Chapter 34: Geometric Optics

• It is all about images
• How we can make different kinds of images using optical devices
• Optical device example: mirror, a piece of glass, telescope, microscope, kaleidoscope, etc
• Always light rays come from or converge to a point called “image point”
• Tools to understand images: ray model of light, laws of reflection and refraction, geometry and trigonometry
• We will study:
  – Plane mirrors
  – Curved mirrors
  – Refracting surfaces
  – Thin lenses
  – Some optical instruments that use these basic devices such as human eye, telescope, microscope, periscope.
Reflection and Refraction at a plane Surface

- **Vocabulary:**
  - **Object:** anything from which light rays radiate
    - Self luminous
    - Non luminous
  - **Point object:** a mathematical model for a dimensionless object (we will study).
  - **Extended object:** an object with length, width, and height (composed of many point objects).
  - **Optical system:** a system that admits the light rays from an object and generates an image of the said object.
  - **Optical axis:** a mathematical line drawn to take advantage of the symmetry of the optical system.
Image formation by reflecting surfaces

Plane mirrors

- The **diverging** rays from an object (P) hit the flat reflecting surface.
- They experience **specular reflection**.
- Reflected rays diverge as if they were coming from a point on the other side of the mirror (p’).
- Our eyes focus those rays onto the retina.
- Retina sends many signals to the brain from the right and left eye.
- The brain constructs not only an image but also depth and distance of the object.
- Rays from **diffused reflection** do not add up to a clear image.
Image formation by refracting surfaces

Interface of two material

- The **diverging** rays from an object (P) hit the flat surface
- They experience **refraction**
- Refracted rays diverge **as if** they were coming from a different point on the same side of the surface (p')
- Our eyes focus those rays on the retina
- Retina sends many signals to the brain from the right and left eye.
- The brain constructs not only an image but also depth and distance of the object which may be different than the position of the real object.
- If the surface was not smooth rays from the object would be **diffused and would not** add up to a clear image.
Types of Image

- **Virtual image**: if the rays constructing an image do not actually pass through the image point.
- **Real image**: if the rays constructing an image actually pass through the image.
- Identify which of these images are virtual and which ones real?
Sign rules for image formation

- **Object distance**: + when the *object* is on the *same side* of the reflecting or refracting surface as the *incoming* light; else -

- **Image distance**: + when the *image* is on the *same side* of the reflecting or refracting surface as the *outgoing* light; else –

- **Radius of curvature of a spherical surface**: + when the *center of curvature* is on the *same side* as the *outgoing* light; else –

- **Real**: + Virtual negative

- **Above optical axis**: + Below optical axis: -
Image location by a plane mirror

- Image is produced by reflection
- At least two rays are needed to form an image
  - Identify the angles of reflection for each ray
  - Draw the reflected rays
  - Wherever they meet is the image point.
- What if they did not intersect?

**Example:**
- Construct image of a candle at 2 meters from a flat mirror
  - parallel to the mirror surface.
  - Perpendicular to the mirror surface
- Identify the object distance, the image distance and the relationship between the two
- Apply the sign convention.
Image of an extended object

Magnification of a plane mirror

- To construct image of an extended object we chop it to point objects and build the image point by point.

- What is the ratio between the object and image size for plane mirror?

- **Lateral magnification:** ratio of the image and object heights

- For plane mirror

  \[ m = y'/y = -s'/s = ? \]
More about images

• **Erect (upright):** both image and object arrows are parallel \((y=y')\)

• **Inverted:** image and object arrows are anti-parallel \((x=-x')\)

• PS and PQ are not inverted but PR is inverted

• **Plane mirrors inverts back and front but they don’t invert up and down.**

• What is the sign of the lateral magnification in each case?

• What is the sign of the longitudinal magnification in each case?
A image can act like an object to produce a secondary image

- P is a real object
- P'₁ is a virtual image formed by mirror 1
- P'₁ acts like a virtual object to form P'₃ by mirror 2
- P is a real object
- P'₂ is a virtual image formed by mirror 2
- P'₂ acts like a virtual object to form P'₃ by mirror 1
Reflection at a spherical surface

- We need magnification and real images. Plane mirrors can’t offer any of these but spherical mirrors offer both.
- Spherical mirror presentation
  - \( R \) radius of curvature
  - \( C \) center of curvature
  - \( V \) vertex (center of the mirror surface)
  - Optic axis: a line connecting \( C \) and \( V \)
- Use laws of reflection construct image of the point \( P \).
- Use the sign convention to determine signs of the \( s, s', C, R \).
- What is type of the image?
- What is a real image good for?
Paraxial imaging by spherical mirrors

The small angle approximation (SAA) or paraxial optics: viewing angle < 10° → \( \tan \alpha = \sin \alpha \approx \alpha \) (in radians) or small and/or distant objects on the optical axis.

