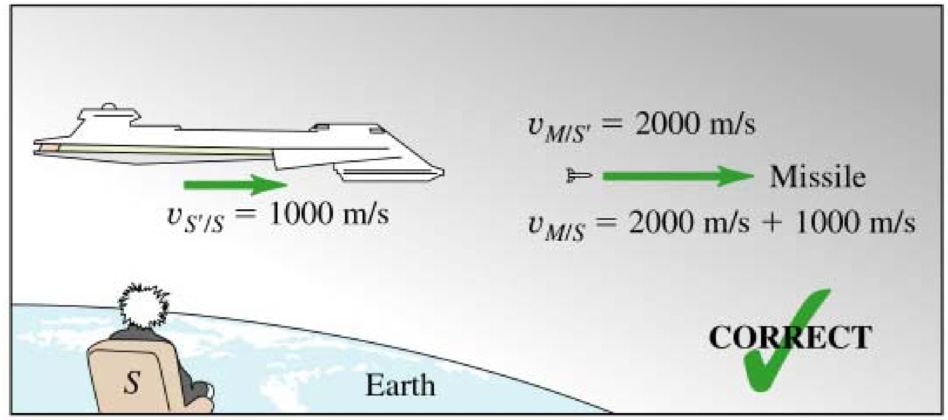
- The laws of physics are the same in all inertial frames of reference.
- A reference frame in which the Newton's first law is valid is an inertial reference frame (a non accelerating reference frame)



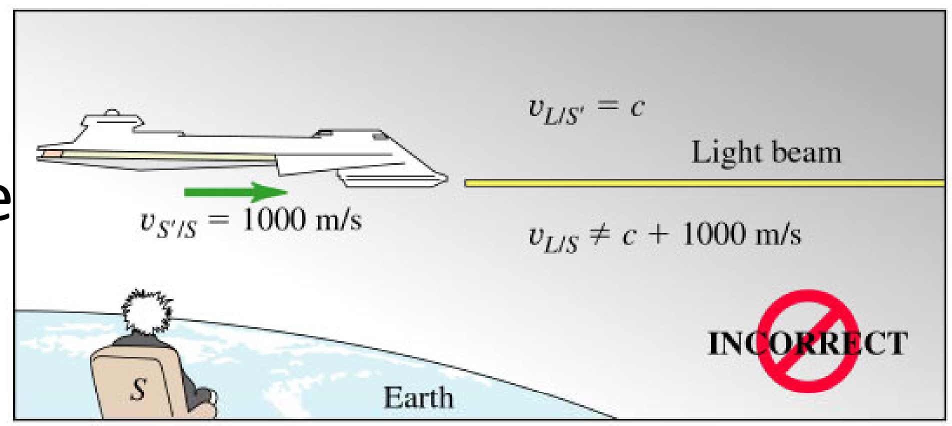
Is the induced emf in the coil is different in a from b?

Einstein's second postulate
Invariance of speed of light

- Speed of light in vacuum is the same in all inertial reference frames and is independent of the motion of source.
- This implies that it is impossible for an inertial observer to travel at c the speed of light in vacuum.



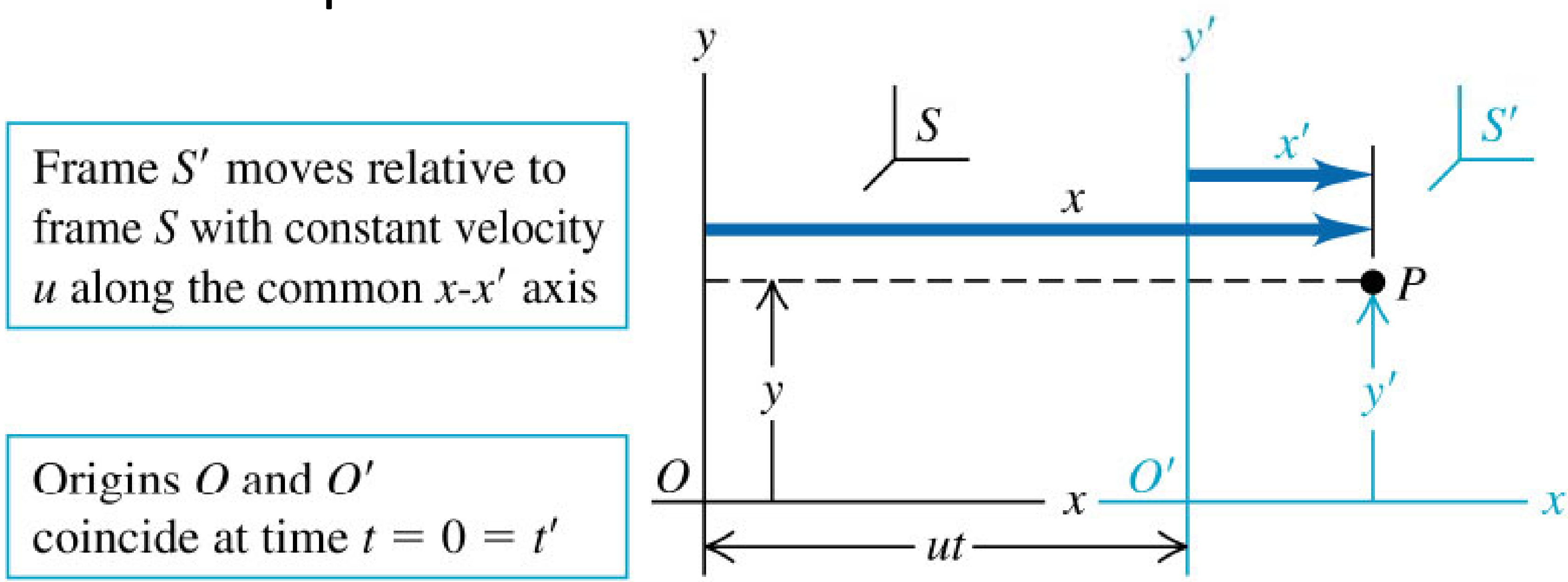
(a)



(b)

The Galilean coordinate transformation

- Two observers are attached to two inertial reference frames S & S’
- Describe motion of a particle at P from the observers at S and S’ point of view.

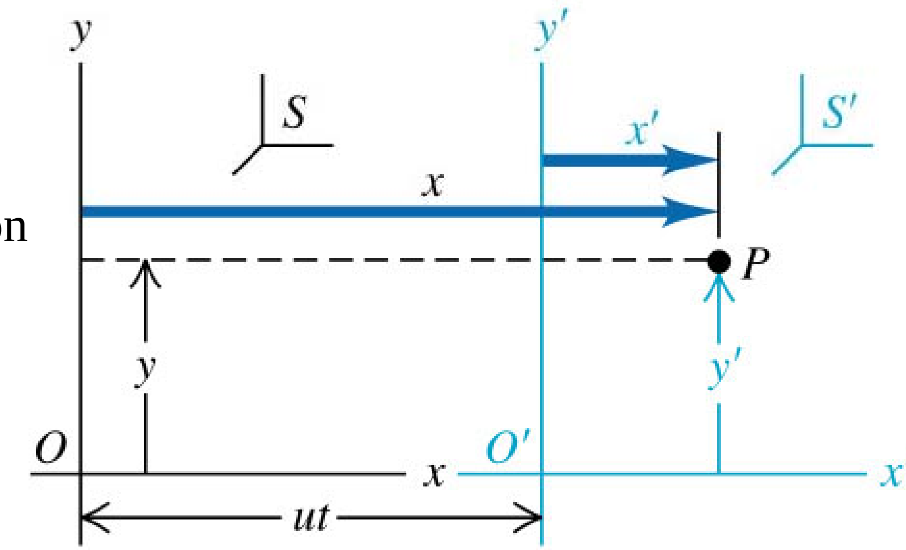


The Galilean coordinate transformation

Coordinates of P in S (attached to earth): (x,y,z)

