Chapter 3
Optical Instrumentation

Lecture Notes for Modern Optics based on
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Spring 2009

Stops, pupils, windows

Stops, pupils, windows are of great importance for control of light in optical instrumentation.

Not all rays leaving the object participate in image formation.

Aperture: an opening defined by a geometrical boundary that creates spatial limitation for the light beams.

Apertures are used to:

- generate sharp boundaries for images
- correct aberrations such as spherical, astigmatism and distortion
- shield the image from undesirable scattered light.

Effects of aperture in an optical system:

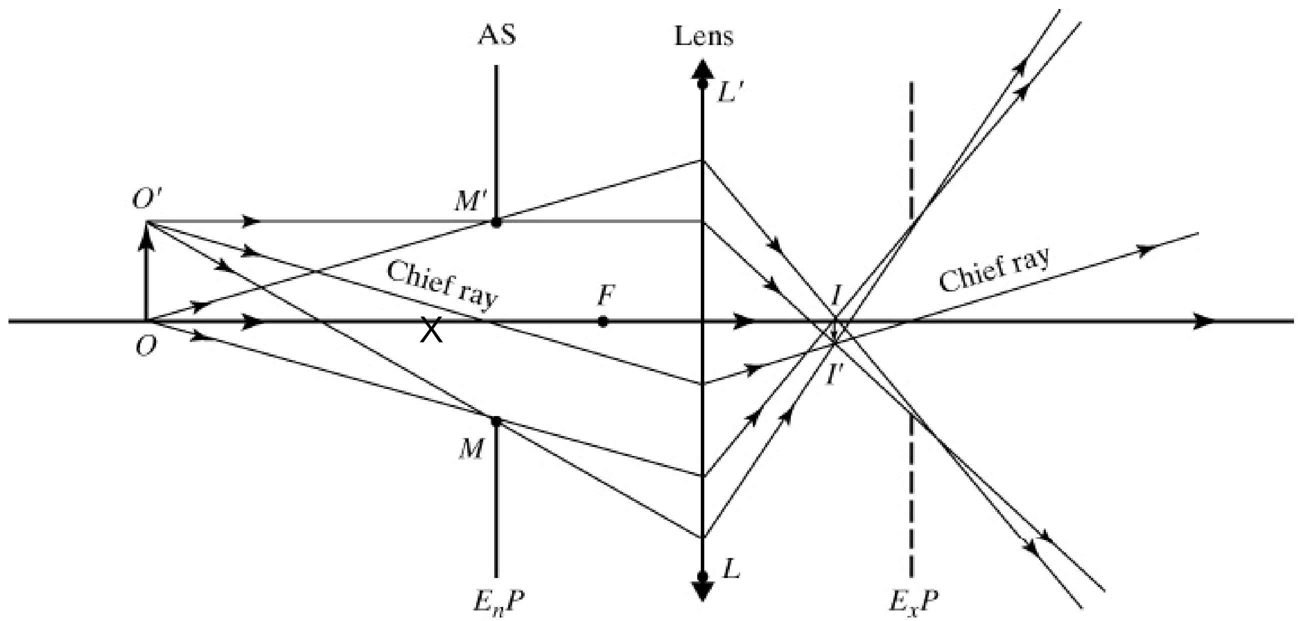
- limiting the field of view
- controlling the image brightness (irradiance W/m^2)

Aperture stop (AS)

The aperture stop of an optical system is the *actual physical component* that limits the size of the maximum cone of rays-from an object point to an image point –that can be processes by the entire optical system.

Example: diaphragm of a camera or iris of the human eye.

The aperture stop (AS) is not always the limiting component. Example for object point X, AS is not the limiting factor. It is the lens rims that cuts the rays.

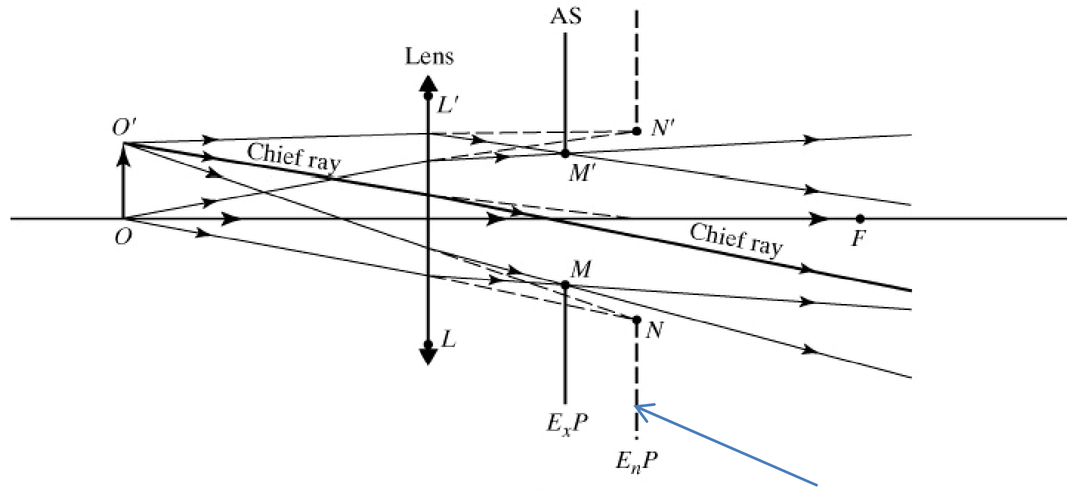


(a)

Entrance Pupil E_nP

Entrance pupil is the <u>limiting aperture (opening) that light rays see looking into the optical system from any object point</u>.

The entrance pupil is the image of the controlling aperture stop formed by the imaging elements preceding it (to the left of AS).

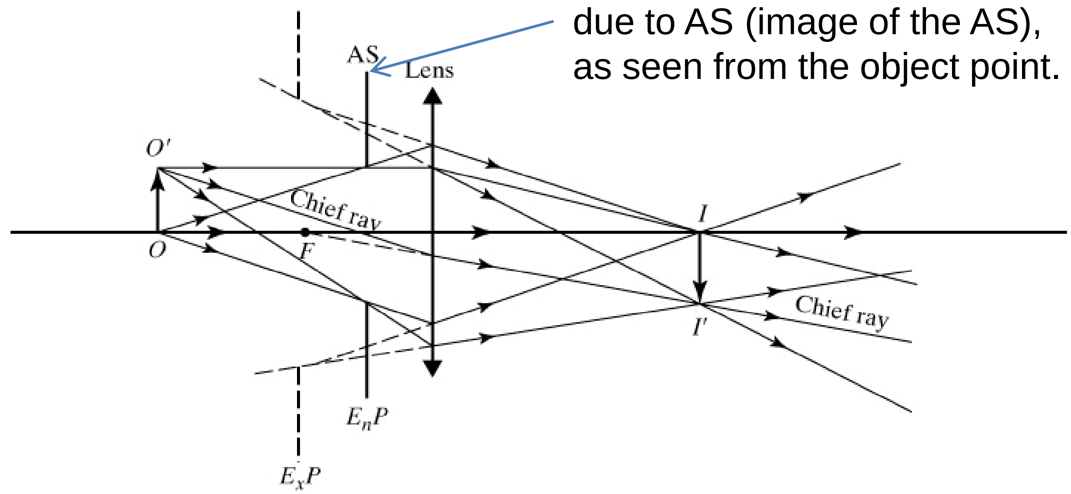


(b)

E_nP is the effective aperture due to AS (image of the AS), as seen from the object point.

Sometimes AS and E_nP are identical but in this figure they differ.

AS and E_nP are conjugate points so E_nP is image of AS.



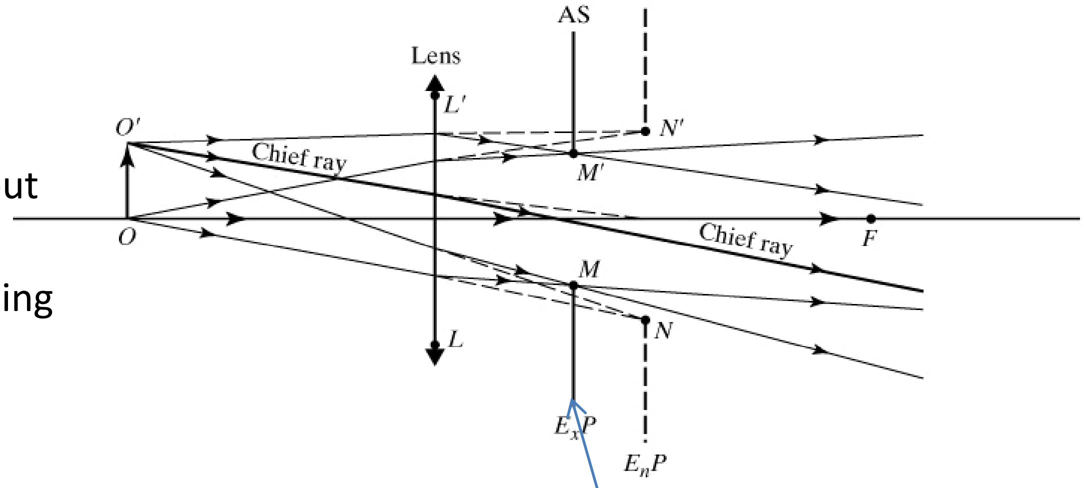
(c)

Exit pupil E_XP

Exit pupil is the image of the AS, as seen from the image point. E_XP limits the output beam size.

The exit pupil is the image of the controlling aperture stop formed by the imaging elements following it (to the right of AS).

E_XP is conjugate of AS and E_nP.



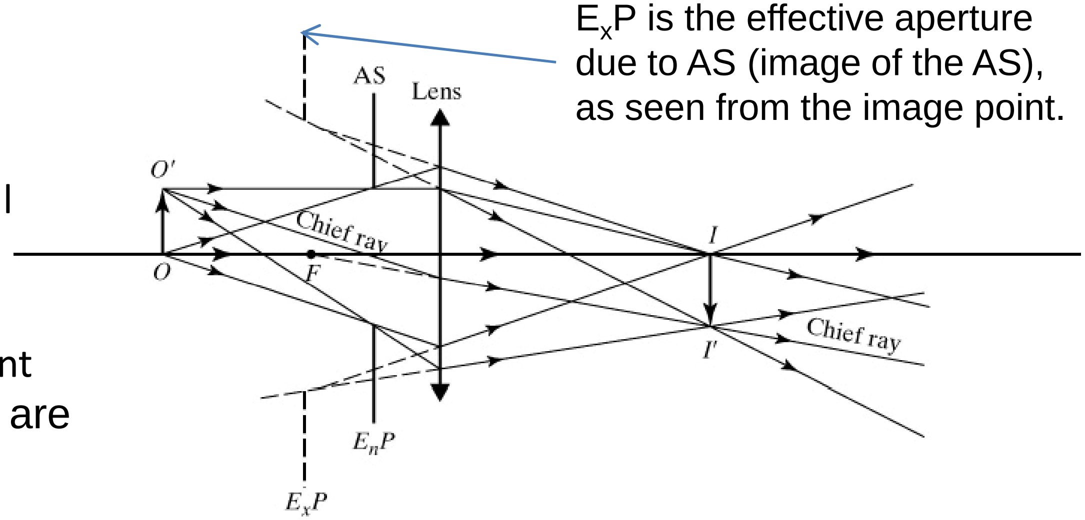
(b)

E_xP is the effective aperture due to AS (image of the AS), as seen from the image point.

Chief ray

The chief or principal ray is a ray from an object point that passes through the axial point, in the plane of the entrance pupil E_nP.

The chief ray must also pass the axial point of the E_XP and AS since these planes are conjugate.



(c)

Example:

Which element serves as the effective AS for the whole system?

The answer is not so obvious . Three candidates L_1, L_2 (or its image L'_2), A (or its image A'_1)

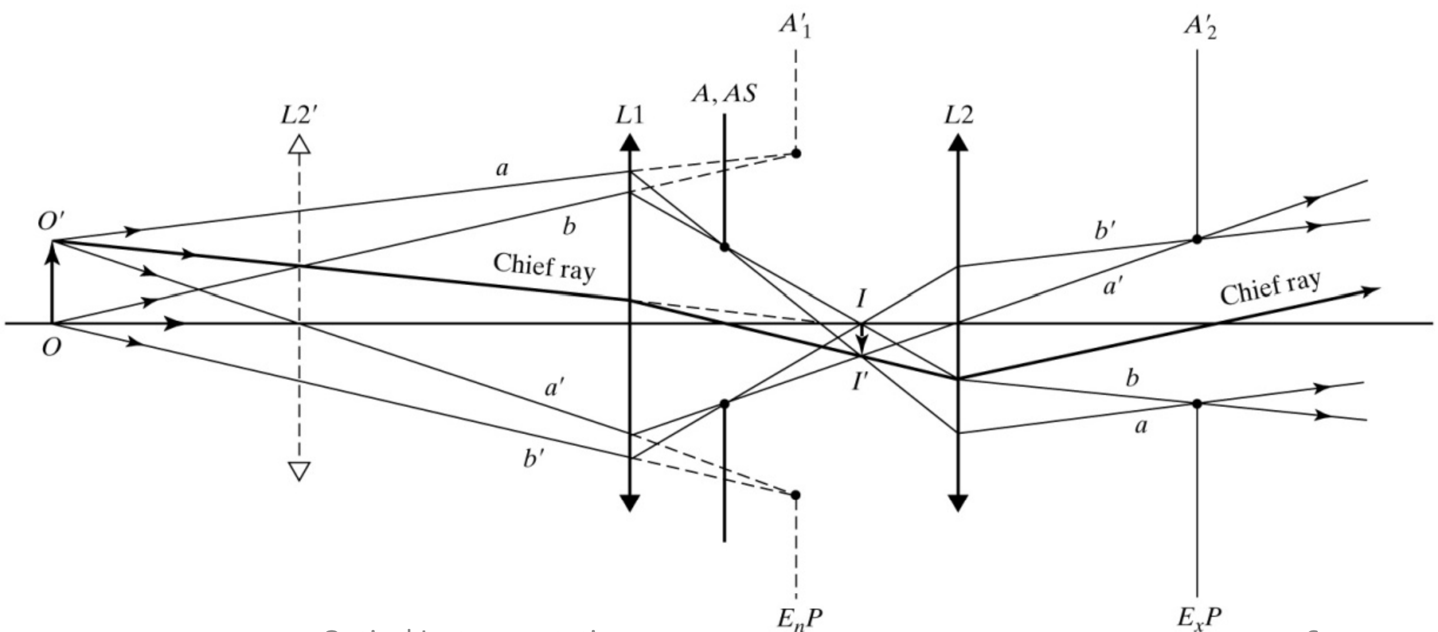
Between the three (L_1, L'_2, A'_1) whichever subtends the smallest angle from the axial object point O is the entrance aperture stop (AS). We can see A'_1 or A is the aperture stop. Now we have AS.

Image of the AS through the elements to the right of it is the exit pupil (E_xP) or A'_2

Image of the AS through the elements to the left of it is exit pupil (E_nP) or A'_1

The cones of light aa' from O' and bb' from O are limitted by the AS, E_nP, and E_xP

The cheif ray passes through the axial points at AS, E_nP, and E_xP



Field of view

Apertures limit the image brightness and field of view like a window does.