Paraxial image location formula by spherical mirrors:

\[
\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}
\]

Away from optic axis (non-paraxial optics), spherical mirrors show **aberrations** i.e. image of a point is not exactly a point. The angles are exaggerated for ease of presentation.

1. \( R = 57.3^\circ \)
2. \( 0.175R = 10^\circ \)

---

(a)

(b)
Paraxial imaging by spherical mirrors

Paraxial approximation:

\[
\begin{align*}
\delta & \ll s \& s' \& R \\
\tan \alpha & \approx \alpha \approx \frac{h}{s} \\
\tan \beta & \approx \beta \approx \frac{h}{s'} \\
\tan \phi & \approx \phi \approx \frac{h}{R} \\
\beta & = \alpha + 2\theta \\
\phi & = \alpha + \theta \\
\frac{h}{s} + \frac{h}{s'} & = 2h \quad \Rightarrow \quad \frac{1}{s} + \frac{1}{s'} = \frac{2}{R}
\end{align*}
\]
Spherical mirrors: focal point and focal length

- **Focal point** is position of the image of an object at infinity.
- **Focal distance** is the distance from the focal point to the vertex of the mirror.

\[
\begin{align*}
\text{Object at infinity: } & \quad \frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \\
\text{Image: } s' &= \frac{R}{2} = f \\
\text{Object at focal point: } & \quad s = \frac{R}{2} = f \\
\text{Image: } s' &= \infty
\end{align*}
\]

Image-object relation:

\[\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}\]

Image and object points are conjugate points.
Spherical mirrors: focal point and focal length

- **Focal point** is position of the image of an object at infinity.
- **Focal distance** is the distance from the focal point to the vertex of the mirror.

\[
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\text{Object at infinity: } & \quad \frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \\
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\end{align*}
\]

\[
\begin{align*}
\text{Object at focal point: } & \quad s = \frac{R}{2} = f \\
\text{Image: } & \quad s' = \infty \\
\end{align*}
\]

Image-object relation:

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

Image and object points are conjugate points.
Image of an extended object
Spherical mirror

- Consider a finite size object PQ
- Construct image of both ends P’Q’
- \( y \) is the object height
- \( y' \) is the image height
- Prove \( m = \frac{y'}{y} = -\frac{s'}{s} \)
- \( m > 0 \) image upright
- \( m < 0 \) image inverted
- \(|m| > 1\) image magnified
- \(|m| < 1\) image downsized
Paraxial images: convex mirrors

Image-object relation by **spherical convex mirrors**

in paraxial approximation:

\[
\begin{align*}
\delta & \ll s \& s' \& R \\
\tan \alpha & \approx \alpha \approx \frac{h}{s} \\
\tan \beta & \approx \beta \approx \frac{h}{s'} \\
\tan \phi & \approx \phi \approx \frac{h}{R} \\
\beta = \alpha + 2\theta & \\
\beta = \phi + \theta & \rightarrow \beta + \alpha = 2\phi \\
\frac{h}{s} + \frac{h}{s'} = \frac{2h}{R} & \rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \\
m = \frac{y'}{y} = -\frac{s'}{s} &
\end{align*}
\]

Everything is the same as convex case if we follow the sing conventions properly.
Focal point of a convex mirror

Follow the sign conventions properly

\[ \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \]

\( s > 0 \)

\( s' < 0 \)

\( R < 0 \)

Why?

\[ \frac{1}{\infty} + \frac{1}{-s'} = \frac{2}{-R} \]

\[ f = s' = \frac{R}{2} \]

\[ \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f} \]
Graphical ray tracing: constructing images by mirrors

Four principal rays’ are traced using the laws of reflection
1. Incoming ray parallel to the optical axis outgoing ray passes through F
2. Outgoing ray parallel to the optical axis incoming ray passes through F
3. A ray to the vertex reflects at the same angle on the other side of OX
4. A ray aiming the center of curvature reflects back at itself
Concave mirrors

a) Object outside center
b) Object at the center
c) Object at the focal point
d) Object between the focal point and vertex
e) Where is the image if object is between the center and focal point?
f) Explain the image properties for the convex mirror?
Refraction at a spherical surface