Coordinates of P in S' (moving): (x',y',z')

Position of the particle in two frames:



$$\left.\begin{aligned} x &= x' + ut \\ y &= y' \\ z &= z' \end{aligned}\right\} \text{Galilean coordinate transformation}$$

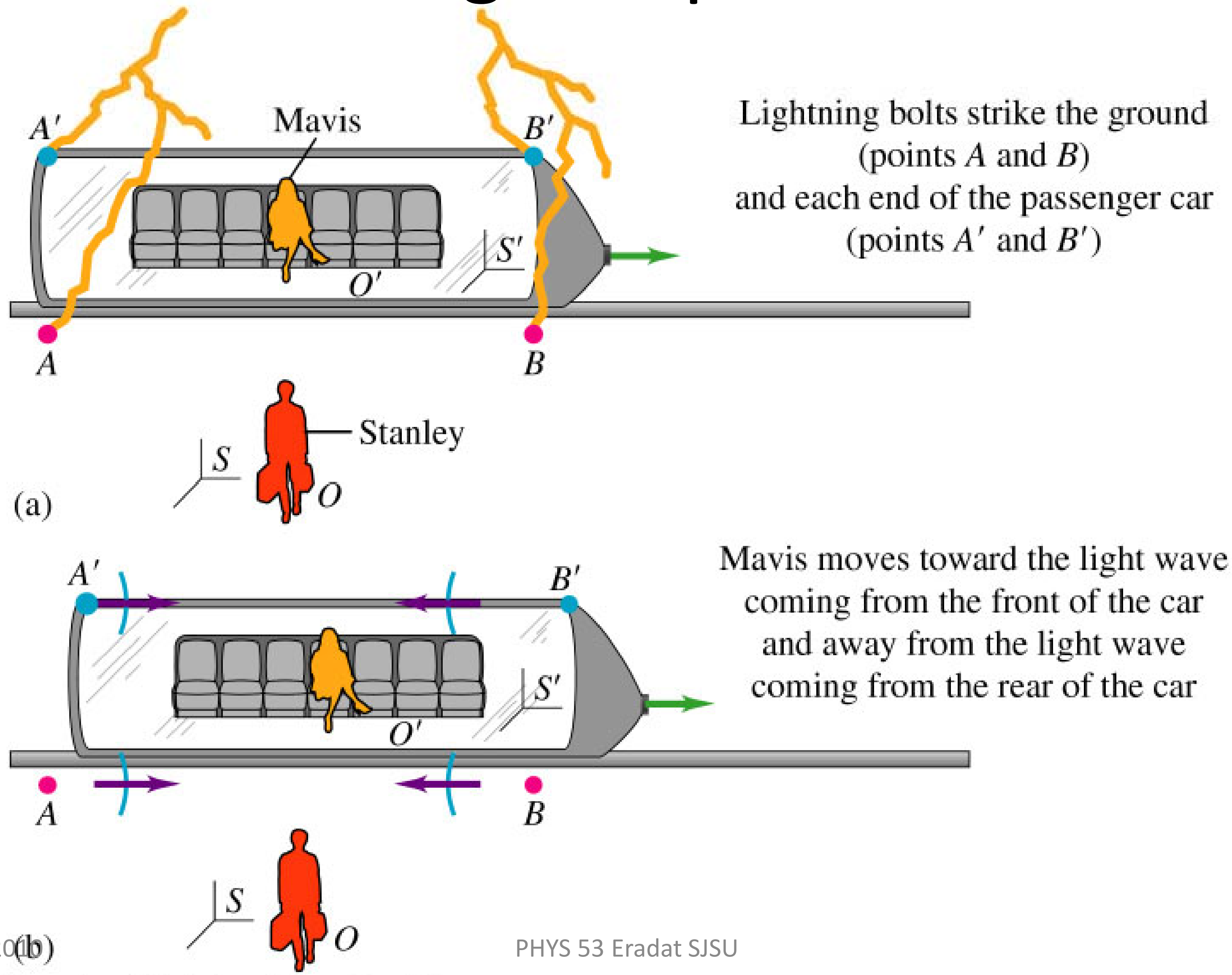
$$v_x = \frac{dx}{dt} = \frac{dx'}{dt} + u \rightarrow$$

$$\left.\begin{aligned} v_x &= v'_x + u \\ v_y &= v'_y = 0 \\ v_z &= v'_z = 0 \end{aligned}\right\} \text{Galilean velocity transformation}$$

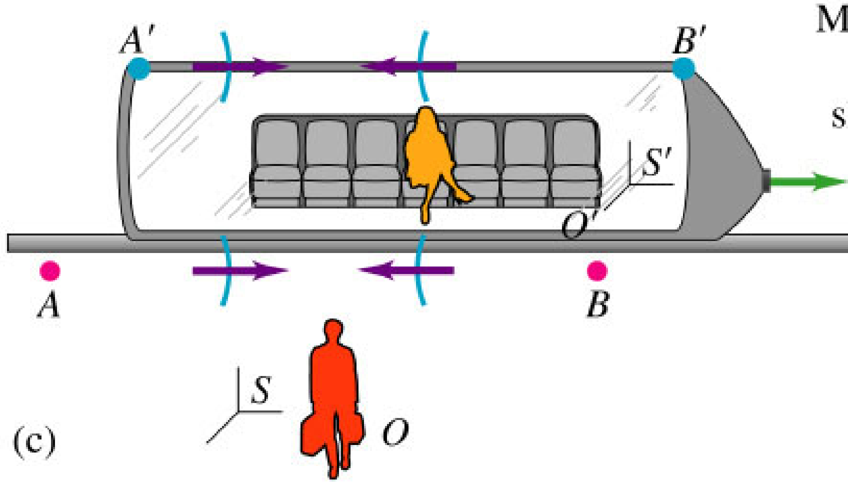
If the particle P is a photon traveling with speed of light c, what would the observer on the earth measure? Do you see a conflict?

Can we assume the two observers have the same time frame? Or $t = t'$?

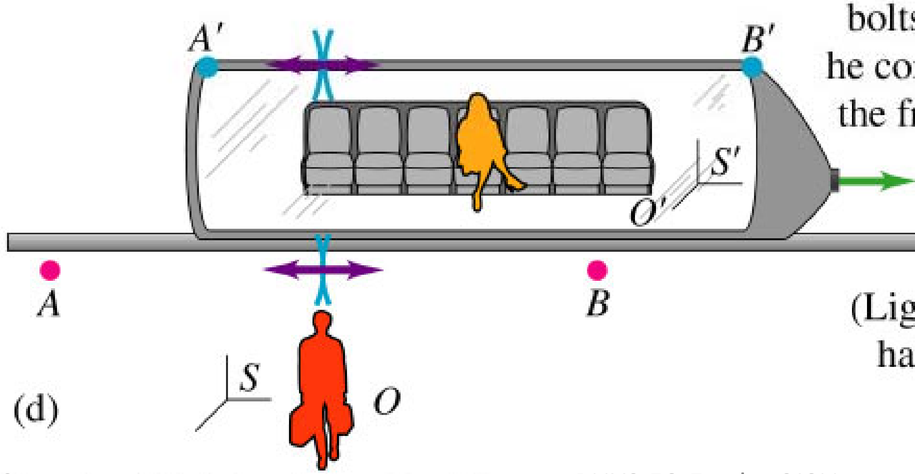
Relativity of simultaneity
A thought experiment I