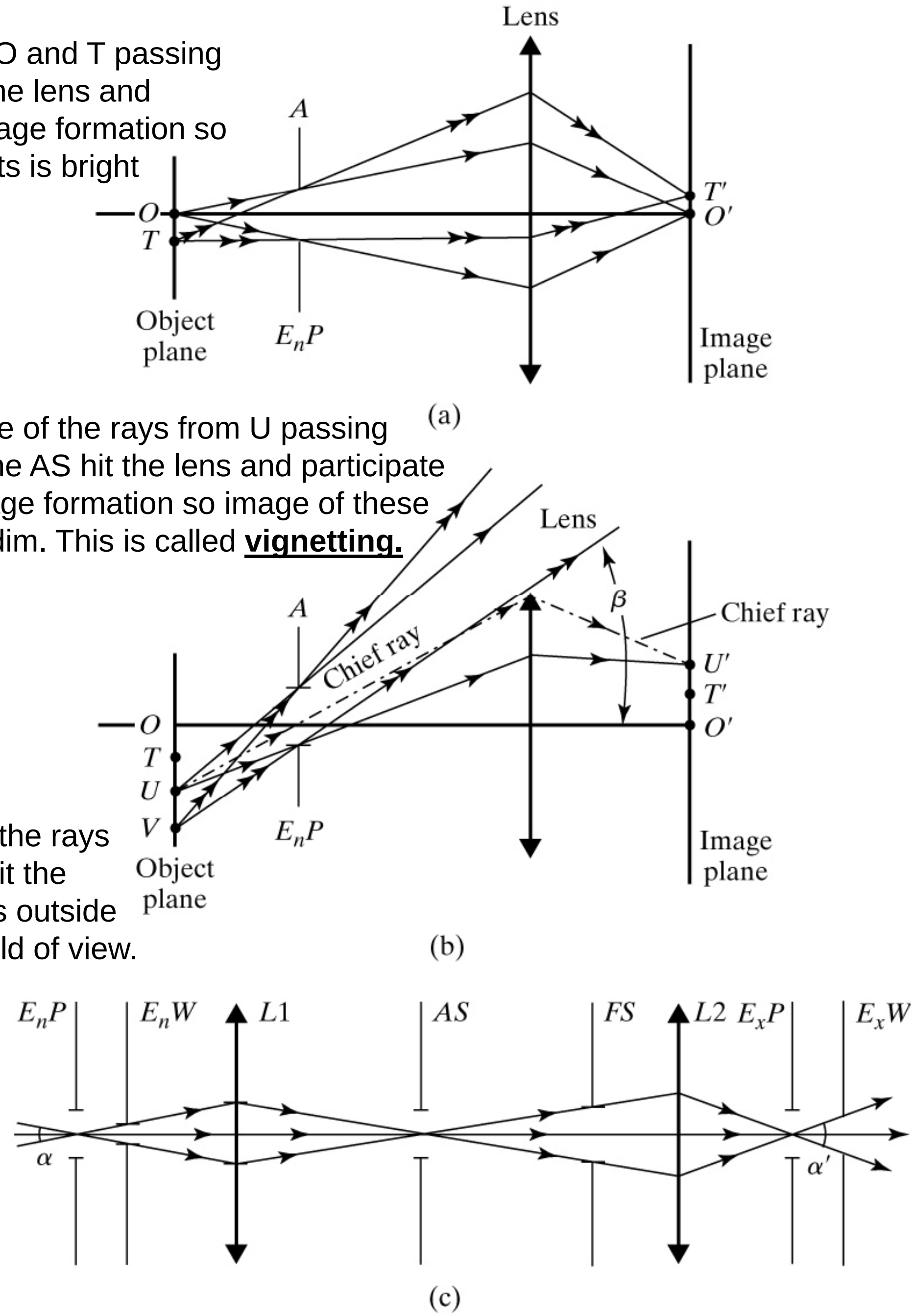
If we consider image points with at least half of brightness (irradiance W/m^2) of the axial image points acceptable then we define **<u>field of view</u>** a circle with radius of OU where U is a point on the object plane that only half of its rays passing through the aperture reaches the image plane (or chief ray from U touches edges of the lens) .

<u>Angular field of view:</u> twice the angle β between the chief ray from the last object point with half brightness and the optical axis.

All of the rays from O and T passing through the AS hit the lens and participate in the image formation so image of these points is bright

Only some of the rays from U passing through the AS hit the lens and participate in the image formation so image of these points is dim. This is called **<u>vignetting.</u>**

None of the rays from V hit the lens. V is outside of the field of view.



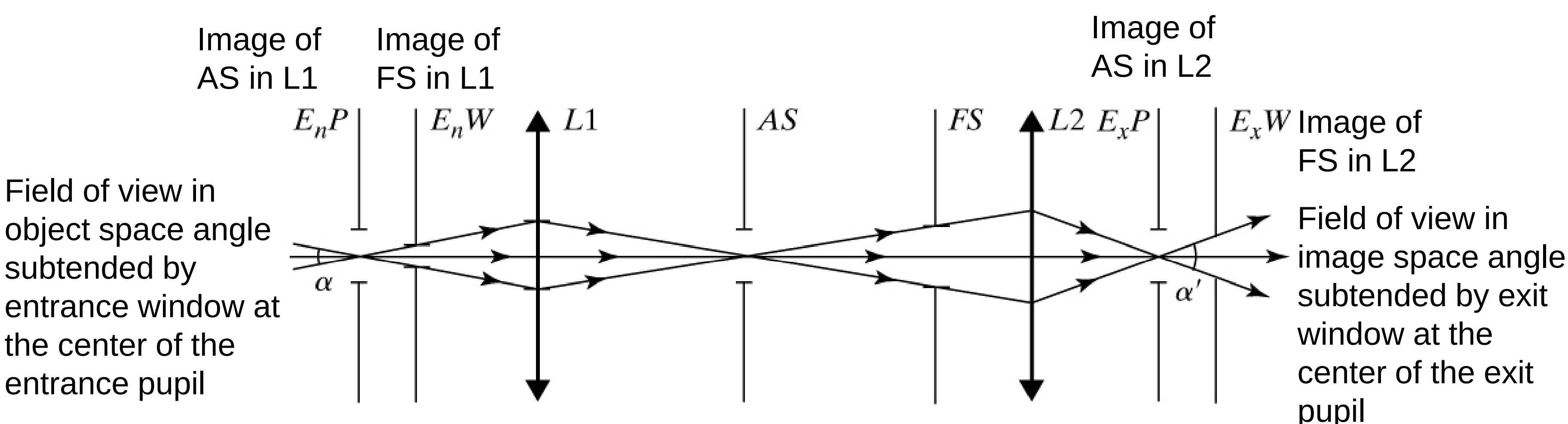
Field stops, Entrance Window, Exit window

Field stop (FS): the aperture that controls the field of view to **eliminate poor quality image points** due to aberration or vignetting.

Practical criteria to determine field stop: as seen from the center of the entrance pupil, the field stop or its image subtends the smallest angle.

Entrance window (E_nW): is the image of the field stop by all optical elements preceding it (to the left of it). It outlines the lateral dimensions of the object being imaged. Conjugate of FS.

Exit window (E_xW): is the image of the field stop by all optical elements following it (to the right of it). This is like a window limiting outside view as seen from inside of a room.



(c)

Summary
Stops, Pupils, Windows

- Brightness
  - Aperture stop AS: The real element in an optical system that limits the size of the cone of rays accepted by the system from an axial object point.
  - Entrance pupil EnP: The image of the aperture stop formed by the elements (if any) that precede it.
  - Exit pupil ExP: The image of the aperture stop formed by the elements (if any) that follow it.
- Field of view
  - Field stop FS: The real element in an optical system that limits the angular field of view formed by an optical system.
  - Entrance window EnW: The image of the field stop formed by the elements (if any) that precede it.
  - Exit window ExW: The image of the field stop formed by the elements (if any) that follow it.

Example 3.1: stops, pupils, windows

The optical system in this figure has a positive thin lens L_1 with f_1=6cm, diameter D=6cm, and a negative thin lens L_2 with f_2=-10cm, diameter D_2=6cm, and an aperture A with diameter D_A=3cm, located 3cm in front of the L_1 which is located 4cm in front of the L_2. The object OP, 3cm high is located 18cm to the left of L_1.

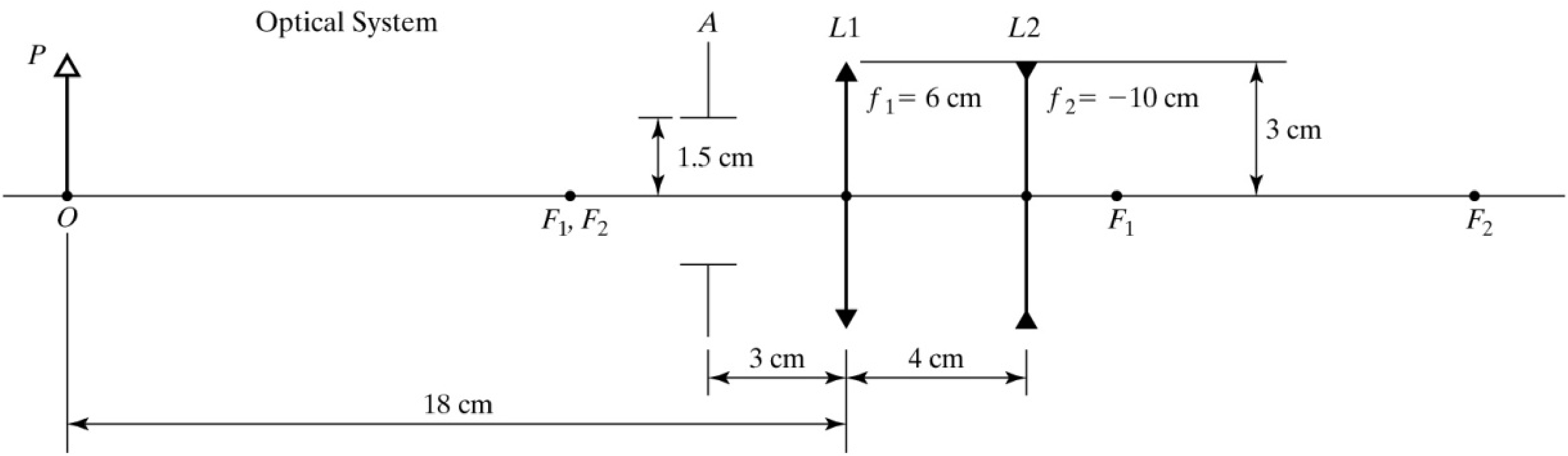
a) Determine which element serves as the aperture stop?

b) Determine size and location of the entrance and exit pupils.

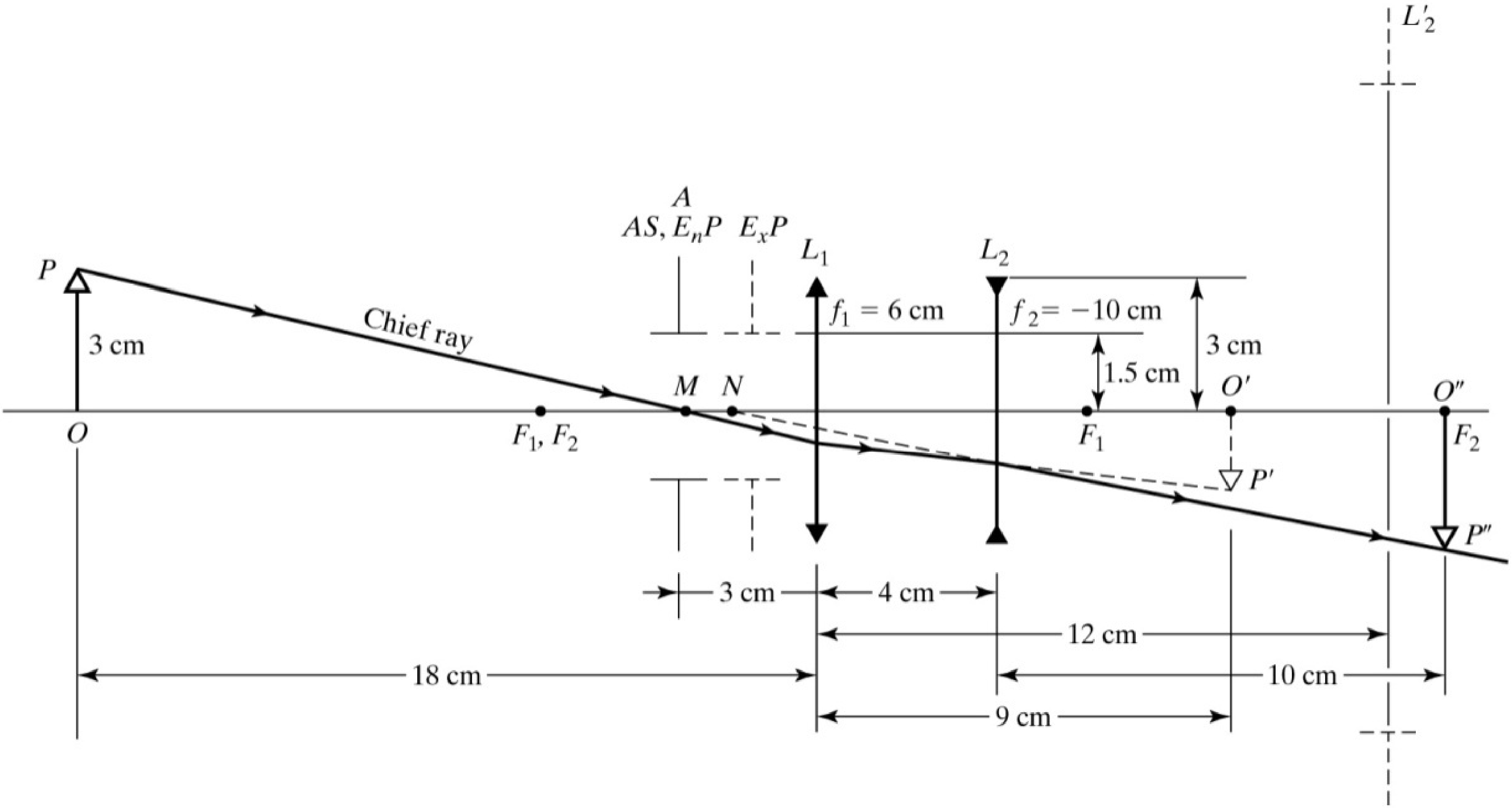
c) Determine location and size of the image OP' of the OP formed by L_1 and the final image OP" formed by the system.

d) Draw a diagram of the system, its pupils and images.

e) Draw the chief ray from object point P to its conjugate in the final image P".