- prove (using paraxial approximation) that all the rays emerging from P and hitting the refracting surface will gather at P'.
- Flat surface is a special case of a spherical surface

What is the radius of curvature for a flat surface?

\[
\frac{n_a}{s} + \frac{n_b}{s'} = \frac{(n_b - n_a)}{R}
\]

\[
m = \frac{y'}{y} = -\frac{n_a}{n_b} \frac{s'}{S}
\]
Lateral magnification of a refracting spherical surface

- Obtain a formula for lateral magnification of a spherical refracting surface
- Discuss each formula for the case of plane refracting surface

\[ m = \frac{y'}{y} = -\frac{n_a s'}{n_b s} \]
Image formation by refraction

• Calculate the image distance and magnification. (s’=+11.3 cm m=-0.929)
• What if the rod is immersed in water (n=1.33)? (s’=-21.3 cm m=+2.33)
• Suggest an easy experiment to see a real image in a tube with spherical head.
Apparent depth in a swimming pool

• Calculate the apparent depth of a 2.00 m deep swimming pool. (1.50 m or $s'=-1.50$ m)

• What is the nature of the image of the pool floor we see?
Properties of thin lenses

- A lens is combination of two refracting surfaces
- Thin lens: radii of curvature >> lens thickness
- Sign rules are the same as single refracting/reflecting surface
- The line that connects two centers of curvature is the optic axis
- There are two focal points on equal distance from the thin lens (center)
- \( F_1 \) is the object focal point (positive and real)
- \( F_2 \) is the image focal point (positive and real)
- Focal lengths are the distances from the foci to the center of the lens
- There are two general category of lenses:
  - Converging lens or positive lens (similar to the concave mirror)
  - Diverging lens or negative lens (similar to the convex mirror)
Constructing image of an extended object with a converging lens

- Construct image of an arrow (PQ) located on the optical axis of a thin converging lens with focal points at F₁ and F₂.
- Derive the object-image and lateral magnification relations for thin lens.
- Discuss the image properties

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{and} \quad m = -\frac{s'}{s}
\]
Converging lens image analysis

• Real
• Inverted about the optic axis or rotated $180^\circ$ about the optic axis.
• No change in right handed or left handedness of the image
Diverging lens or negative lens

- There are two focal points on equal distance from the thin lens (center)
- \(F_1\) is the object focal point
- \(F_2\) is the image focal point
- Pay attention to the location of the object and image focal points.
- Negative focal lengths
- Virtual focal points
- Same equations apply

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \quad m = -\frac{s'}{s}
\]
Recognition of converging and diverging lens

- Edges thicker than center: **diverging**
- Center thicker than edges: **Converging**
Image formation for thin lens converging and diverging

- Use three or two principal rays to construct an image

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

\[ m = -\frac{s'}{s} \]
Example:
Images for diverging lens

1. Imagine an object is approaching from infinity towards a diverging lens with focal length of 20.0 cm. How the image moves? Is it real or virtual? Get the image distances for \( s = 100.0 \) cm, \( s = 40.0 \) cm, \( s = 20.0 \) cm, \( s = 10.0 \) cm, \( s = 1.0 \) cm, \( s = 0.001 \) cm.

2. How the magnification changes in each case?

3. Draw a diagram for location of the object and image when the object is at 40 cm.

4. Answer the same questions for the case of converging lens with the same focal length.
Answer to the part 3 of the Quiz
A lens and its radii of curvature $R_1$ & $R_2$

- A lens has two spherical refracting surfaces.
- The first surface with radius of curvature $R_1$ that incoming rays hit is called surface 1.
- The second surface with radius of curvature $R_2$ that incoming rays hit is called surface 2.
Lensmaker’s equation for thin lens in paraxial approximation (small angles)

A lens is combination of two spherical refracting surfaces. Use the spherical refracting surface equation twice and the following facts to derive the lensmaker’s equation.

Paraxial approximation is used.

\[
\frac{n_a + n_b}{s_1} = \frac{n_b - n_a}{R_1}, \quad \frac{n_b + n_c}{s_2} = \frac{n_c - n_b}{R_1}
\]

\[
n_a = n_c = 1; \quad n_b = n; \quad s_2 = -s_1'
\]

\[
\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]
Images for converging lens

1. Imagine an object is approaching from infinity to the $F_1$ (the object focal point) of a converging lens. How the image moves? Is it real or virtual?