Relativity of simultaneity
A thought experiment II



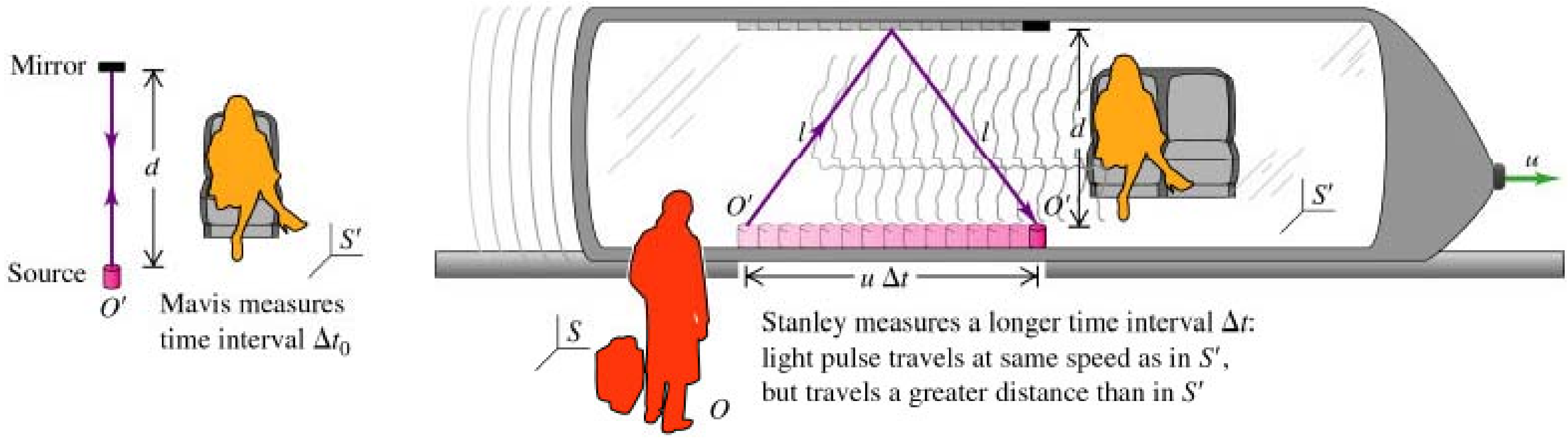
Mavis sees the light from the front of the car first—
she concludes that the bolt at the front of the car struck first



Stanley sees the light from the two bolts arrive at the same time—
he concludes that the bolts struck the front and the back of the car simultaneously

(Light wave from the rear bolt has not yet reached Mavis)

37.3 Relativity of time intervals



(a) (b)

$$\Delta t_0 = \frac{2d}{c}$$

$$\Delta t = \frac{2l}{c} = \frac{2}{c}\sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2} = \frac{2}{c}\sqrt{\left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

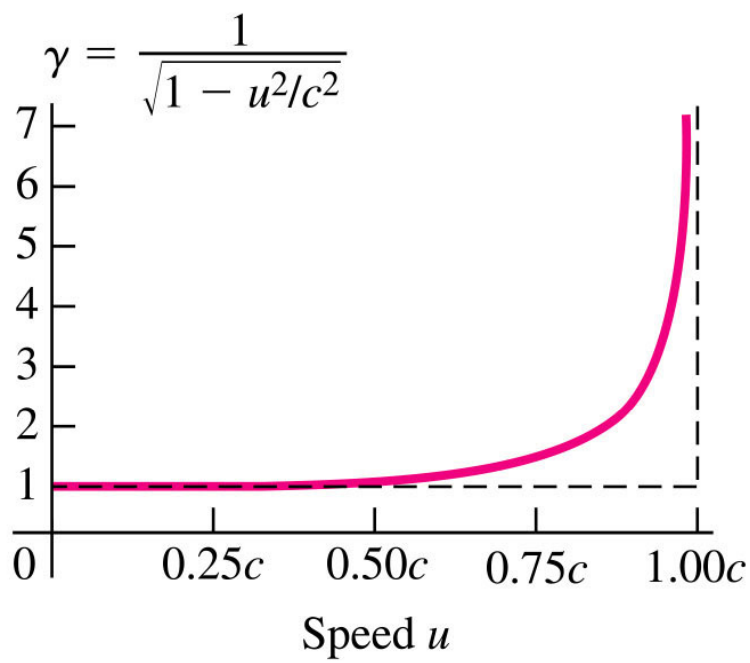
$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$$

$$\gamma = 1/\sqrt{1 - u^2/c^2}$$

$$\boxed{\Delta t = \gamma \Delta t_0}$$

Relativistic and non-relativistic speeds

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}}$$



Non-relativistic speeds: $\gamma \cong 1$ or $\beta << 1$

Example: u=0.000200c $\rightarrow \gamma$=1.00000002

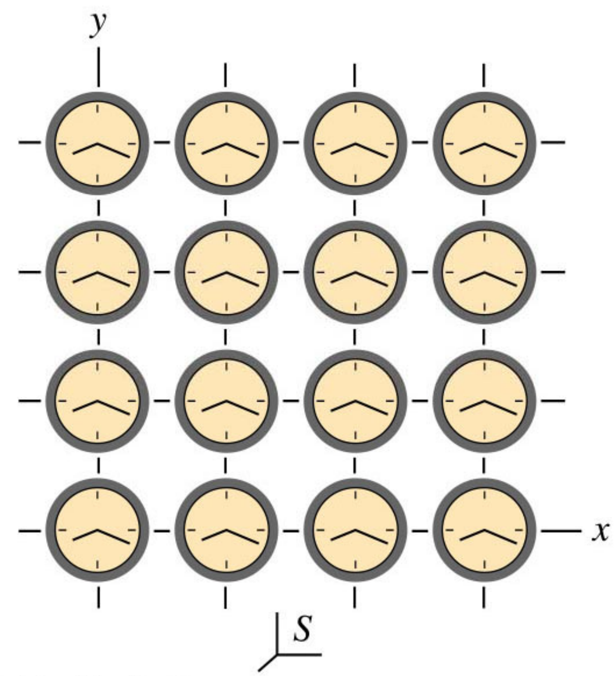
Relativistic speeds: $\gamma > 1$ or $\gamma >> 1$ which happens as $\beta \rightarrow 1$ or u $\rightarrow$ c

Example: u=0.2c $\rightarrow \gamma$=1.02

Exaxt boundaries do not exist and it all depends of accuracy of our measurements.

Proper time Δt_0

- Proper time is measured by the clock that is at rest with respect to a reference frame.
- The time interval between two events that occur at the same point. Δt_0
- Whenever the we compare the time measured by observers in different inertial frames we come across a concept of time dilation.
- The time measured with prope clocks is always the shortest or moving clocks measure a longer time interval.