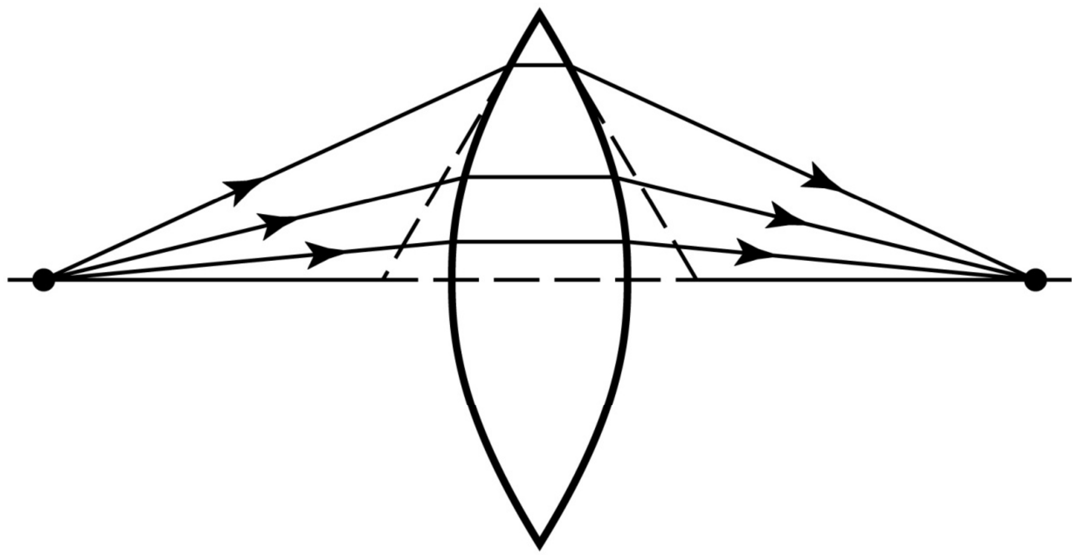
Solution to the example 3.1 : stops, pupils, windows



Prisms

A double concave lens con be modeled as combination of prism.

We will derive relationships that models propagation of light through a prism.



Angle of deviation and prism parameters

A monochromatic ray of light hits a prism with index of refraction n and prism angle A.

θ_1 & θ'_1 are angles of incidence and refraction at the first face

θ'_2 & θ_2 are angles of incidence and refraction at the second face

δ the total angular deviation of the incident ray due to the action of prism.

δ_1 and δ_2 are the angular deviation of the incident ray due to the action of the first and second faces.

Applying the Snell's law at both surfaces: $\begin{cases} (1)\sin\theta_1 = n\sin\theta'_1 \\ n\sin\theta'_2 = (1)\sin\theta_2 \end{cases}$

Gemetrical relations between the angles: $\begin{cases} \delta_1 = \theta_1 - \theta'_1 \\ \delta_2 = \theta_2 - \theta'_2 \\ B + \theta'_1 + \theta'_2 = 180;\ \text{sum of the angles of a quadrilateral is } 180^0 \\ A + B = 180;\ \text{sum of the angles of a quadrilateral is } 360^0 \end{cases}$

From the last two relations: $A = \theta'_1 + \theta'_2$

Finding the δ, the angle of deviation:

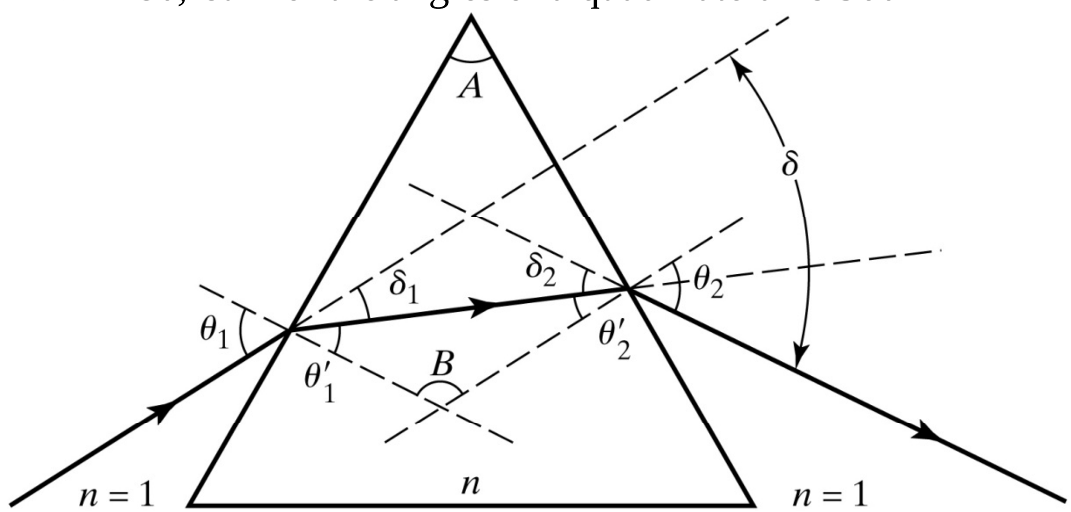
$$\theta'_1 = \sin^{-1}\left(\frac{\sin\theta_1}{n}\right)$$

$$\delta_1 = \theta_1 - \theta'_1$$

$$\theta'_2 = A - \theta'_1$$

$$\theta_2 = \sin^{-1}\left(n\sin\theta'_2\right)$$

$$\boxed{\delta = \delta_1 + \delta_2 = \theta_1 + \theta_2 - \theta'_1 - \theta'_2}$$



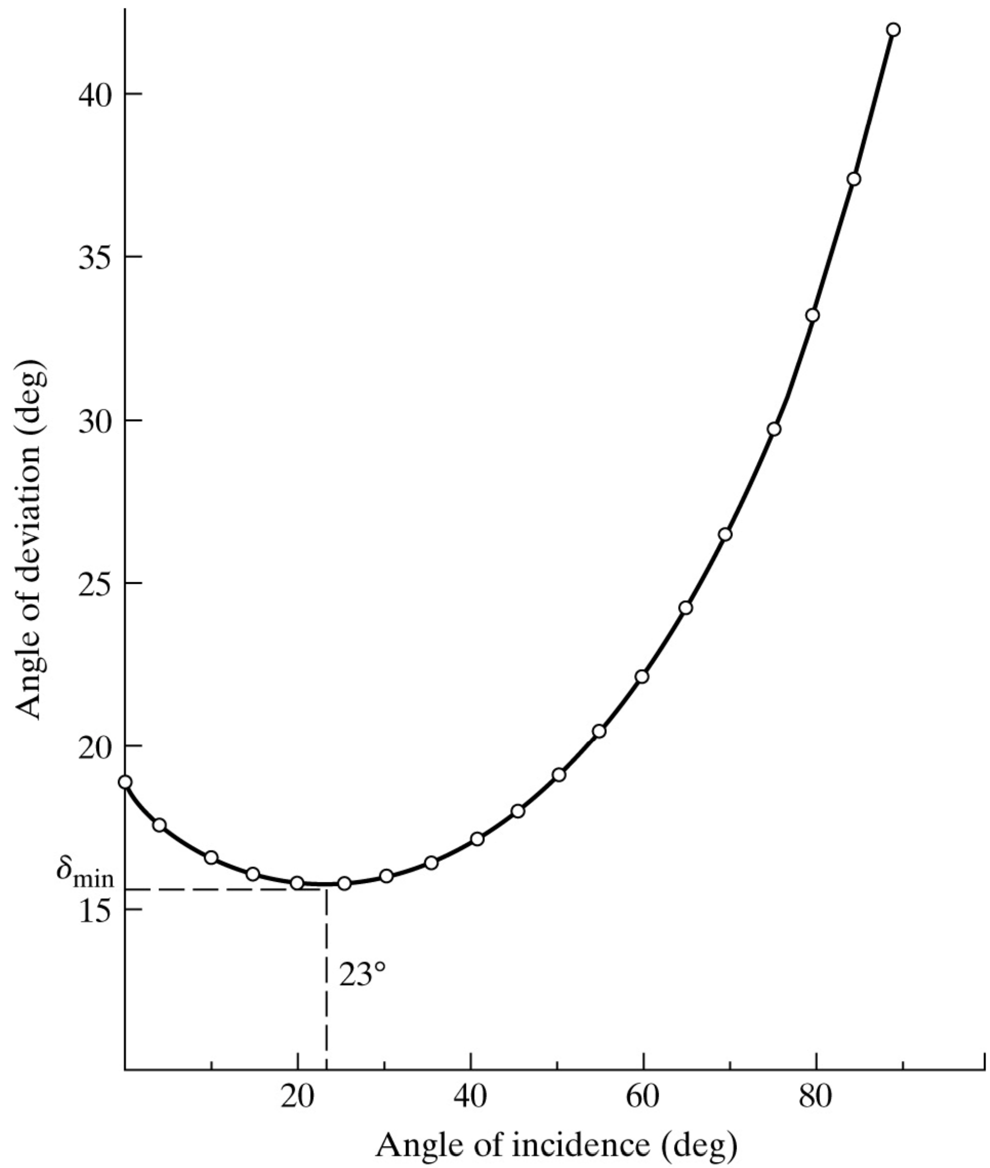
Minimum deviation angle of a prism

If we plot δ vs. θ_1 we will observe that for a given prism angle A, and index of refraction n, there is always a minimum angle of deviation.

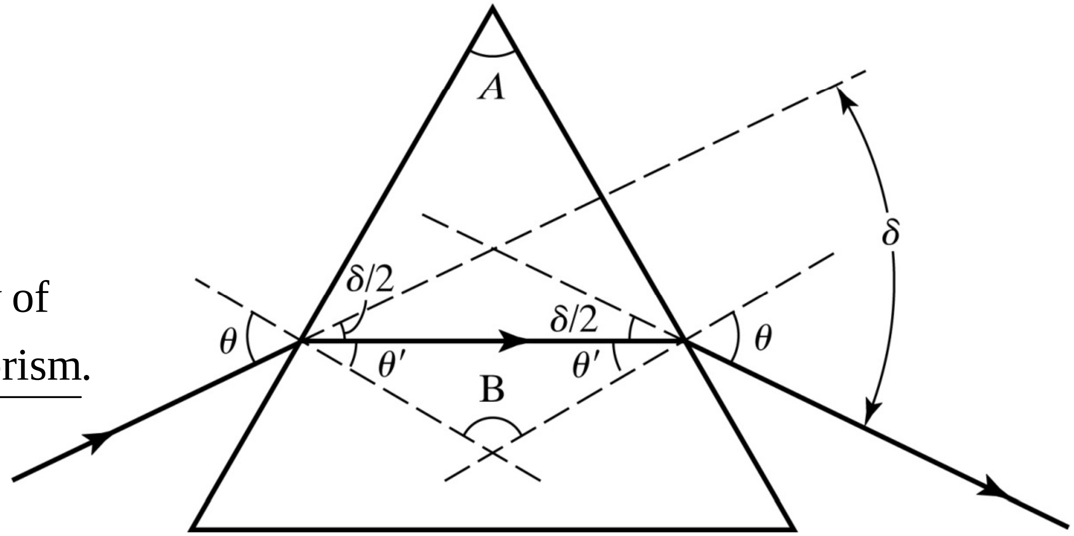
Example:

for $A = 30^0, n = 1.50,$

we get $\theta_{1,\min} = \theta_{2,\min} = 23^0$



Finding index of refraction of the prism material



$\delta = \theta_1 + \theta_2 - \theta'_1 - \theta'_2$

Since θ_1 and θ_2 appear summetrically in the equation we conclude that minimum deviation occurs when the ray of light passes symmetrically through the prism.

Now we can write: $\delta_{\min} = 2\theta_{\min} - 2\theta'_{\min}$

$$A = 2\theta'_{\min} \rightarrow \theta_{\min}' = \frac{A}{2} \; and \; \theta_{\min} = \frac{\delta_{\min} + A}{2}$$

$$\sin\theta_{\min} = n\sin\theta'_{\min} \quad \rightarrow \sin\left(\frac{\delta_{\min} + A}{2}\right) = n\sin\left(\frac{A}{2}\right) \rightarrow \boxed{n = \frac{\sin\left[\left(A + \delta_{\min}\right)/2\right]}{\sin\left(A/2\right)}}$$

we can use this equation to determine refractive index of a material by measuring deviation angle of a ray of light passing through a prism.

For small prism angles that cause small deviation angles we can approximately write:

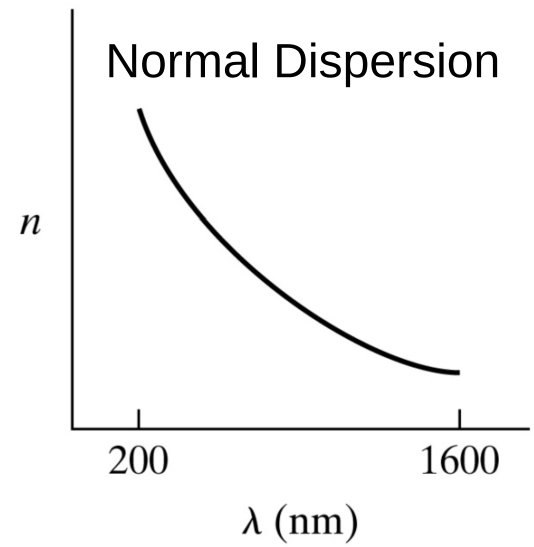
$$n = \frac{\left(A + \delta_{\min}\right)/2}{A/2} \rightarrow \boxed{\delta_{\min} \cong A\left(n-1\right)}$$ minimum deviation angle of a small angle prism.

Dispersion

Since index (n_λ) of a material varies for different wavelengths, a prism produces different deviation angles (δ_λ) for different wavelengths. That is why prisms disperse a white light to its constituent colors. This effect is called dispersion.

A material has normal dispersion if $\frac{dn}{d\lambda} < 0$

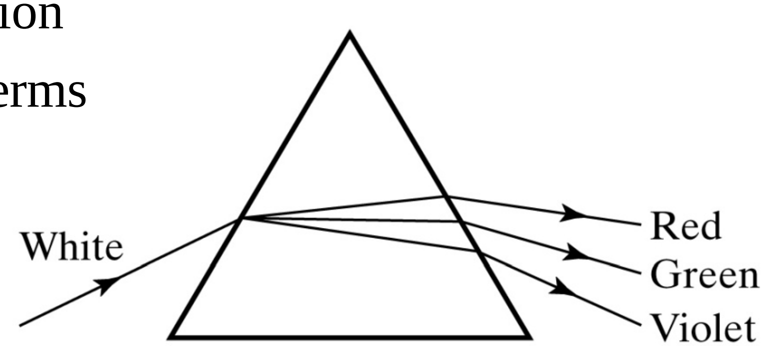
A material has anomalous dispersion if $\frac{dn}{d\lambda} > 0$



The impirical relation that approximates the dependence of n in λ by Augustin Cauchy:

$n_\lambda = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$ where A, B, C are inpirical constants to be fitted to the experimental dispersion data of the material. For most material firs two terms provide a good fit. Then the disperion $dn/d\lambda$ is:

$$\frac{dn}{d\lambda} = -\frac{2B}{\lambda^3}$$



Dispersive power of a prism

The three wavelengths used to characterize dispersion are called Fraunhofer lines that appear in the solar spectrum:

F: due to absorption by hydrogen atom, blue, 486.1 nm

D: due to absorption by sodium atom, yellow, 589.2 nm

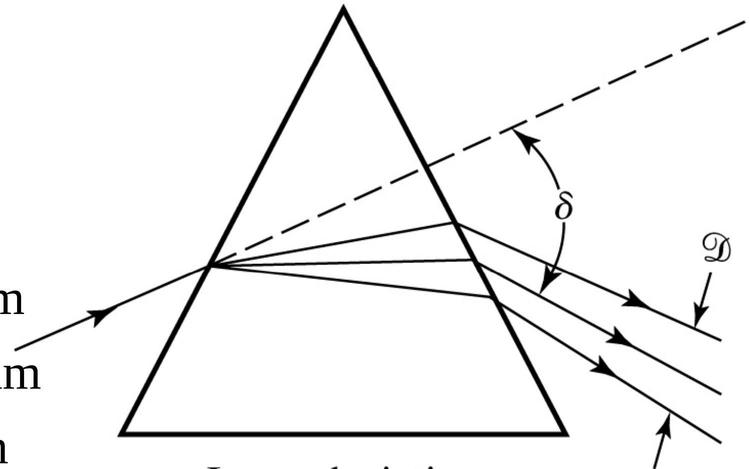
C: due to absorption by hydrogen atom, red, 656.3 nm

Using a thin prism at minimum deviation for the sodium D line, we define **dispersive power** of a prism as:

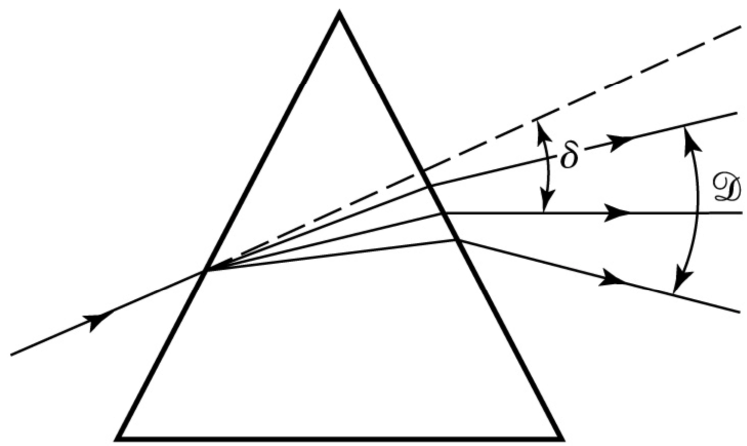
$$\textit{Dispersive power} = \frac{\textit{dispersion of the F and C lines}}{\textit{deviation for the sodium D line}}$$

$$\boxed{\Delta = \frac{D}{\delta} = \frac{n_F - n_C}{n_D - 1}}$$

$$\textit{Abbe number} = \frac{1}{\textit{Dispersive power}} = \frac{1}{\Delta}$$



Large deviation
Small dispersion



Small deviation
Large dispersion

Example

Calculate dispersive power and Abbe Number of the crown and flint glass.