2. How image moves when the object moves from $F_1$ towards the lens? Is image real or virtual?

3. What is the relative direction of motion of the object and image for the converging lenses?
Images for converging lens

(a) Object $O$ is outside focal point: image $I$ is real

(b) Object $O$ is closer to focal point: image $I$ is real and farther away

(c) Object $O$ is even closer to focal point: image $I$ is real and even farther away

(d) Object $O$ is at focal point: image $I$ is at infinity

(e) Object $O$ is inside focal point: image $I$ is virtual and larger than object

(f) A virtual object $O$ (light rays are converging on lens)
Optical instruments

• Single lens systems
  – Cameras
  – Eyeglasses
  – Magnifiers

• Two-lens or compound systems composed of a primary or objective lens forms a real image that is used by a secondary lens or eyepiece or ocular to make a magnified virtual image
  – Microscope
  – Telescope
Camera
an optical instrument or an optical device

- A real image of an object is recorded on a film (in old cameras) or electronically (array of detectors in digital cameras)
- Net effect of the combined lens system is a converging lens
Camera parts

A complicated lens and aperture system is used for correcting various aberrations such as wavelength dependence of the refractive index (chromatic), astigmatism, coma, etc. allowing only light enter at angles it is designed for.

The lens is moved back and forth for focusing.
Camera parts: Exercise

When a camera is focused for an object the real image is exactly on the recording material.

Exercise: Assume the lens is focused for an object at a given distance. When we want to take picture of an object, at a larger distance do we move the lens away from the recording material or bring it closer?
Camera angle of view & field of view for a given lens-film combination

- Field of view is defined as the largest angle of view along the diagonal dimension.
- **Example:** Compare angle of view of two lenses with $f_1=28\text{mm}$ and $f_2=300\text{mm}$ with a 24X35mm film for a distance of 10m. ($\alpha_1=75^0$, $\alpha_2=8^0$)

![Diagram showing camera angle of view and field of view](image)
Exposure

- **Exposure**: total light energy per unit area reaching the film (intensity)
- Proper image brightness requires certain limit of exposure
- Exposure is controlled by:
  - Shutter time (usually 1 s to 1/1000 s)
  - Lens aperture diameter (D): larger lenses provide more exposure and brighter pictures.
- Digital cameras control the exposure automatically due to sensitivity of their pixels.
Aperture; \( f \# \); Exposure

Exposure: amount of admitted light into the camera.
Exposure is controlled by two elements:

1) Irradiance: \( E_e = \frac{\text{light power incident at the image plane}}{\text{area of the film or CCD or CMOS imager}} \)

2) Shutter speed: \( 1/t_{\text{shutter}} \) (automatic in digital cameras)

\[
E_e \propto \text{relative aperture of a lens} \propto \frac{\text{area of aperture}}{\text{area of image}} = \frac{D^2}{d^2} \propto \left( \frac{D}{f} \right)^2
\]

Image size is proportional to the focal length of the lens \( d = f \alpha \)

\[
f \# = \frac{f}{D}
\]

\[
E_e \propto \frac{1}{(f \#)^2}
\]
F-number and irradiance

Commercial cameras have selectable apertures that provide irradiance changes by a factor of 2.

The corresponding $f\#$ changes by a factor of $\sqrt{2}$.

Larger $f\# \rightarrow$ smaller light gathering power $(D/f)^2$

$\rightarrow$ longer exposure times

\[
\text{Total exposure} = \text{irradiance} \times \text{time} = E_e \left( \frac{J}{m^2 \cdot s} \right) \times t(s)
\]

For a given film speed or ISO-number variety of $f\#s$ and shutter speed combinations can provide satisfactory exposure.
F-number and irradiance

Example:

We want to make two images with different shutter speeds.

For case 1: \( f \#_1 = 8 \) the shutter speed is \((1/50)\)s.

Find the equivalent \( f \) / stop for shutter speed \((1/100)\)s that provides the same exposure.