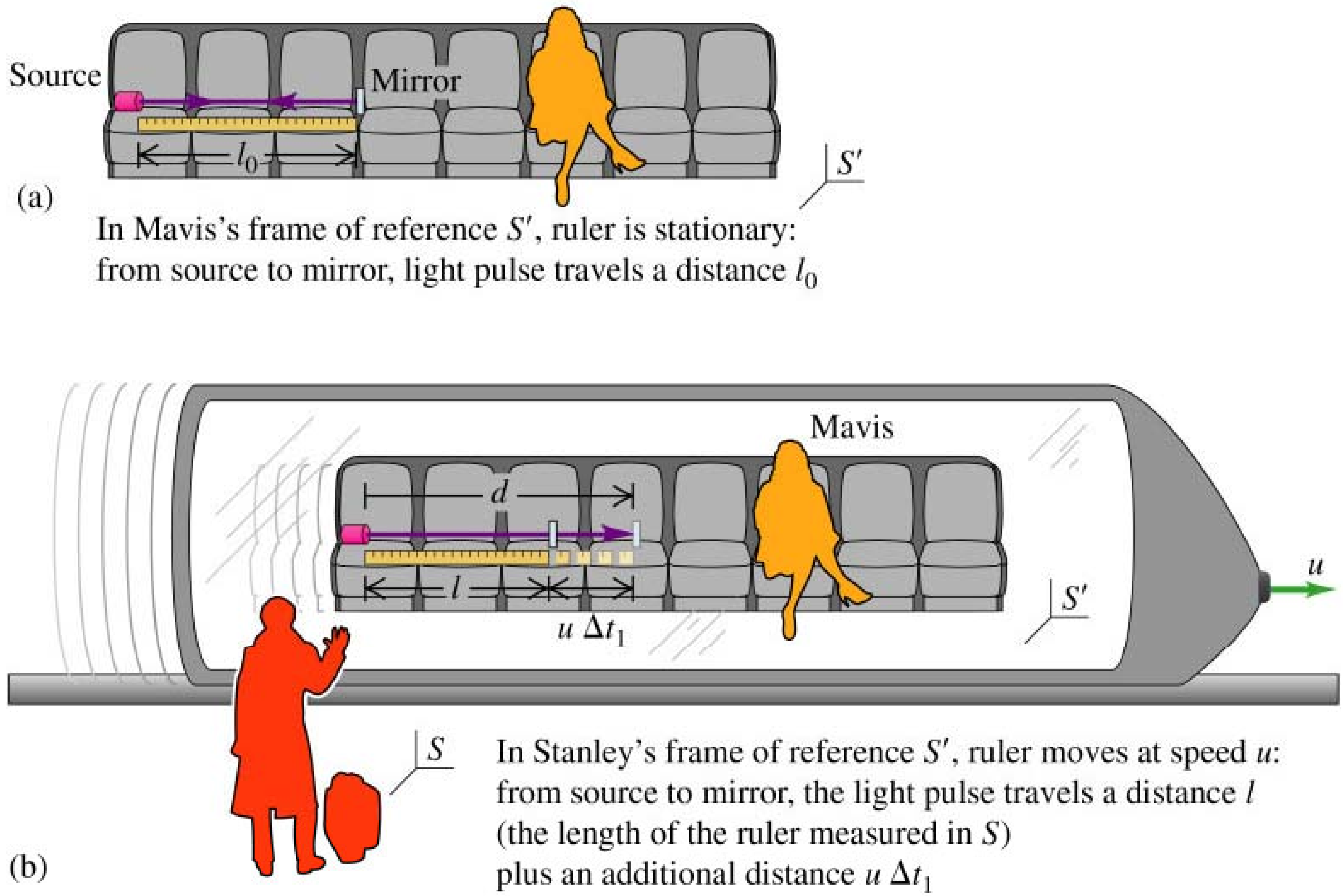
Example

- Time dilation at 0.990c
- Time dilation at Jetliner speed
- Just when is it proper?

Twin pardox

- One twin leaves on a spaceship and travels at 0.8 c. The other remains home. They each see the other as moving at relativistic speeds, so both assume the other is not aging as fast. Which one really is younger?
  - Neither
  - The one who stayed home
  - The one who took the trip
- The one who took the trip is in a non-inertial frame, and has experienced acceleration relative to original frame.

Relativity of Length
Parallel to the direction of motion



Length contraction

In S' the ruler is at rest with length l_0.

Length traveled between source and mirror: l_0

Time of travel: $\Delta t_0 = \dfrac{2l_0}{c}$ is the proper time.

In S the ruller is moving with length l.

Length traveled between source and mirror: $d = l + u\Delta t_1$

On the other hand speed of light: c so $d = c\Delta t_1$

$$c\Delta t_1 = l + u\Delta t_1 \rightarrow \Delta t_1 = \frac{l}{c+u}$$

On the way back: $d = l - u\Delta t_2 = c\Delta t_2 \rightarrow \Delta t_2 = \dfrac{l}{c-u}$

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{l}{c+u} + \frac{l}{c-u} = \frac{2l}{c(1-u^2/c^2)} = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \frac{2l_0}{c\sqrt{1-u^2/c^2}}$$

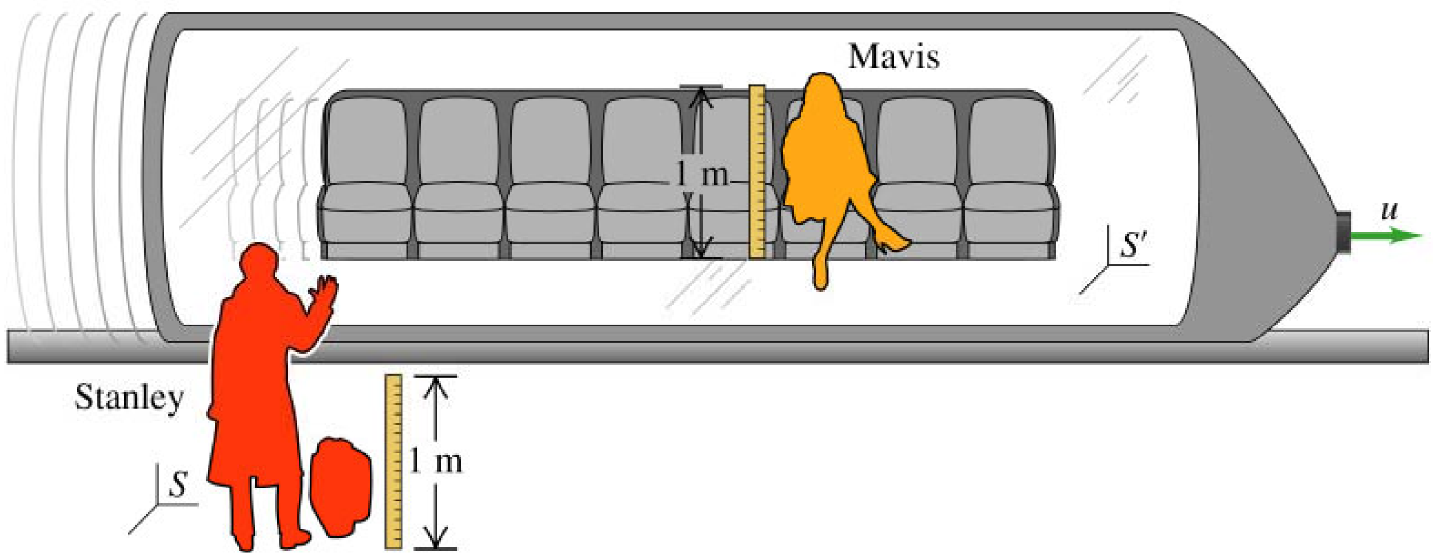
We arrive at length contraction formula : Lengths in motion are shorter.

$$\boxed{l = l_0\sqrt{1-u^2/c^2} = l_0/\gamma}$$

Proper length l_0 is the length measured in the frame in which the body is at rest.

Lengths perpendicular to the direction of relative motion

- There is no length contraction in the direction perpendicular to the direction of motion.