Answer:

Dispersive power of crown= 1/65, Abbe Number=65

Dispersive power of flint 1/29

Abbe Number=29

TABLE 3-1 FRAUNHOFER LINES

		n	
λ (nm)	Characterization	Crown glass	Flint glass
486.1	F, blue	1.5286	1.7328
589.2	D, yellow	1.5230	1.7205
656.3	C, red	1.5205	1.7076

Prism spectrometer

Spectrometer: an analytical instrument employs a prism as a dispersive element, to measure **deviation angles** of various wavelengths of the incident light.

Spectroscope: only used for visual inspection without capability for measurement.

Spectrograph: only used for recording.

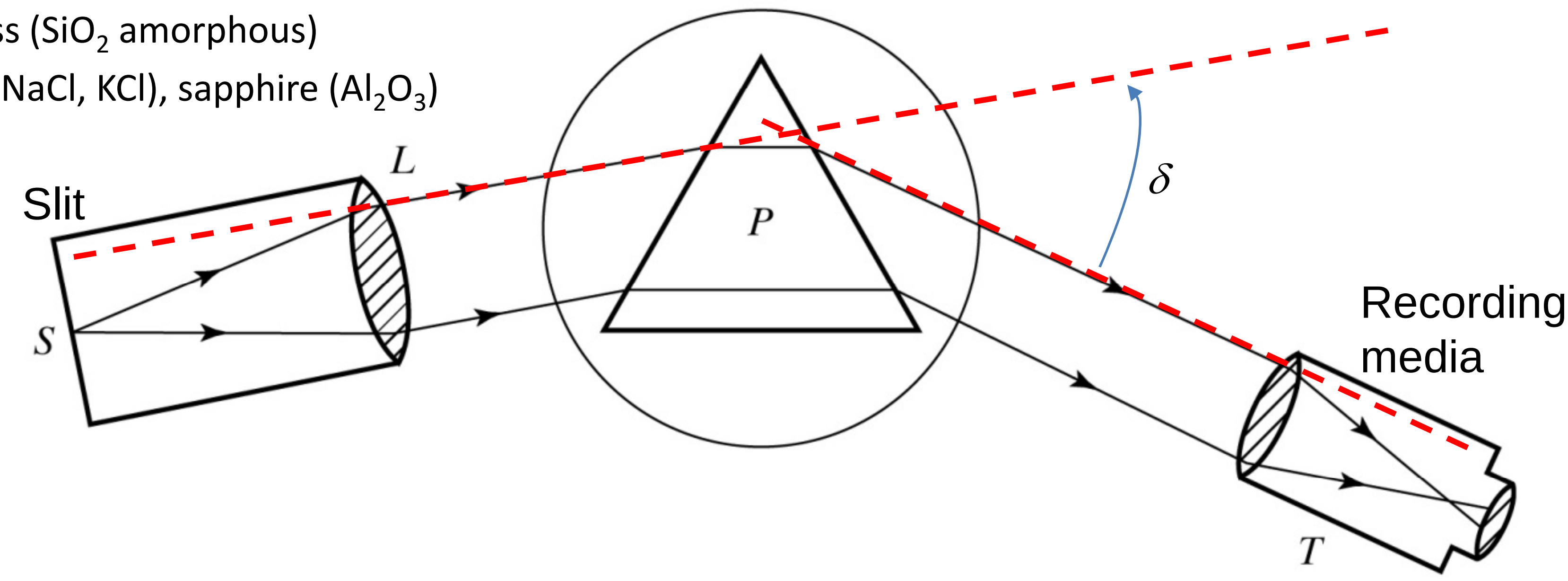
Spectrometer: provides quantitative data and measurements of the spectrum.

Material are used for different parts of the spectrum:

UV: quartz (SiO_2 tetrahedra), fluorite (CaF_2)

VIS: glass (SiO_2 amorphous)

IR: salt (NaCl, KCl), sapphire (Al_2O_3)



Chromatic Resolving Power

Goal: finding the smallest $\Delta\lambda$ that a spectrograph can resolve.

a) A monochromatic parallel beam of light (λ) is incident on a prism (n) such that it fills the face of it.

b) If a second wavelength (λ') is present in the beam, it will have a different index (n'), such that:

$\lambda' = \lambda + \Delta\lambda$ and $n' = n - \Delta n$ (Assuming normal dispersion)

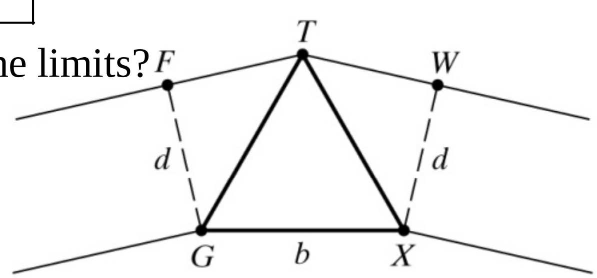
Based on Fermat's principle: $\begin{cases} FT + TW = nb \\ FT + TW - \Delta s = (n - \Delta n)b \end{cases} \rightarrow \Delta s = b\Delta n = b\dfrac{dn}{d\lambda}\Delta\lambda$

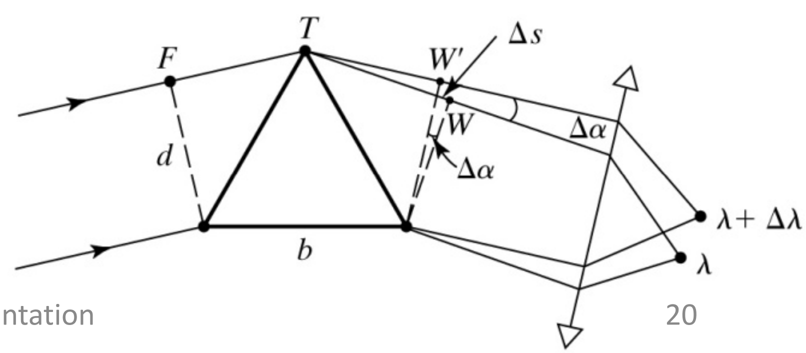
Angular difference between emerging wavefronts: $\boxed{\Delta\alpha = \dfrac{\Delta s}{d} = \left(\dfrac{b}{d}\right)\left(\dfrac{dn}{d\lambda}\right)\Delta\lambda}$.

Rayleigh's criterion for limit of resolution of diffraction-limited line images: minimum angular separation between the two wavefronts for just barely resolwing two lines is given by $\Delta\alpha = \lambda / d$. Then we have:

$\dfrac{\lambda}{d} = \left(\dfrac{b}{d}\right)\left(\dfrac{dn}{d\lambda}\right)\Delta\lambda \rightarrow$ $\boxed{(\Delta\lambda)_{\min} = \dfrac{\lambda}{b(dn/d\lambda)}}$ $\leftarrow$ The resolution limit of a prism

$\boxed{\Re = \dfrac{\lambda}{(\Delta\lambda)_{\min}} = b\dfrac{dn}{d\lambda}}$ $\leftarrow$ The resolving power of a prism. Name way to increase the resolving power of a prism. What are the limits?





Example 3.2

Determine the resolving power and minimum resolvable wavelength difference for a prism made of flint glass with base of 5 cm.

Using table 3.1,

$$\frac{\Delta \mathrm{n}}{\Delta\lambda} = \frac{\mathrm{n_F} - n_D}{\lambda_\mathrm{F} - \lambda_D} = \frac{1.7328\text{-}1.7205}{486\text{-}589} = -19\times10^{-4}\,\mathrm{nm}^{-1}$$

The resolving power,

$$\Re = d\left(\frac{dn}{d\lambda}\right) = 5971$$

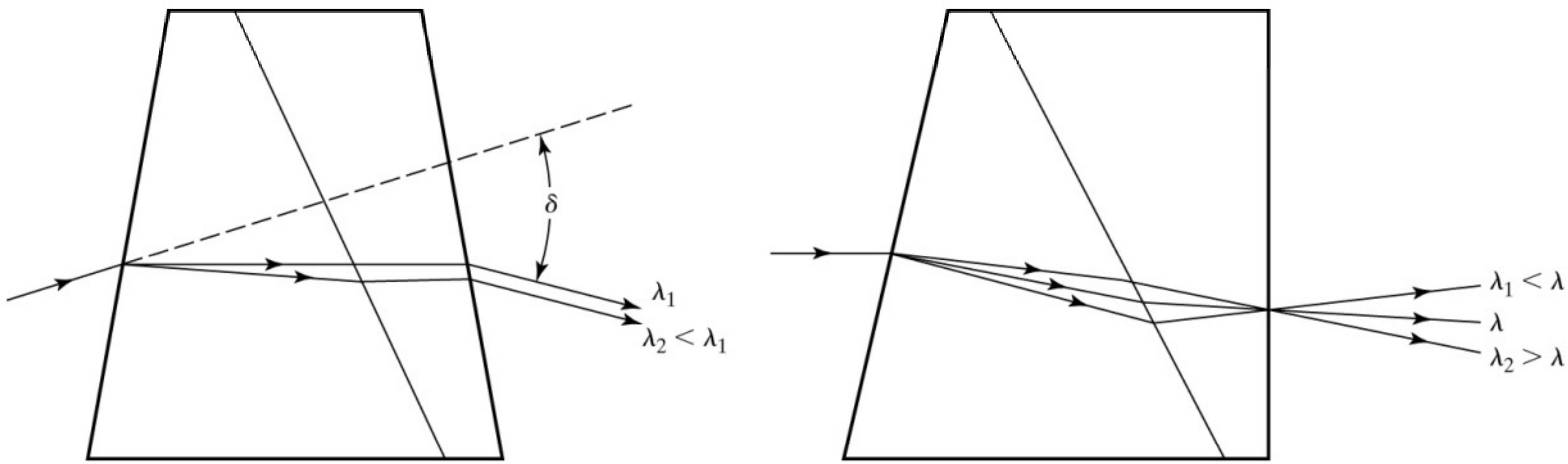
The minimum resolvable wavelength difference in the region aroung 550 nm,

$$\left(\Delta\lambda\right)_{\min} = \frac{\lambda}{\mathrm{R}} = \frac{5550\,\overset{0}{\mathrm{A}}}{5971} \cong 1\ \overset{0}{\mathrm{A}}$$