Solution:

Total exposure for case 1:

\[
\text{Total exposure for case 1} = \left( \frac{1}{f \#_1} \right)^2 t(s) = \left( \frac{1}{8} \right)^2 \frac{1}{50} = 1.28
\]

Total exposure for case 2:

\[
\text{Total exposure for case 2} = 1.28 = \left( \frac{1}{f \#_2} \right)^2 \frac{1}{100} = \left( \frac{1}{f \#_2} \right)^2 \frac{1}{2 \cdot 50}
\]

\[
\left( \frac{1}{f \#_2} \right)^2 = 2 \left( \frac{1}{f \#_1} \right)^2 \rightarrow \frac{1}{f \#_2} = \sqrt{2} \frac{1}{f \#_1} = 1.4 \left( \frac{1}{8} \right) \rightarrow f \#_2 = 5.6
\]
### TABLE 3-2  STANDARD RELATIVE APERTURES AND IRRADIANCE AVAILABLE ON CAMERAS

<table>
<thead>
<tr>
<th>$A = f$-number</th>
<th>$(A = f$-number$)^2$</th>
<th>$E_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$E_0$</td>
</tr>
<tr>
<td>1.4</td>
<td>2</td>
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</tr>
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<td>2</td>
<td>4</td>
<td>$E_0/4$</td>
</tr>
<tr>
<td>2.8</td>
<td>8</td>
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</tr>
<tr>
<td>4</td>
<td>16</td>
<td>$E_0/16$</td>
</tr>
<tr>
<td>5.6</td>
<td>32</td>
<td>$E_0/32$</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
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</tr>
<tr>
<td>22</td>
<td>512</td>
<td>$E_0/512$</td>
</tr>
</tbody>
</table>

Aperture size decreases, Irradiance decreases
Examples

• The light intensity reaching a film is inversely proportional to the square of the f-number and exposure time.

• Which lens has more light gathering capacity?
  a) $f=50\text{ mm}$, $D=25\text{ mm}$ ($D/f$)
  b) $f=50\text{ mm}$, $D=50\text{ mm}$
  c) $f=100\text{ mm}$, $D=50\text{ mm}$
  d) $f=100\text{ mm}$, $D=100\text{ mm}$

• What is the effective diameters of the f/2, f/2.8, f/4, f/5.6, f/8, f/11, f/16 apertures (f-stops with factor of 2 decrease in intensity)? (2f, 2.8f, …)

• Compare these two exposures.
  – $D=f/4$ and $1/500\text{ s}$
  – $D=f/8$ and $1/125\text{ s}$
The eye

• The eye (camera) is an optical machine with variable focal length that can produce a real image of the outside objects at various distances on a sensitive screen retina (film or imager).

• Change of the effective focal length of the eye is called accommodation and it depends on the age and how well the ciliary muscle can change the curvature of the crystalline lens.

• Far point: the furthest point that eye can see. Infinity for a healthy eye

• Near point: the closest point that eye can construct a clear image of it (~25 cm for a healthy eye)

• What is the lens type of our eyes?

• What are the image properties?
Data related to the eye

http://hyperphysics.phy-astr.gsu.edu/hbase/vision/eyescal.html
Problematic eyes

- **Myopic or near sighted** eye has a shorter far point than a healthy eye, as a result it can’t see the far objects clearly.
  - It has a lens with shorter focal length.
  - Image of far objects fall in a short distance than retina.

- **Hyperopic or far sighted** eye has a longer near point than a healthy eye, as a result it can’t see the close objects clearly.
  - It has a lens with longer focal length.
  - Image of the near objects fall behind the retina.

- **Presbyopic** eye has lost the accommodation power due to aging near point moves from 10 cm at age 10 to about 200 cm at 60.
Correcting a hyperopic & presbyopic eye: Near point has moved further.

(a) Nearby object

(b) Converging lens forms a virtual image at or beyond the eye’s far point: acts as a distant object for the eye.
Correcting a myopic eye
Far point has moved closer

(a) Distant object

(b) Diverging lens forms a virtual image at or inside the eye’s near point; acts as a nearby object for the eye

Image not focused on retina

Myopic eye

Image focused on retina

Diverging lens
Astigmatism

- Surface of the cornea is not spherical. It can be for example cylindrical.
- Image of the horizontal and vertical lines are not on the same plane.

(a) Vertical lines are imaged in front of the retina

(b) A cylindrical lens corrects for astigmatism
Examples for the problematic eye (bring the solution to the class)

a) Near point of an eye is 150 cm. What is the eye’s problem? To see an object at 25 cm what contact lens is required? (f=? Cm ) Express your answer in diopters which is the inverse of the focal length in meters. This is the number appearing in your prescription when you see an optometrist.

b) The glasses are usually worn 2.5 cm in front of the eye. Far point of an eye is 550 mm. What is the patient’s condition? What eyeglasses are required to see an object at infinity? (focal length or diopters)
Example: An image as an object (bring the solution to the class)

• **A two lens system**: an object with 8.0 cm height is placed at 12.0 cm to the left of a converging lens with a focal length of 8.0 cm. A second converging lens with focal length of 6.0 cm is placed at 15.0 cm to the right of the first lens. Both lenses have the same optic axis. Find the image position, size, and orientation for the two lens system.