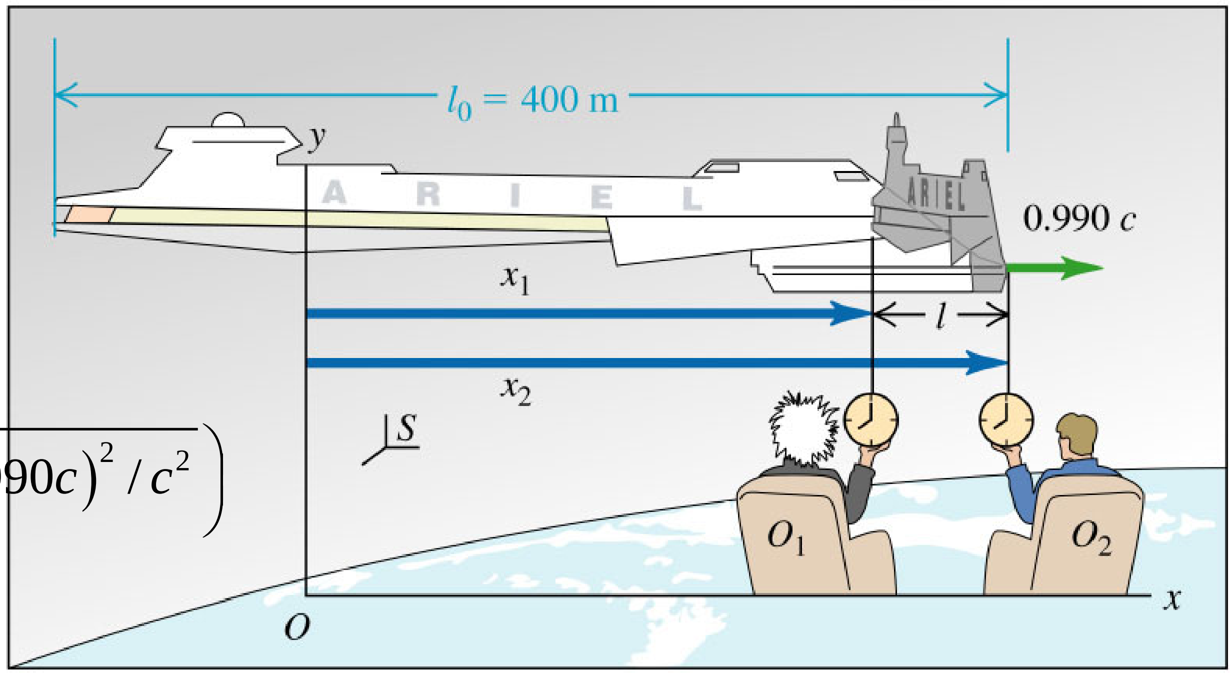
How long is the spaceship?

- A spaceship passes by earth at speed of 0.990c. Length of the ship measured by a crew on it is 400m. How long is the ship measured by the two observers on the earth?

$$l = l_0\sqrt{1-u^2/c^2}$$

$$l = (400m)\left(\sqrt{1-(0.990c)^2/c^2}\right)$$

$$l = 56.4m$$



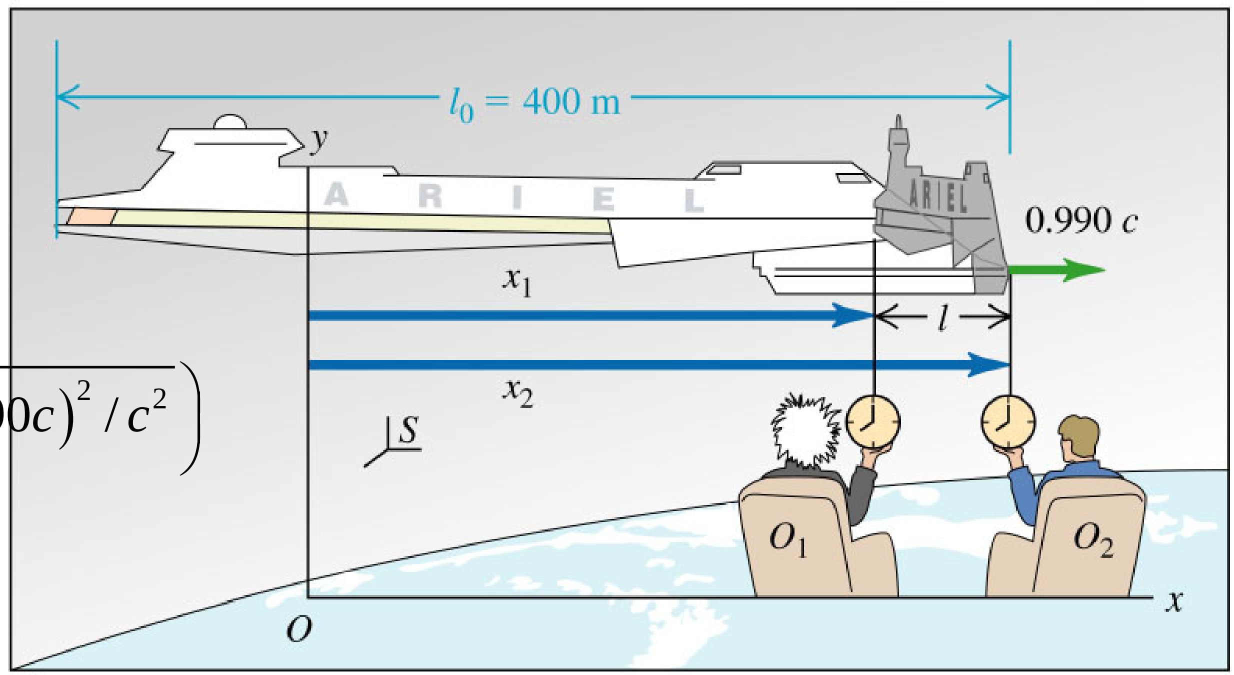
How far apart are the observers?

- The two observers are 56.4 m apart on the earth. How far apart are they according to the spaceship crew?

$$l = l_0\sqrt{1-u^2/c^2}$$

$$l = (56.4m)\left(\sqrt{1-(0.990c)^2/c^2}\right)$$

$$l = 7.96\ m$$



Moving with a muon

- A muon has on average a proper lifetime of $2.20X10^{-6}$s and a dilated lifetime of $15.6X10^{-6}$s in a frame with speed of v=0.990c. The distance traveled by muon is
- $X'=vt_0$ =653 m in its own frame
- X=vt=4630 m in earth people frame.
- How?
  – Answer: Muon sees the distance of 4630m to earth contracted since the earth is moving

$$l = l_0\sqrt{1-u^2/c^2}$$

$$l = (4630m)\left(\sqrt{1-(0.990c)^2/c^2}\right)$$

$$l = 653m$$

The Lorentz Coordinate Transformation (LCT)

Goal: to relate the coordinates of the moving reference frame (x',y',z',t') to the coordinates of the stationary one (x,y,z,t).

Galilean transformation: $t = t'$;   Lorentz transformation: $t \neq t'$

Distance from O to O' at t seen by S observer is: $s = ut$

x' is proper length at S'

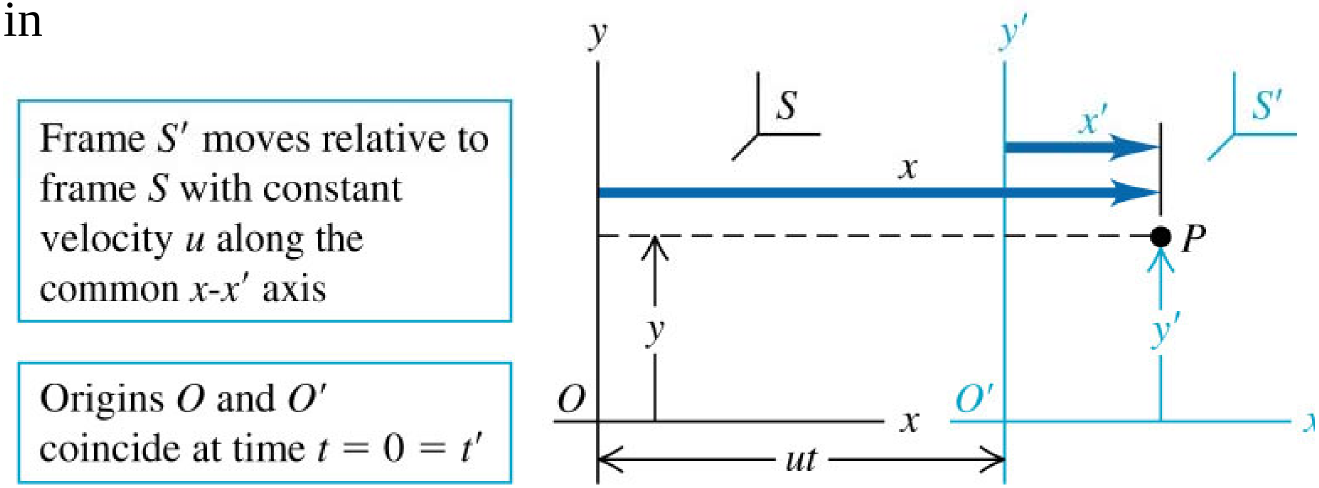
s is contracted as seen in S'

$$s' = s\sqrt{1 - u^2/c^2}$$

The distance from O to P in

S is: $x = ut + x'\sqrt{1 - u^2/c^2}$

$$\boxed{x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}}$$



The Lorentz time Transformation

Goal: to relate the coordinates of the moving reference frame (x',y',z',t') to the coordinates of the stationary one (x,y,z,t). Lorentz transformation: $t \neq t'$