Prisms with special applications

No dispersion, only deviation
One prism cancels the other ones dispersion

No deviation for a particular wavelength



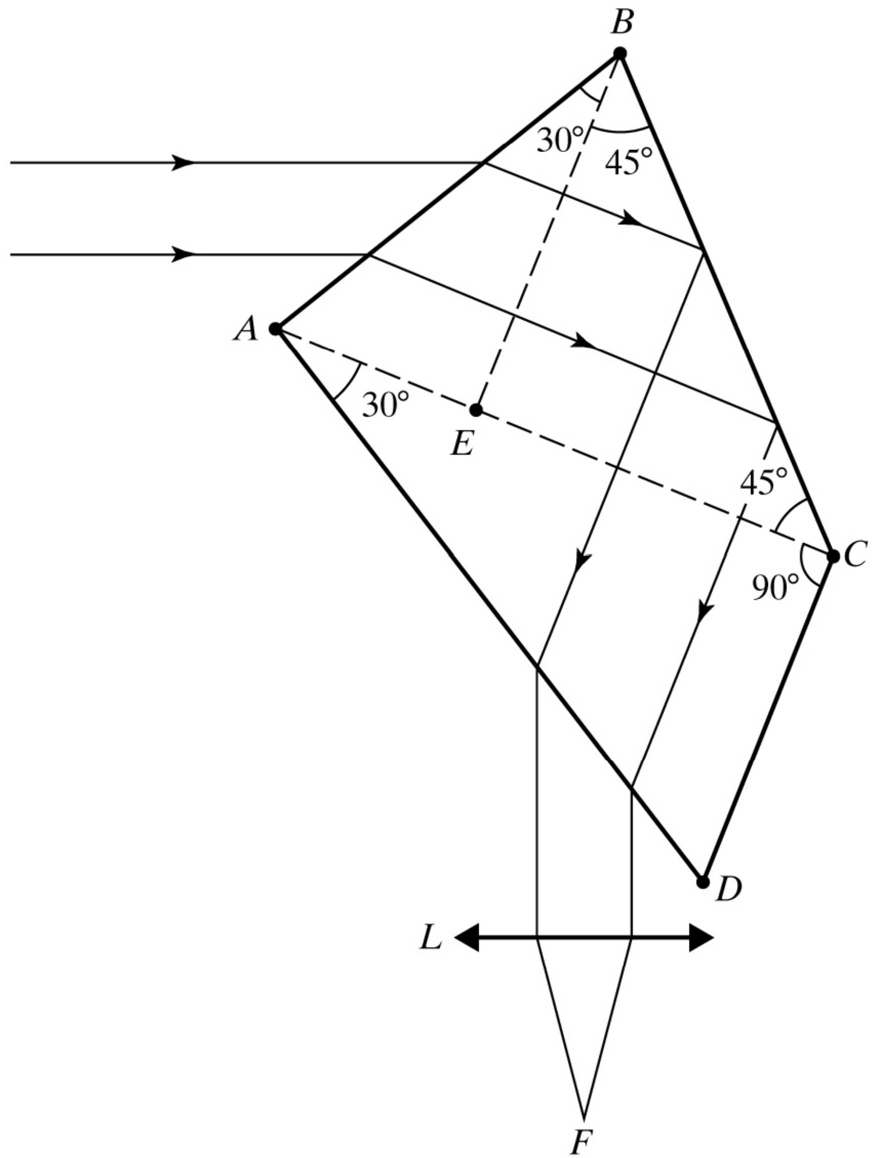
(a) Achromatic prism

(b) Direct-vision prism for wavelength λ

Prisms with special applications

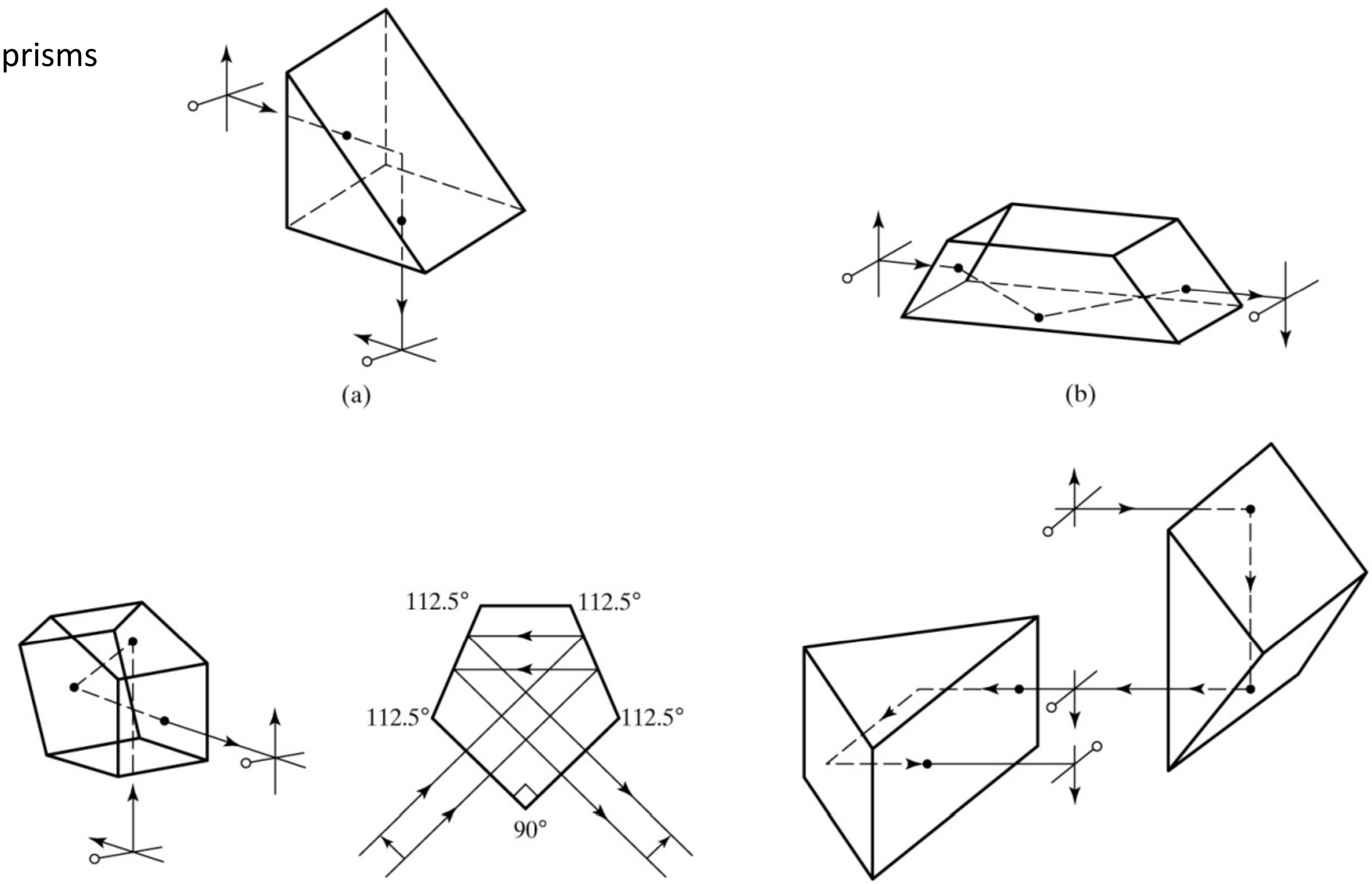
Constant deviation for all wavelengths.

Pellin-Broca prism of constant deviation



Prisms with special applications

Reflecting prisms



(a) (b) (c) (d)

Imaging by a pinhole camera

The pinhole camera

Simplest form of camera is a pinhole camera. There is no focusing and every point of image is constructed by the rays that are approximately coming from a point is the pinhole is small enough.

Smaller pinholes cause diffraction.

Optimal pinhole size: 0.5 mm

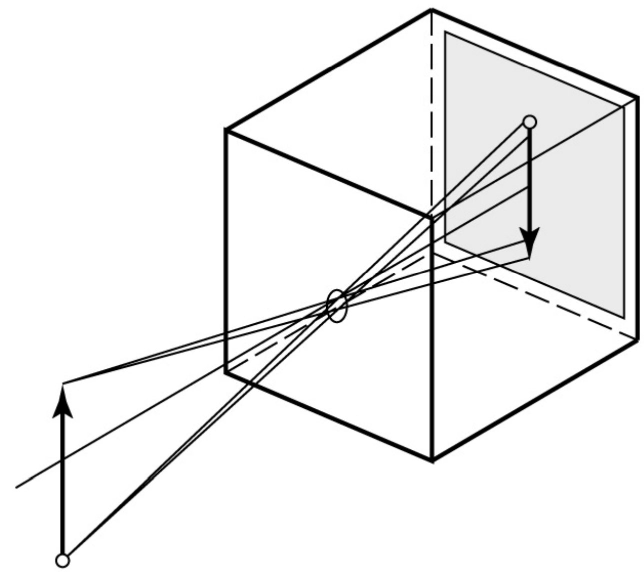
Optimal film distance: 25 cm

Image properties:

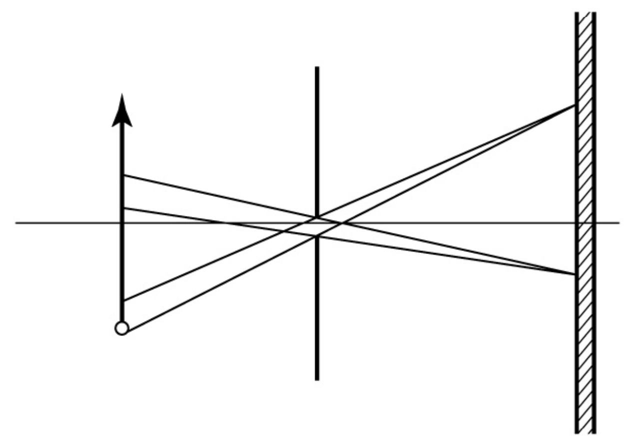
Unlimited depth of field: since there is no focusing element so all the objects appear sharp.

Limited image brightness

Limited image sharpness



(a)



(b)

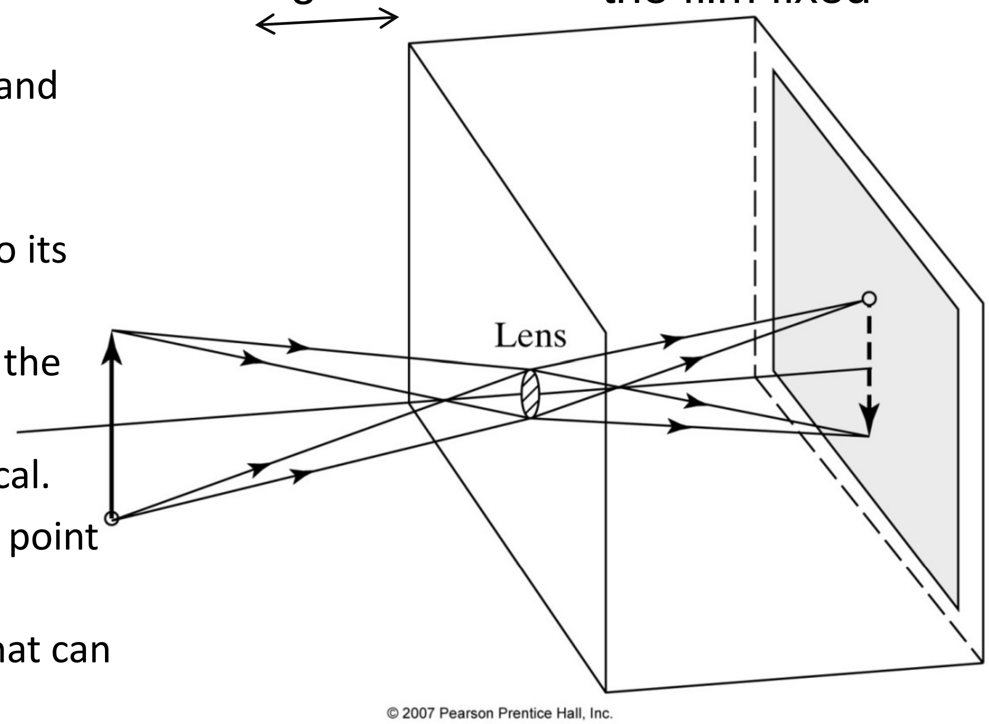
The simple camera

By enlarging the aperture in a pinhole camera and placing a lens in it several changes happen:

1) **Increase in brightness** of the image due to focusing all the rays from an object point to its conjugate on the film
2) **Increase in sharpness** of the image due to the focusing power of the lens.
3) The lens-to-film distance is now more critical.
4) For object at infinity the film is at the focal point of the lens.
5) Close-ups: lenses with short focal length that can handle near objects.
6) Telephoto: lenses with long focal length that images far object at the expense of subject area.
7) Wide-angle: lenses with short focal length and large field of view.
8) Combination of positive and negative lenses is used to avoid a long camera tube .

The lens is moved back and forth for focusing

Position of the film fixed



Camera aperture and f-number

Two elements controle the amount of admitted light into the camera:

1) The aperture size

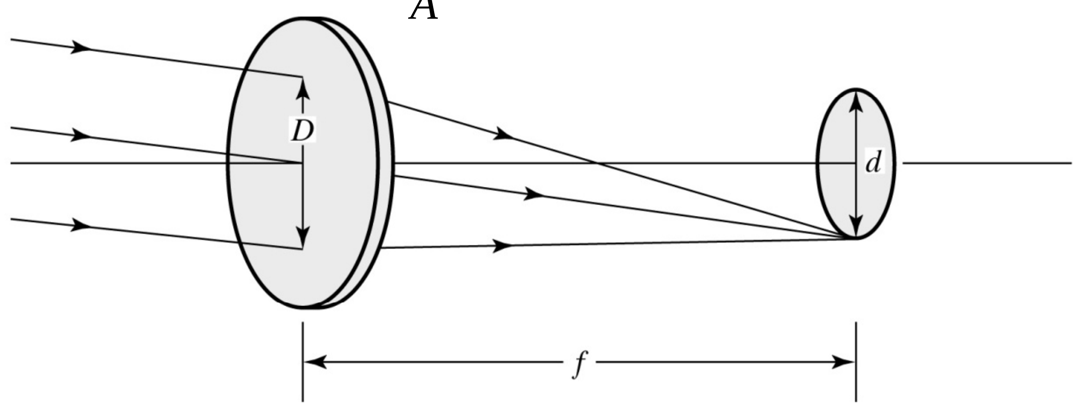
2) Shutter speed

$$\text{Irradiance} = \frac{\text{light power incident at the image plane}}{\text{area of the film or CCD or CMOS imager}}$$

$$E_e \propto \frac{\text{area of aperture}}{\text{area of image}} = \frac{D^2}{d^2} = \left(\frac{D}{f}\right)^2$$

Since image size is proportional to the focal length of the lens

$f - number$ or relative aperture of a lens: $A = f / D \rightarrow E_e \propto \frac{1}{A^2}$



F-number and irradiance

Selectable apertures in cameras usually provide steps that change irradiance by a factor of 2, the corresponding $f-number$ changes by a factor of $\sqrt{2}$.

Larger $f-number \rightarrow$ smaller exposure (J/m^2)

$$\text{Total exposure} = irradiance\left(\frac{J}{m^2 \cdot s}\right) \times time(s)$$

So for a given film speed or ISO-number variety of f - $number$ and $shutter\ speed$ combinations can provide satisfactory exposure.

Example:

Shutter speed: $shs_1 = \frac{1}{50s}$ & $A_1 = 8$

$$\text{Total exposure} = \left(\frac{1}{8}\right)^2 \frac{1}{50} = 1.28$$

Find equivalent $f\#$ for $shs_2 = \frac{1}{100s}$

Speed is half so E_e has to be twice as much:

$(1/A_2)^2 = 2(1/A_1)^2$

$1/A_2 = \sqrt{2}(1/A_1) = 1.4(1/8) \rightarrow \boxed{A_2 = 5.6}$

Or we could use the table to pick the next f/stop.

TABLE 3-2 STANDARD RELATIVE APERTURES AND IRRADIANCE AVAILABLE ON CAMERAS

$A = f$-number	$(A = f\text{-number})^2$	E_e
1	1	E_0
1.4	2	$E_0/2$
2	4	$E_0/4$
2.8	8	$E_0/8$
4	16	$E_0/16$
5.6	32	$E_0/32$
8	64	$E_0/64$
11	128	$E_0/128$
16	256	$E_0/256$
22	512	$E_0/512$

Aperture size decreases ↓

Irradiance decreases ↓

Aperture size and depth of field

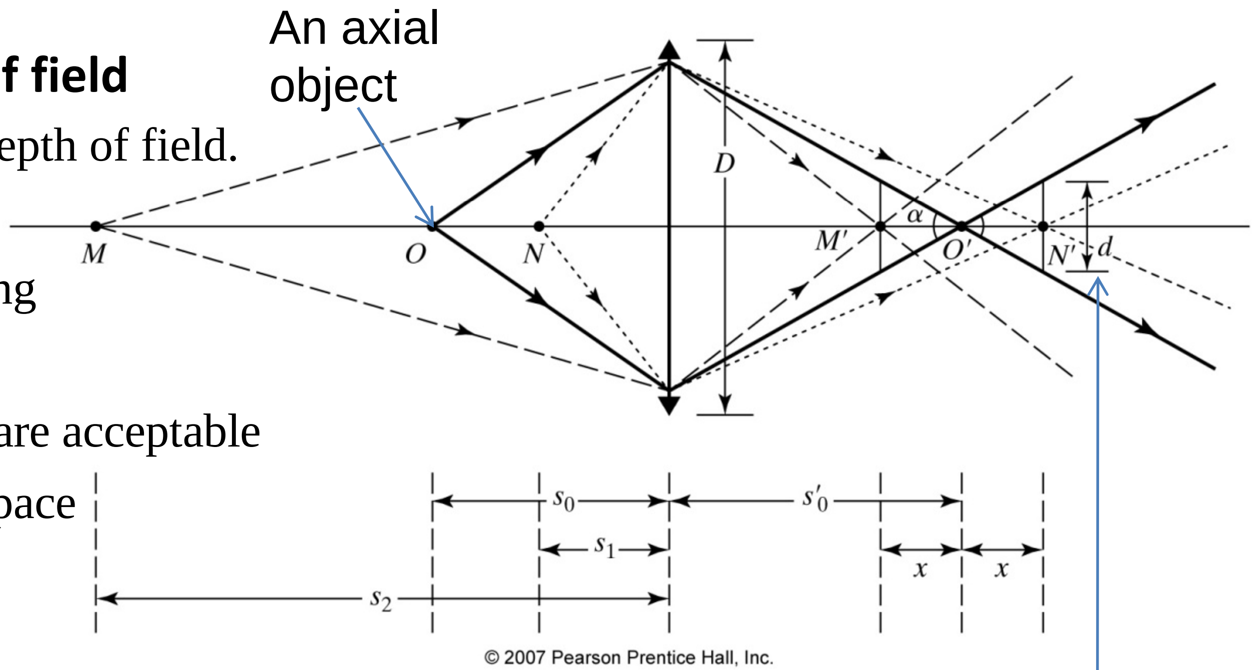
Apertur selection affects the depth of field.