• Make the second lens diverging and solve the problem
Two lens system
Apparent and angular size

- **Apparent size**: depends on the size of object’s image on the retina
- **Angular size**: is the angle subtended by the object
- When an object is closer to the eye, its angular size is bigger so is the apparent size. But what happens to objects closer than near point of the eye?
The magnifier: a paraxial analysis

- How we can increase angular and apparent size of the smaller objects and yet construct a clear image on the retina?

Angular size without magnifier:
\[ \theta = \tan \theta = \frac{y}{25\text{cm}} \]

Angular size with magnifier:
\[ \theta' = \tan \theta' = \frac{y}{f} \]

Angular magnification for a simple magnifier:
\[ M = \frac{\theta'}{\theta} = \frac{25\text{ cm}}{f} \]
The magnifier: Example

Limit on the angular magnification is aberration, usually 4× for simple lenses and 20× for the corrected lenses.

Difference between the lateral, m, and angular, M, magnification. For image at infinity the lateral magnification is meaningless.

Example: A 2mm insect is being observed by a 4× magnifier in a comfortable setting for the eye.

a) What is the focal length of the magnifier?
b) What is the angular size of the insect's image?
c) What is the lateral magnification?
Microscope

• Offers a much greater magnification than a simple magnifier.
• Two converging or positive lenses
• Object placed just beyond the first lens’s focal point makes a real inverted image
• This image is just inside the object focal point of the second lens creating a virtual, upright, and magnified image
Microscope

- Two converging or positive lenses
- Object placed just beyond the first lens’s focal point makes a real inverted image
- This image is just inside the object focal point of the second lens creating a virtual, upright, and magnified image
- For relaxed viewing the image from the primary is set on the focal point of the secondary lens.
- This creates the final image at infinity.
The microscope: a paraxial analysis

Overall angular magnification of a compound microscope:

\[ M = \text{lateral mag. of objective} \times \text{angular mag. of ocular} = m_1 \times M_2 \]

\[ m_1 = -\frac{s_1'}{s_1} \text{ is the lateral magnification of the objective (usually } s_1 = f_1) \]

\[ M_2 = \frac{\theta_2'}{\theta_2} = \frac{25 \text{ cm}}{f_2} \text{ is the angular magnification of the ocular (like the magnifier case)} \]

So \[ |M| = |m_1 \times M_2| = \left| \left( -\frac{s_1'}{f_1} \right) \left( \frac{25\text{ cm}}{f_2} \right) \right| = \left( \frac{(25\text{ cm})s_1'}{f_1f_2} \right) \]

Final image is inverted and virtual. The shorter the focal lengths of the objective and ocular, the higher the magnification.
Telescope

- Essentially similar to the compound microscope
- Used to view the large objects at far distances
- Many telescopes use curved mirrors as an objective (why?)
- No spherical or chromatic aberrations, can be made large and robust (largest in Hawaii 10m diameter)
The telescope: a paraxial analysis

Telescope length for most comfortable viewing situation

\[ L = f_1 + f_2 \quad \text{(image at infinity)} \]

Angular magnification of a telescope (since both object and image are at \( \infty \)):

\[ M = \frac{\theta'}{\theta} \quad \theta = \frac{-y'}{f_1} \quad (- \text{since image is inverted}) \]

\[ \theta' = \frac{y'}{f_2} \quad (+ \text{image is not inverted by this lens}) \]

\[ M = \frac{\theta'}{\theta} = \frac{-y' f_2}{f_1 y'} = -\frac{f_1}{f_2} \quad (- \text{final image is inverted}). \]

Why we can't use a telescope as a microscope or vice versa?

The longer the objective focal length the higher the magnification

The shorter the eyepiece focal length the higher the magnification
Example: reflecting telescope

- Radius of curvature for a reflecting telescope objective is 1.30 m. Focal length of the eyepiece is 1.10 cm. The final image is at infinity.
  
a) What should be the distance between the eyepiece lens and the vertex of the mirror? (66.10 cm)
b) What is the angular magnification? (-59.1)
Example: Cassegrain telescope

- Focal length of the primary: 2.5 m
- Focal length of the secondary: -1.5
- Distance from the vertex of the primary to the detector: 15 cm
- What should be the distance of vertexes of two mirrors?