Principle of relativity: form of transformation from S to S' is identical to that from S' to S with a change in the sign of the relative velocity u.

$$\left.\begin{aligned} x' &= -ut' + x\sqrt{1-u^2/c^2} \\ x' &= \frac{x-ut}{\sqrt{1-u^2/c^2}} \end{aligned}\right\} -ut' + x\sqrt{1-u^2/c^2} = \frac{x-ut}{\sqrt{1-u^2/c^2}} \rightarrow t' = \frac{t-ux/c^2}{\sqrt{1-u^2/c^2}}$$

Lengths perpendicular to the direction of motion are not affected so

Lorentz coordinate transformation
$$\begin{cases} x' = \dfrac{x-ut}{\sqrt{1-u^2/c^2}} = \gamma\left(x-ut\right) \\ y' = y \\ z' = z \\ t' = \dfrac{t-ux/c^2}{\sqrt{1-u^2/c^2}} = \gamma\left(t-ux/c^2\right) \end{cases}$$

The Lorentz Velocity Transformation

Goal: to relate the velocities of the moving reference frame (v_x', v_y', v_z') to the velocities of the stationary one (v_x, v_y, v_z).

In frame S: in time dt a particle moves by dx.

The corresponding distance and time in S' is:

$$\left.\begin{aligned} dx' &= \gamma\left(dx - u\,dt\right) \\ dt' &= \gamma\left(dt - u\,dx/c^2\right) \end{aligned}\right\} v_x' = \frac{dx'}{dt'} = \frac{\frac{dx}{dt} - u}{1 - \frac{u}{c^2}\frac{dx}{dt}} = \frac{v_x - u}{1 - u v_x / c^2}$$

The Lorentz one-dimentional velocity transformation:

$$\boxed{v_x' = \frac{v_x - u}{1 - u v_x / c^2}}$$

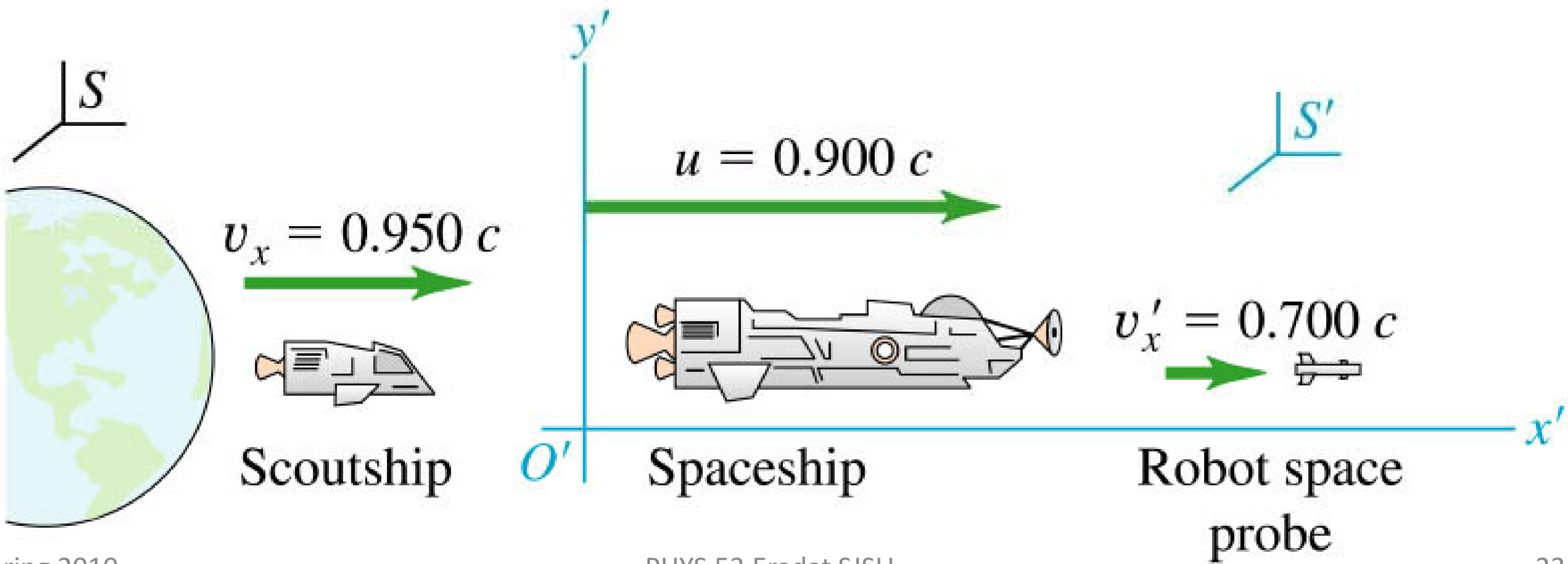
Using the principle of relativity we write the inverse transformation as:

$$v_x = \frac{v_x' + u}{1 + u v_x'/c^2}$$

Relative velocities

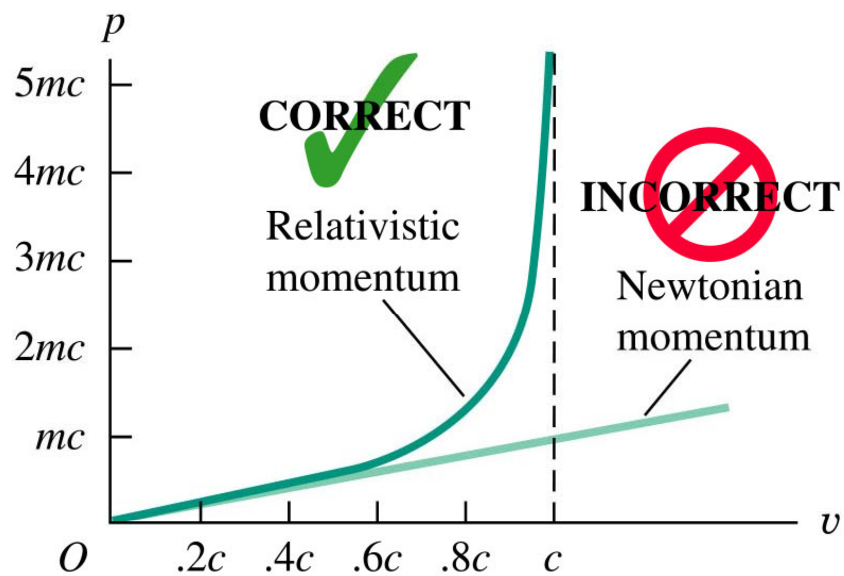
A. A spaceship (S') moving away from the earth (S) with speed 0.900c fires a robot at speed 0.700c relative to itself. What is the robot's velocity relative to earth? u=0.900c

B. A scoutship (S) with velocity 0.950c relative to earth tries to catch up with the spaceship (S'). What is the relative velocity of the scoutship and spaceship? u=0.900c



Relativistic momentum I

- Principle of conservation of momentum: When two bodies interact in inertial frames, in absence of external forces, the total momentum is conserved.
- Conservation of momentum is a valid physical law so it should be the same in all inertial reference frames.
- Using the Lorentz velocity transformation and **P**=m**V** does not lead to conservation of momentum is a moving reference frame.
- We have to modify our definition of the momentum.