d: the largest acceptble image point diameter depending on the desired image quality.

x: distance of the images that are acceptable

MN: depth of field in object space

M'N': conjugateof the depth of field in image space



Blurring diameter d: the largest acceptable image size for a point object

Near-point s_1 of the depth of field is the object for image at $s'_0 + x$

Far-point s_2 of the depth of field is the object for image at $s'_0 - x$

$$\tan\alpha \cong \frac{D}{2s'_0} \text{ and } \tan\alpha \cong \frac{d}{2x} \rightarrow x \cong \frac{ds'_0}{D}$$

$$s_1 = \frac{s_0 f\left(f + Ad\right)}{f^2 + Ads_0} \text{ and } s_2 = \frac{s_0 f\left(f - Ad\right)}{f^2 - Ads_0}$$

Where $A = f / D$ and the depth of the field $MN = s_2 - s_1$

$$\boxed{\text{depth of the field} = \frac{2Ads_0\left(s_0 - f\right) f^2}{f^4 - A^2 d^2 s_0^2}}$$

Aperture size and depth of field (continued)

$$\frac{1}{s_0}+\frac{1}{s'_0}=\frac{1}{f}\rightarrow s'_0=\frac{fs_0}{s_0-f} \text{ with } x\cong\frac{ds'_0}{D} \text{ and } A=\frac{f}{D}$$

$$\frac{1}{s_1}+\frac{1}{s'_0+x}=\frac{1}{f}\rightarrow s_1=\frac{f\left(s'_0+x\right)}{s'_0+x-f}=\frac{f\left(s'_0+\frac{ds'_0}{D}\right)}{s'_0+\frac{ds'_0}{D}-f}=\frac{fs'_0\left(1+\frac{Ad}{f}\right)}{s'_0\left(1+\frac{Ad}{f}-\frac{f}{s'_0}\right)}=\frac{f\left(1+\frac{Ad}{f}\right)}{\left(1+\frac{Ad}{f}-\frac{f\left(s_0-f\right)}{fs_0}\right)}$$

$$s_1=\frac{\left(f+Ad\right)}{\frac{1}{fs_0}\left(\frac{Ads_0}{1}+\frac{f^2}{1}\right)}\rightarrow \boxed{s_1=\frac{s_0 f\left(f+Ad\right)}{f^2+Ads_0}}$$

$$\frac{1}{s_2}+\frac{1}{s'_0-x}=\frac{1}{f}\rightarrow s_2=\frac{f\left(s'_0-x\right)}{s'_0-x-f}=\frac{f\left(s'_0-\frac{ds'_0}{D}\right)}{s'_0-\frac{ds'_0}{D}-f}=\frac{fs'_0\left(1-\frac{Ad}{f}\right)}{s'_0\left(1-\frac{Ad}{f}-\frac{f}{s'_0}\right)}=\frac{f\left(1-\frac{Ad}{f}\right)}{\left(1-\frac{Ad}{f}-\frac{f\left(s_0-f\right)}{fs_0}\right)}$$

$$s_2=\frac{\left(f-Ad\right)}{\frac{1}{fs_0}\left(-\frac{Ads_0}{1}+\frac{f^2}{1}\right)}\rightarrow \boxed{s_2=\frac{s_0 f\left(f-Ad\right)}{f^2-Ads_0}}$$

Where $A=f/D$ and the depth of the field $MN=s_2-s_1$

$$\boxed{\text{depth of the field}=\frac{2Ads_0\left(s_0-f\right)f^2}{f^4-A^2d^2s_0^2}}$$

Example 3.3 calculating depth of field

A 5 cm focal length lens with f/16 aperture is used to image an object 9 ft away. The blurring diameter in the image is chosen to be d=0.04 mm. determine the location of the near point and far point and the depth of the field.

The data: $s_0 = 9\,ft \simeq 275cm; \quad d = \ 0.004\ ; \quad f = 5cm;$

$f\,/16 \rightarrow A = 16 = f\,/\,D \ \rightarrow D = 5/16 = 0.3125$

Find the near point: $s_1 = \dfrac{s_0 f\left(f + Ad\right)}{f^2 + Ads_0}$ we have all the parameters on the RHS $\rightarrow$

$\boxed{s_1 = 163.5cm = 4.45\,ft}$

Find the far point: $s_2 = \dfrac{s_0 f\left(f - Ad\right)}{f^2 - Ads_0}$ we have all the parameters on the RHS $\rightarrow$

$\boxed{s_2 = 1103cm = 36.19\,ft}$

Depth of the field $= s_2 - s_1 = 1103 - 163.5 = 939.5cm \simeq 30.824\,ft$

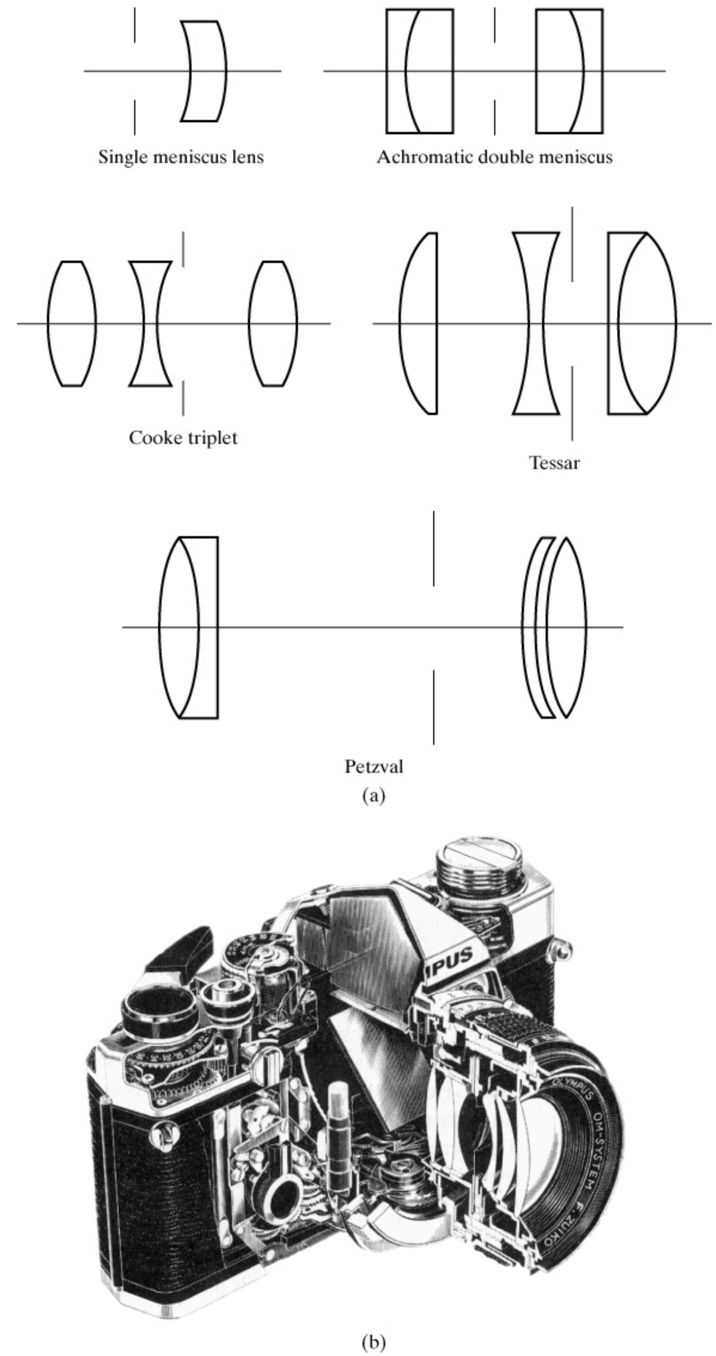
$\boxed{\text{Depth of the field} = 30.824\,f}$

As we select different apertures and change f# then we affect the depth of field. Cameras have a scale for depth of field. The depth of field $\dfrac{2Ads_0\left(s_0 - f\right)f^2}{f^4 - A^2d^2s_0^2}$ increases with f# or A i.e. decreases with the aperture size D since $A = f\,/\,D$

Requirements on camera lenses

- Large field of view 35^o-65^o for normal lenses and 120^o for wide angle lenses.
- Free from aberration over entire area of the film at focal plane.
- All 5 Seidle aberrations (spherical, coma, curvature of the field, astigmatism, and distortion) plus chromatic must be corrected.
- Computational techniques have relaxed some of these requirements but still designing a good cameral lens requires human ingenuity.
- Usually there is more than one solution. The choice depends on compromises.

Various stages of a camera lens design

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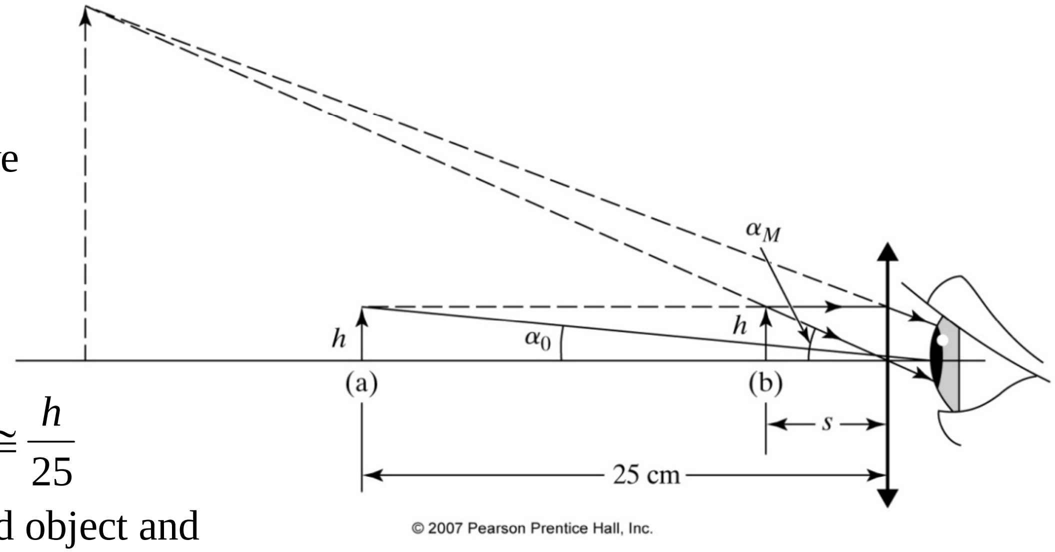
Magnifiers and eyepieces paraxial treatment



Simple magnifier: essentially is a **positive lens used to read fine prints**.

Near point of the eye: 25 cm for a healthy eye

Far point of the eye: ∞ for a healthy eye

For an object of height h at the near point of the eye,

the angular magnification of the eye is: $\alpha_{\mathrm{eye}} \cong \frac{h}{25}$

We insert a positive lens between the eye and object and place the object at or just inside the focal point of the lens.

The angular magnification of the lens is: $\alpha_{\mathrm{L}} \cong \frac{h}{s}$

Angular magnification of a simple magnifier: $M = \frac{\alpha_L}{\alpha_{eye}}$

If the virtual image is at infinity, the total magnification is $M = \frac{\alpha_L}{\alpha_{eye}} \cong \frac{h/f_L}{h/25} \rightarrow \boxed{M_\infty = \frac{25}{f_L}}$

If the virtual image is at the near point of the eye, the total magnification is $M = \frac{\alpha_L}{\alpha_{eye}} \cong \frac{h/s}{h/25}$

where $s = \frac{25f}{25+f}$ then $M = \frac{\alpha_L}{\alpha_{eye}} \cong \frac{25}{s} = \frac{25(25+f)}{25f} \rightarrow \boxed{M_{25} = \frac{25}{f} + 1}$

Magnifiers and eyepieces paraxial treatment

Occulars or *eyepieces* are lenses that aid the eye in viwing images formed by other components.

A magnifier is an occular.

From $2\times$ up to $10\times$ we may get acceptable images with a simple magnifier.

For higher magnifications we need to take the aberrations serious.

One major correction is the transverse chromatice aberration.

For two thin lenses with separation L (chapter 18), $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{L}{f_1 f_2}$

Using the lensmakers equation:

$$\frac{1}{f_1}=(n-1)\left(\frac{1}{R_{11}}-\frac{1}{R_{12}}\right)=(n-1)K_1;\quad \frac{1}{f_2}=(n-1)\left(\frac{1}{R_{21}}-\frac{1}{R_{22}}\right)=(n-1)K_2$$

$$\frac{1}{f}=(n-1)K_1+(n-1)K_2-LK_1K_2(n-1)^2$$

To eliminate the chromatic aberration we require the f to be independent of index of refraction n

$$\frac{d(1/f)}{dn}=K_1+K_2-2LK_1K_2(n-1)=0\rightarrow L=\frac{1}{2}\left[\frac{1}{K_1(n-1)}+\frac{1}{K_2(n-1)}\right]\rightarrow \boxed{L=\frac{1}{2}(f_1+f_2)}$$

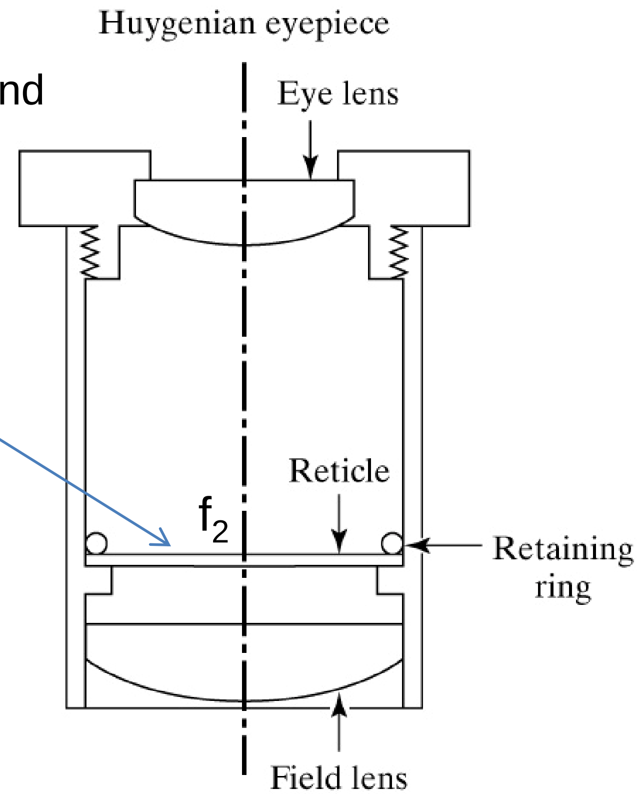
This condition is independent of the lens shapes so we can use the shpe to correct the other aberrations. For example a minimum Coddington factor for minimizing the spherical aberration etc.

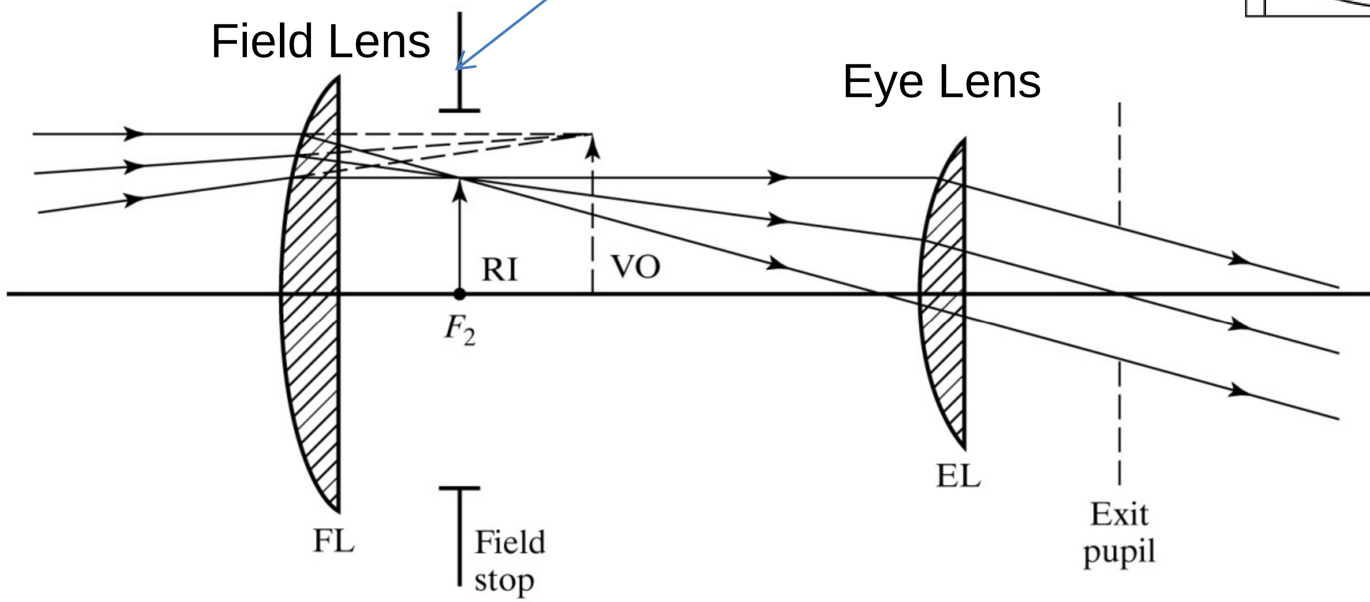
Huygens eyepiece

Note **Huygens eyepiece: two plano-convex lenses separated by half the sum of their focal lengths**. It cannot be used as an ordinary magnifier. It is designed to view the virtual objects

f_1=1.7 f_2

Quality of the image and reticle is not the same because only the eye lens participate in imaging of the reticle.



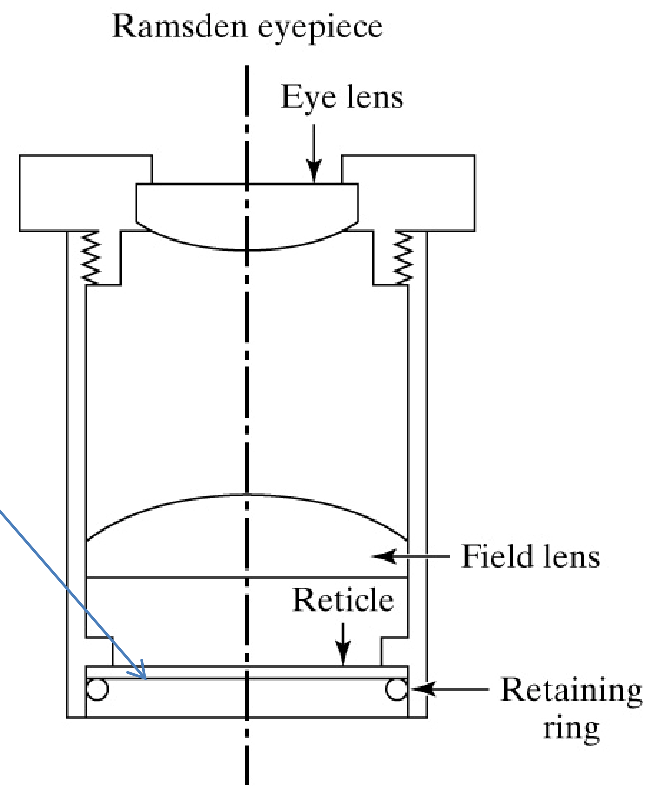


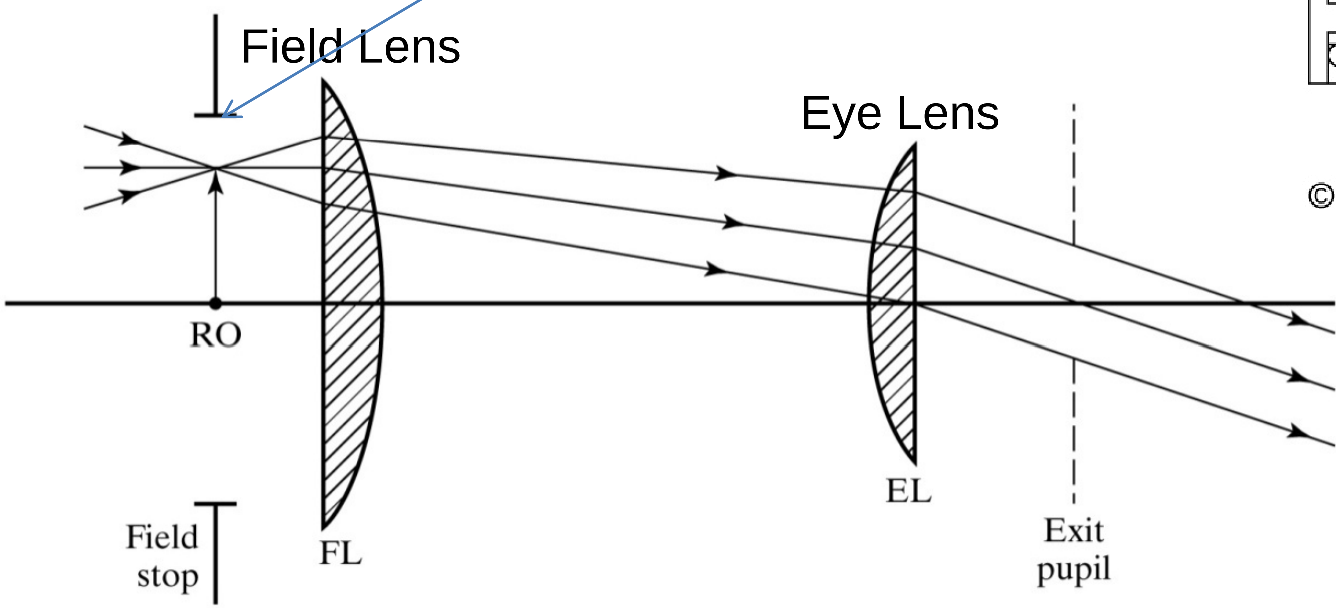
Ramsden eyepiece

Ramsden eyepiece: two plano-convex lenses separated by half the sum of their focal lengths. It can be used as an ordinary magnifier. It is designed to view the real objects

$f_1 = 1.7\, f_2$

Quality of the image and reticle is the same because both the eye lens and field lens participate in imaging of the reticle.





Example 3.4

Huygens eyepiece uses two lenses f1=6.25cm, f2=2.50 cm. Determine their optimum separation for minimum chromatic aberration, their equivalent focal length, their angular magnification when viewing image at infinity.

The optimum separation: $L = \frac{1}{2}\left(f_1 + f_2 \right) = 4.375cm$

The effective focal lengh: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2} \rightarrow f = 3.57cm$

The angular magnification: $M = \frac{25}{f} = \frac{25cm}{3.57cm} = 7$

We usually match the exit pupil to the size of the eye pupil so the radiance is not lost.

$$\text{magnification} = \frac{\text{diameter of the exit pupil}}{\text{diameter of the entrance pupil}} = \frac{\text{diameter of the eye pupil}}{\text{diameter of the entrance pupil}} = \frac{25}{f} > 1$$

$f = \frac{25 \times \text{diameter of the entrance pupil}}{\text{diameter of the eye pupil}}$ has ro be much less than 25cm near point of the eye to have a meaningful magnification. We can't make the diameter of the eye pupil very small so there is a lower limit for f. Exit pupil is the image of the entrance pupil as formed by occular.

Available eyepieces based on these limitations:

$M = 4\times$ to $25\times$

$f = 6.25$ to $1cm$

Eye releif, the distance from the eye lens to exit pupil: 6 to 25 mm

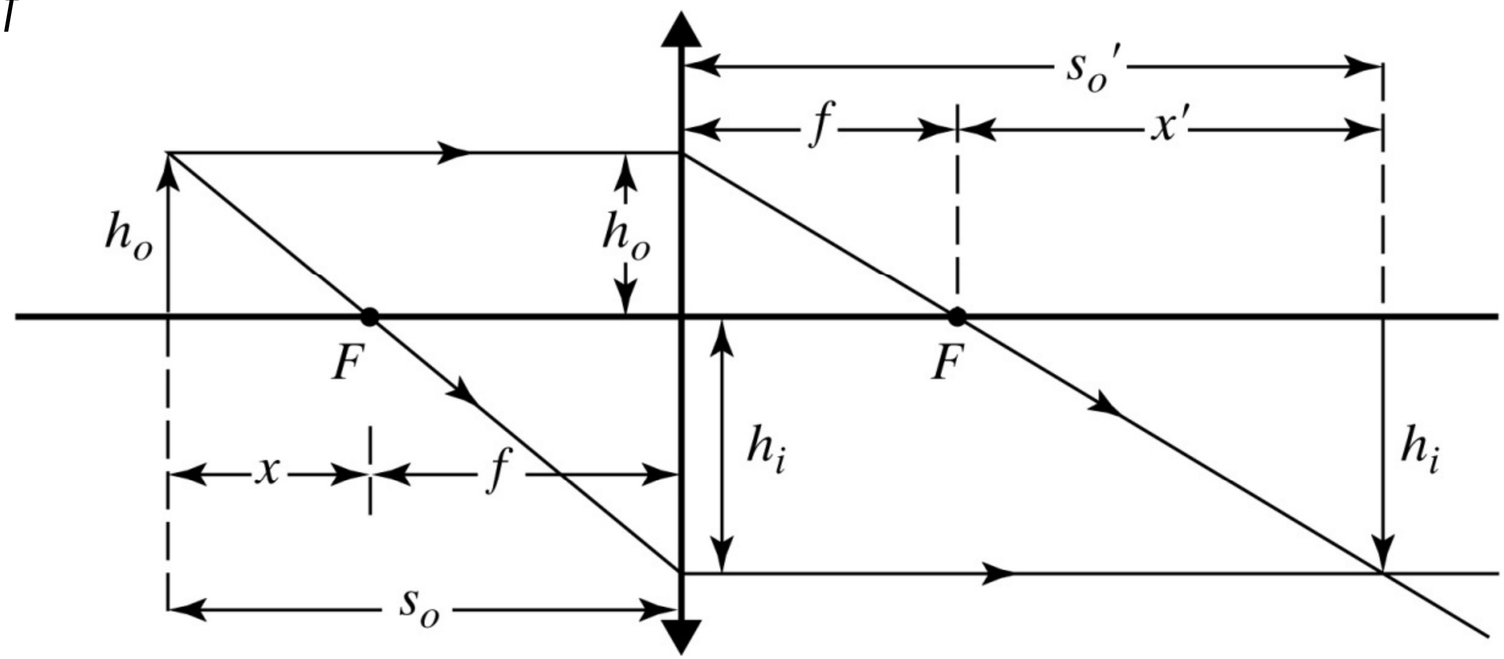
Field of view, size of the primary image that can be covered by the eyepiece: 6 to 30 mm

Newtonian equation for the thin lens

The object and image distances are measured from the focal points like the picture.

The equation is simpler and is used in certain applications

$$m = \frac{h_i}{h_o} = \frac{f}{x} = \frac{x'}{f} \rightarrow xx' = f^2$$



Microscopes

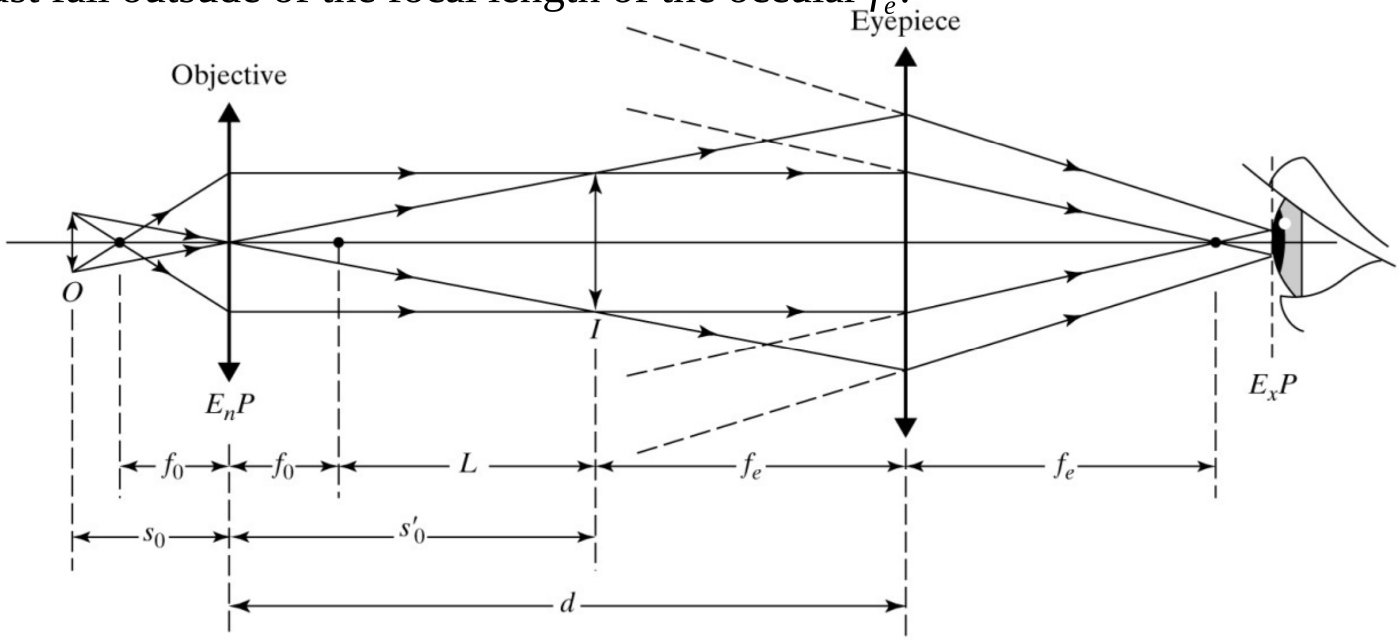
Compound microscopes are used for viewing very small objects when magnification of the simple magnifiers is not enough. It is composed of two lenses

Objective: a small focal length positive lens, forms a real image of the object.