Relativistic momentum II

To make conservation of momentum valid in all inertial frames we redefine the mass:

When an object is at rest with respect us we measure a mass of m for it.

We call this the <u>rest mass</u>. When an object is moving with speed of v with respect to us its momentum measures:

$$\boxed{\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}}$$ rather than $\vec{p} = m\vec{v}$

$$\lim_{v \to c} \vec{p} \to \infty$$

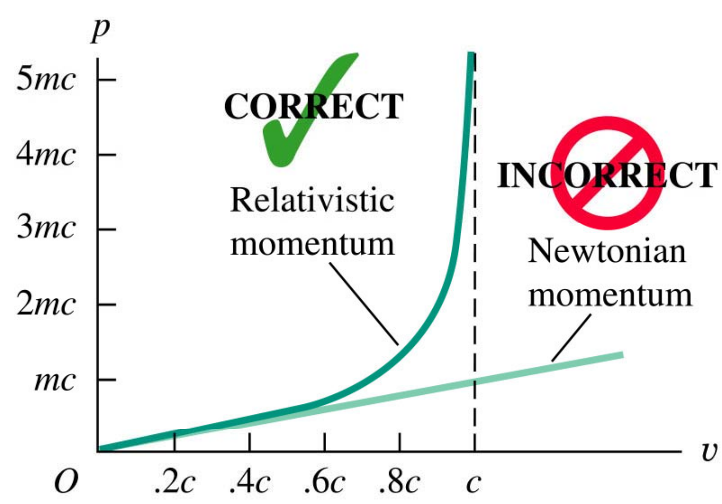
<u>relativistic momentum > nonrelativistic momentum</u>

From new definition of momentum we conclude

$$\boxed{\vec{p} = \gamma m \vec{v}}$$

Material particles have non-zero rest mass.

Photons have zero rest mass.



Newton's second law and relativistic mass

The most general form of the Newton's second law (when masses are changing)

$$\underbrace{\vec{F}}_{\text{Net force on the particle}} = \underbrace{\frac{d\vec{p}}{dt}}_{\text{Rate of change of momentum}}$$

So if we use the relativistic momentum instead of the classic one we should be fine.

For the simple case of <u>velocity and force along the same line</u> we get:

$$\vec{F} = \frac{d\vec{p}}{dt} = \underbrace{\frac{d}{dt}\frac{m\vec{v}}{\sqrt{1-v^2/c^2}}}_{\text{Take the derivative}} = \frac{m}{\left(1-v^2/c^2\right)^{3/2}}\frac{d\vec{v}}{dt} = \frac{m}{\left(1-v^2/c^2\right)^{3/2}}a$$

The acceleration when $\vec{F} \parallel \vec{v} \rightarrow \boxed{\vec{F} = \gamma^3 ma}$

The acceleration when $\vec{F} \perp \vec{v} \rightarrow \boxed{\vec{F} = \gamma ma}$

For the arbitrary $\vec{F}$ & $\vec{v}$ directions we can decompose them to $\parallel$ & $\perp$ components.

As v $\uparrow$ the particle's a $\downarrow$ when $v = c$, then $a = 0$

The conclusion is that once the speed has reached the speed of light it is impossible to accelerate the particle to a speed of higher than c.

Dynamics of an electron

An electron is moving opposite to an electric field of $E = 5.00 \times 10^5\ N/C$.

No other force is acting on it.

$m_e = 9.11 \times 10^{-31 kg},\ q_e = -1.60 \times 10^{-19} C$

a) Find magnitude of momentum and acceleration when v=0.010c, 0.90c, 0.99c

$F = qE \rightarrow F = \left(-1.60 \times 10^{-19} C\right)\left(5.00 \times 10^5\ N/C\right) = 8 \times 10^{-14} N$

$p = \gamma m v$ & $a = F / \left(\gamma^3 m\right)$ when $F \parallel v$

$v_1 = 0.010c \ \rightarrow p_1 = 2.7 \times 10^{-24} kg.m/s$ and $a_1 = 8.8 \times 10^{16} m/s^2$

$v_2 = 0.090c \ \rightarrow p_2 = 5.6 \times 10^{-22} kg.m/s$ and $a_1 = 7.3 \times 10^{15} m/s^2$, 8.3% of the non-relativistic a

$v_3 = 0.990c \ \rightarrow p_1 = 1.9 \times 10^{-21} kg.m/s$ and $a_1 = 2.5 \times 10^{14} m/s^2$, 0.28% of the non-relativistic a

b) Find the corresponding accelerations if a net force of the same magnitude acting perpendicular to the velocity.

When $F \perp v \rightarrow a = F / \left(\gamma m\right) \ \rightarrow a_1 = 8.8 \times 10^{16} m/s^2$ & $a_2 = 3.8 \times 10^{16} m/s^2$

& $a_3 = 1.2 \times 10^{16} m/s^2$ Larger accelerations than the parallel case by a factor of γ^2

Circular accelerators are more efficiennt using the same amount of field.

Relativistic work and energy

Generalization of work and energy equations: If $F \parallel x \rightarrow W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} \frac{m a_x dx}{\left(1 - v_x^2/c^2\right)^{3/2}}$

Kinetic energy of a particle $=$ Net work done on it in briging it to the speed of v from rest

$$v_x = \frac{dx}{dt} \quad a_x = \frac{dv_x}{dt} \rightarrow a_x dx = \frac{dv_x}{dt} dx = \frac{dx}{dt} dv_x \rightarrow a_x dx = v_x dv_x$$

$$W = K = \int_{x_1}^{x_2} \frac{m v_x dv_x}{\left(1 - v_x^2/c^2\right)^{3/2}}$$

If we change variables to $u = 1 - v_x^2/c^2$ and $du = -2 v_x dv_x / c^2$ then

the integral becomes $K = W = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$

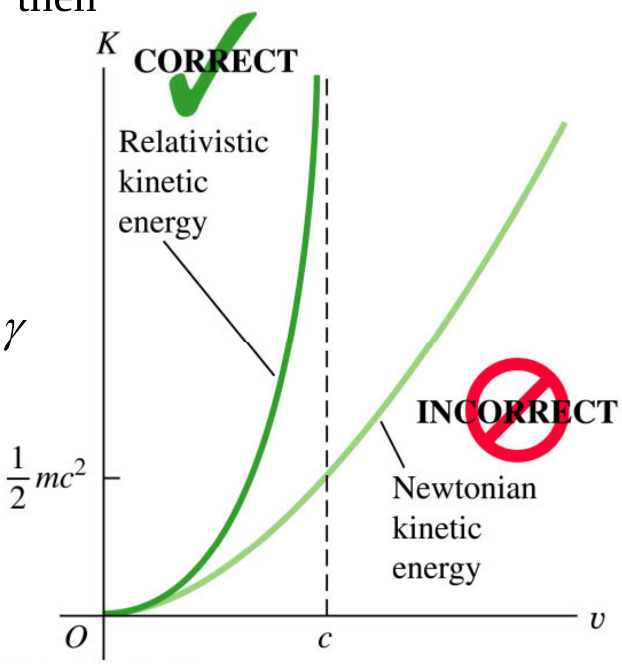
$\boxed{K = (\gamma - 1) mc^2}$ Relativistic Kinetic Energy $\boxed{\lim_{v \to c} K \rightarrow \infty}$

When $v << c$ we can use the binomial expansion to expand γ

$$(1+x)^n = 1 + nx + n(n-1)x^2/2! + n(n-1)(n-2)x^3/3! + \ldots$$

$$K = \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \ldots - 1\right) mc^2 = \frac{1}{2} mv^2 + \frac{3}{8}\frac{mv^4}{c^2} + \ldots$$

$\rightarrow \boxed{K \simeq mv^2/2 \text{ for } v << c}$ non-relativistic kinetic energy



Rest energy and E=mc²

$$\underbrace{K}_{\text{Kinetic energy}} = (\gamma - 1) mc^2 = \underbrace{\gamma mc^2}_{\text{Depends on speed}} - \underbrace{mc^2}_{\substack{\text{Independent of speed} \\ \text{Energy at rest}}} \rightarrow \underbrace{\gamma mc^2}_{\text{Total energy}} = \underbrace{K + mc^2}_{\text{KInetic energy and energy at rest}}$$

$$\boxed{E_{total} = K + mc^2 = \frac{mc^2}{\sqrt{1 - v^2 / c^2}} = \gamma mc^2}$$

Evidence for rest mass is shown in creation and anhilation of the elementary particles.

Now with these findings the independently developed conservation of energy and conservation of mass become related to create the law of conservation of mass & energy.

Generation of power through nuclear reactions is an experimental manifestation of this law.

$\boxed{E^2 = (mc^2)^2 + p^2c^2}$ Relation between the total energy, rest energy, and momentum

For particles with zero rest mass that always have to travel with speed of light we have

$$\boxed{E = pc}$$

Energetic electrons

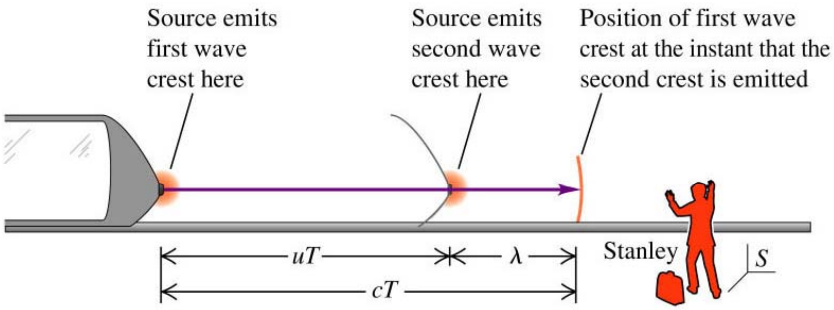
a) Find rest energy of an electron ($m_e = 9.11\times10^{-31kg},\ q_e = -1.60\times10^{-19}C$) in jules and electronvolts.

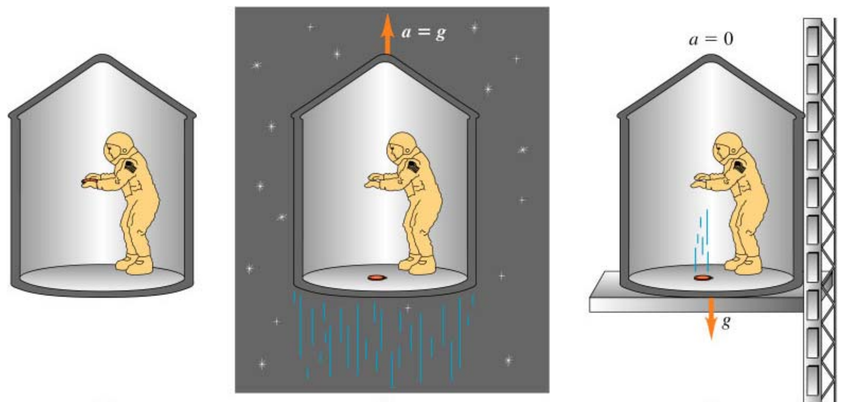
$$\mathrm{mc}^2 = 8.187\times10^{-14}\,J$$

$$1eV = 1.602\times10^{-19}\,J$$

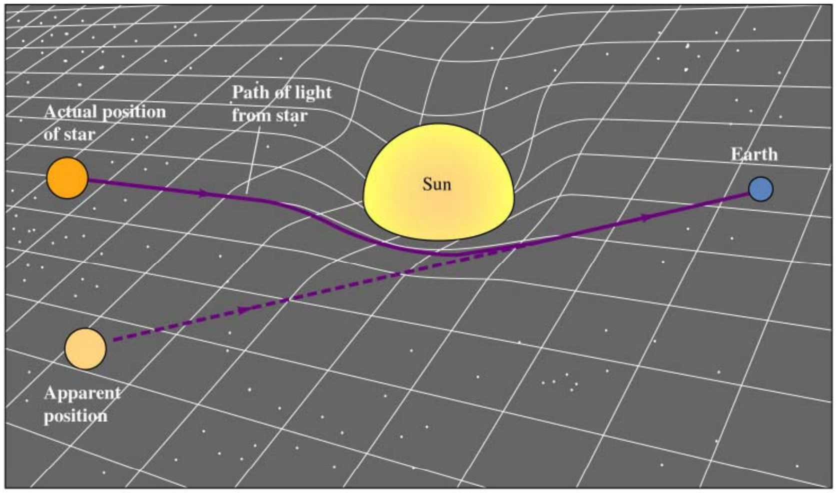
$$\mathrm{mc}^2 = 5.11\times10^{5}\,eV$$

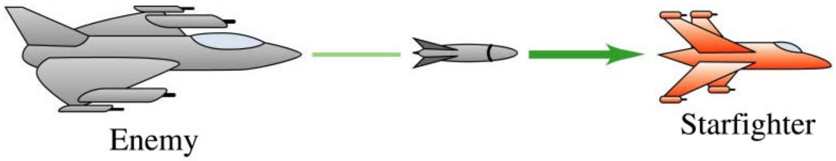
b) Find speed of an electron that has been accelerated from rest through potential increase of 20.0KV or 5.0MV.

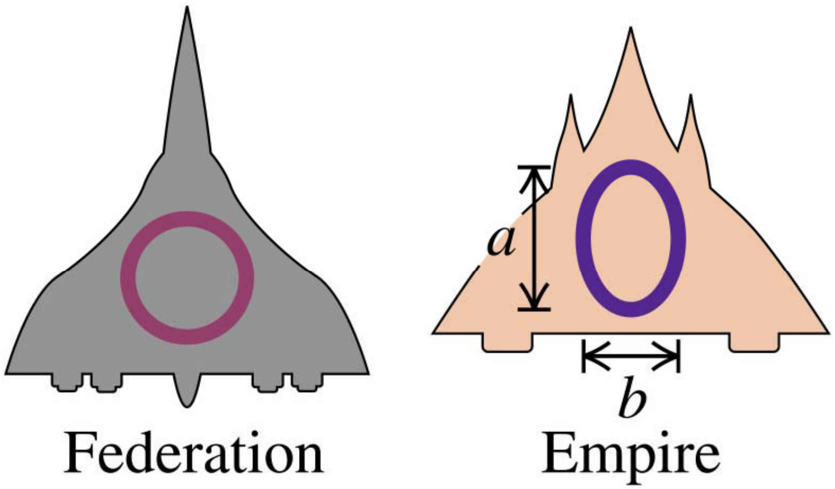

Source emits
first wave
crest here
Source emits
second wave
crest here
Position of first wave
crest at the instant that the
second crest is emitted
uT
λ
cT
Stanley
S



(a) (b) (c)


Path of light
from star
Actual position
of star
Sun
Earth
Apparent
position


Enemy
Starfighter



a
b
Federation
Empire