Eyepiece: a magnifier that creates a larger virtual image from the real image formed by the objective.

For viewing the image with human eye, a virtual image is requird so the real image of the objective must be at the focal point or inside of the focal length of the eyepiece f_e.

For capturing the image on a camera, the image from eyepiece must be real thus the intermediate image must fall outsude of the focal length of the occular f_e.



Magnification of the compound microscope

Magnification of the compound microscope when viewing the object at infinity:

Total magnification $M = \dfrac{25}{f_{eff}}$

$$\frac{1}{f_{eff}} = \frac{1}{f_o} + \frac{1}{f_e} - \frac{d}{f_o f_e} = \frac{f_e - f_o - d}{f_o f_e}$$

$$\Bigg\} \; M = \frac{25\left(f_e - f_o - d\right)}{f_o f_e}$$

$$\frac{1}{s_0} + \frac{1}{s'_o} = \frac{1}{f_0} \rightarrow s_o = \frac{f_o s'_o}{s'_o - f_0} \rightarrow \frac{s'_o}{s_o} = \frac{d - f_e - f_o}{f_o} \quad \text{where } s'_o = d - f_e$$

$$\Bigg\} \; M = -\underbrace{\left(\frac{s'_o}{s_o}\right)}_{\text{Linear magnification of the objective}} \overbrace{\left(\frac{25}{f_e}\right)}^{\text{Angular magnification of the eyepiece}}$$

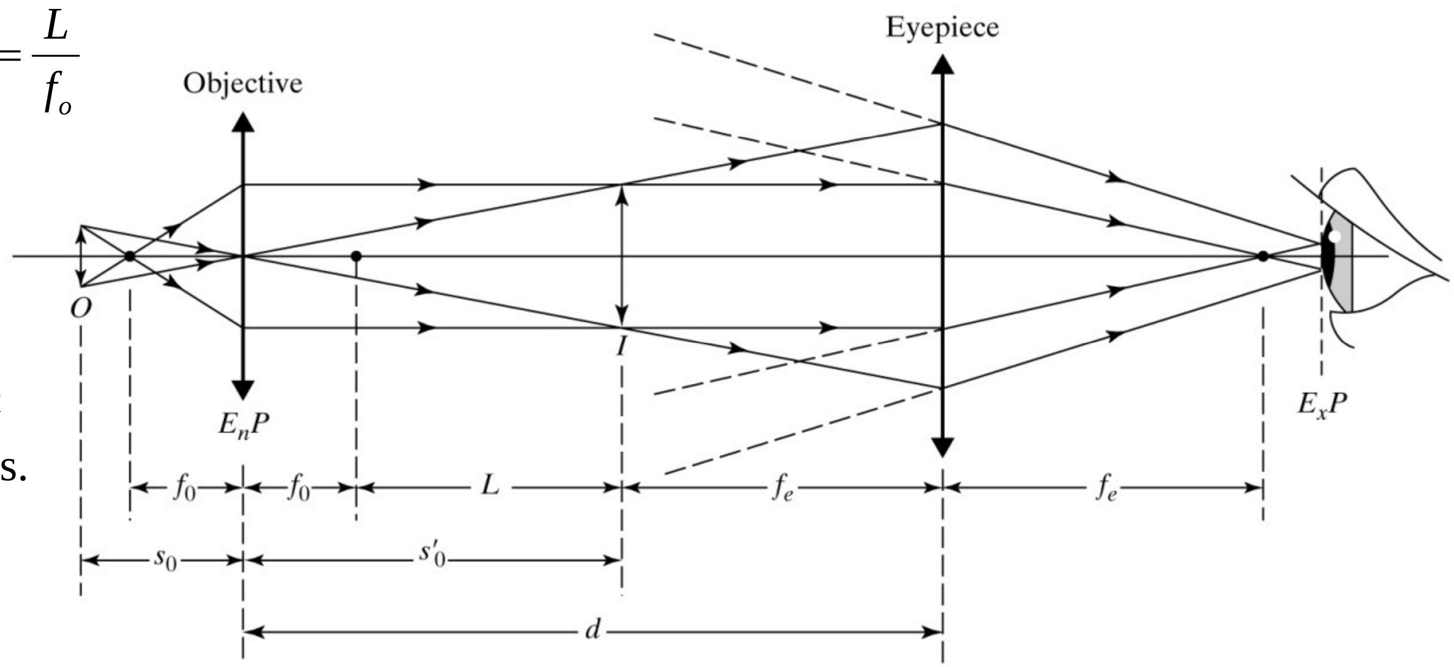
Using the Newton's equation for a thin lens for magnitude of the lateral magnification we get:

$$|m| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s'_o}{s_o}\right| = \frac{x'}{f_o} = \frac{L}{f_o}$$

then we can write:

$$\boxed{M = -\left(\frac{25}{f_e}\right)\left(\frac{L}{f_o}\right)}$$

Standard of $L = 16cm$ for many microscopes.



Example 3.5

A microscope has an objective of 3.8 cm focal length. If the distance between the lenses is 16.4 cm, find the magnification of the microscope.

$$L = d - f_o - f_e = 16.4 - 3.8 - 5 = 7.6cm$$

$$M = -\left(\frac{25}{f_e}\right)\left(\frac{L}{f_o}\right) = -\left(\frac{25}{5}\right)\left(\frac{7.6}{3.8}\right) = -10$$

A magnification of $10\times$

Numerical aperture

For producing brighter images we need to maximize light collection by an optical system. Admittance of light is limited by the aperture stop. To produce bright images AS must be as large as possible.

As M $\uparrow$, forcal lengths & diameters of the objectives $\downarrow$, solid angle of useful rays from the object $\downarrow$
By increasing the refractive index of the object space we can increase the light gathering capability of the objective and produce brighter images (see the figure).

Numerical Aperture is defined as a measure of the light gathering capability of an optical system.

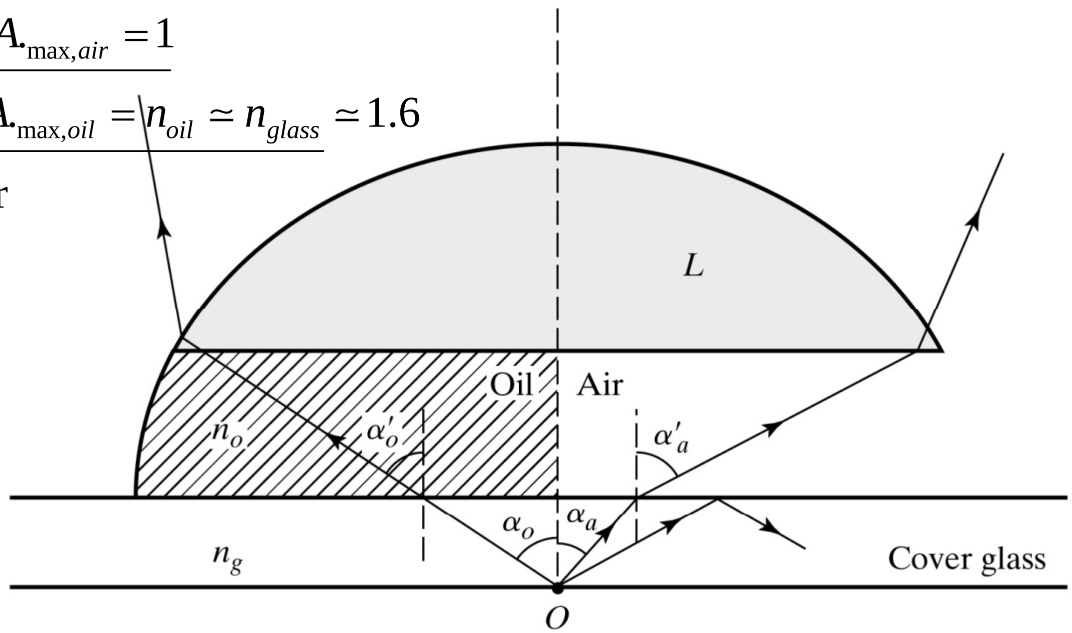
$\boxed{N.A. = n\sin\alpha}$, where α is the half-angle of the cone of light entering the system and n is the index of the object space. The numerical aperture is an invariant in object space due to Snell's law:

$$N.A._{air} = n_{glass}\sin\alpha_{glass} = n_{air}\sin\alpha'_{air} \rightarrow \underline{N.A._{\max,air} = 1}$$

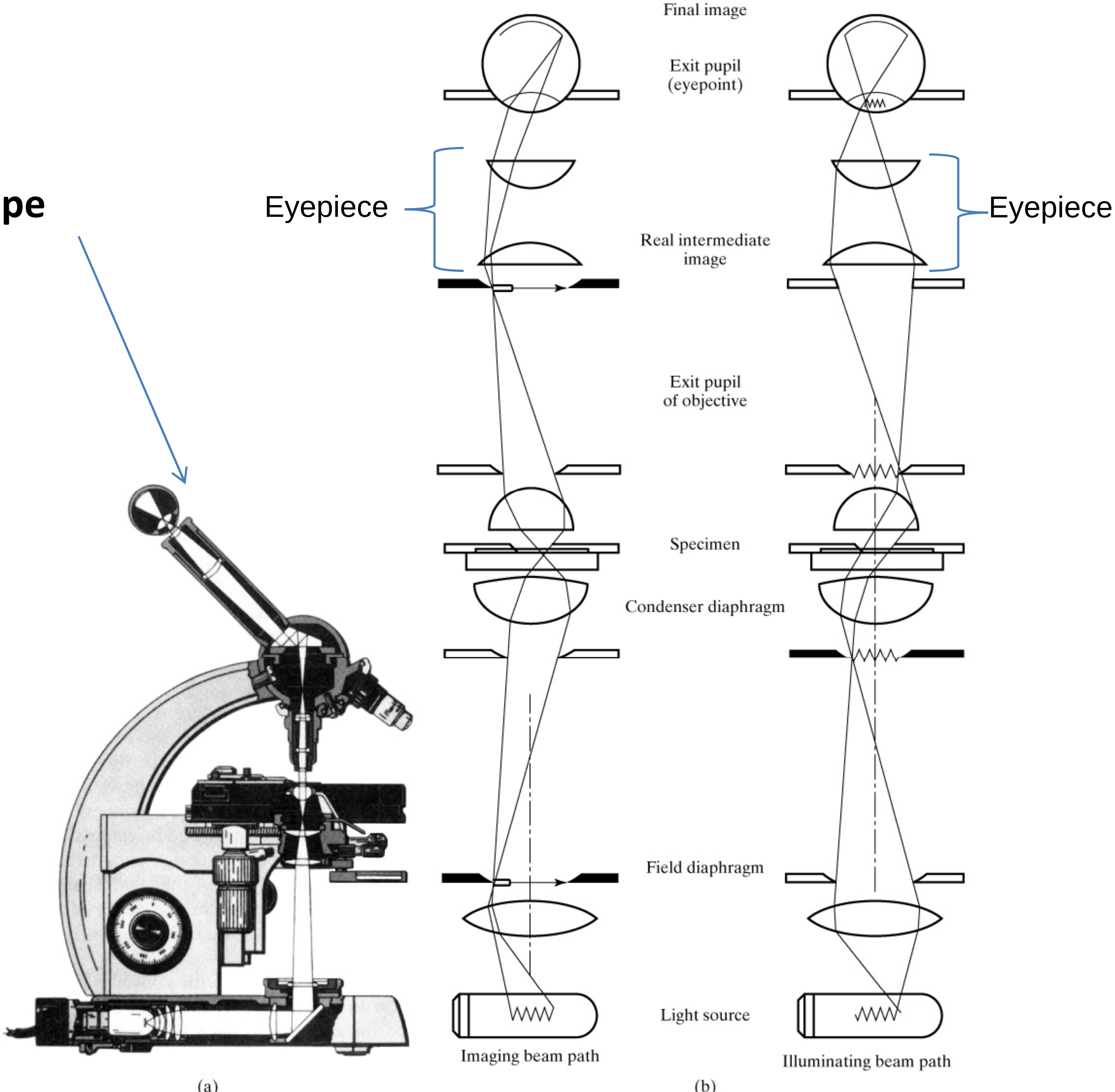
$$N.A._{oil} = n_{glass}\sin\alpha_{glass} = n_{oil}\sin\alpha'_{oil} \rightarrow \underline{N.A._{\max,oil} = n_{oil} \simeq n_{glass}} \simeq 1.6$$

$N.A.$ is an alternative to relative aperture or $f\,\#$ describing "fastness" of a lens.

$$\begin{cases} \text{Image brightness} \propto \dfrac{1}{(f\,\#)^2} \propto (N.A.)^2 \\ \text{Resolving power} \propto N.\text{A.} \\ \text{Depth of focus} \propto 1/N.A. \end{cases}$$

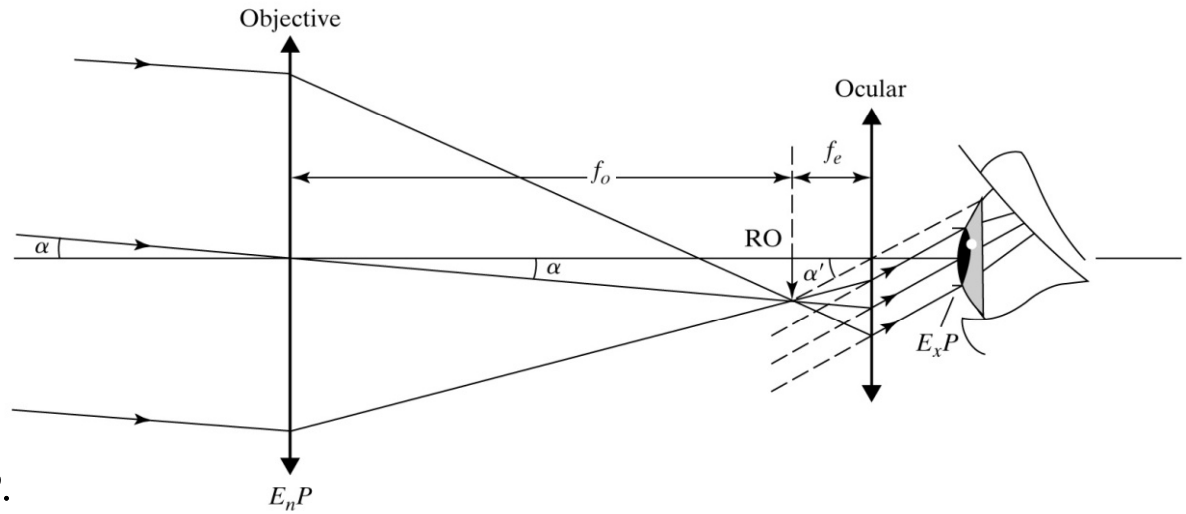


Microscope


Eyepiece
Eyepiece
Final image
Exit pupil
(eyepoint)
Real intermediate
image
Exit pupil
of objective
Specimen
Condenser diaphragm
Field diaphragm
Light source
Imaging beam path
Illuminating beam path
(a)
(b)

Refracting telescopes

Keplerian or astronomical telescope generates inverted image



Parallel distant rays collected by a positive objective and produce a real image at focal point of the ocular.

The angular magnification:

$$M = \frac{\alpha'}{\alpha} = \frac{-f_o}{f_e}$$

Length of the telescope is: $L = f_o + f_e$

The objective lens acts as AS and E_nP.

The $E_xP \geq$ eye pupil,

located just outside of the eyepiece.

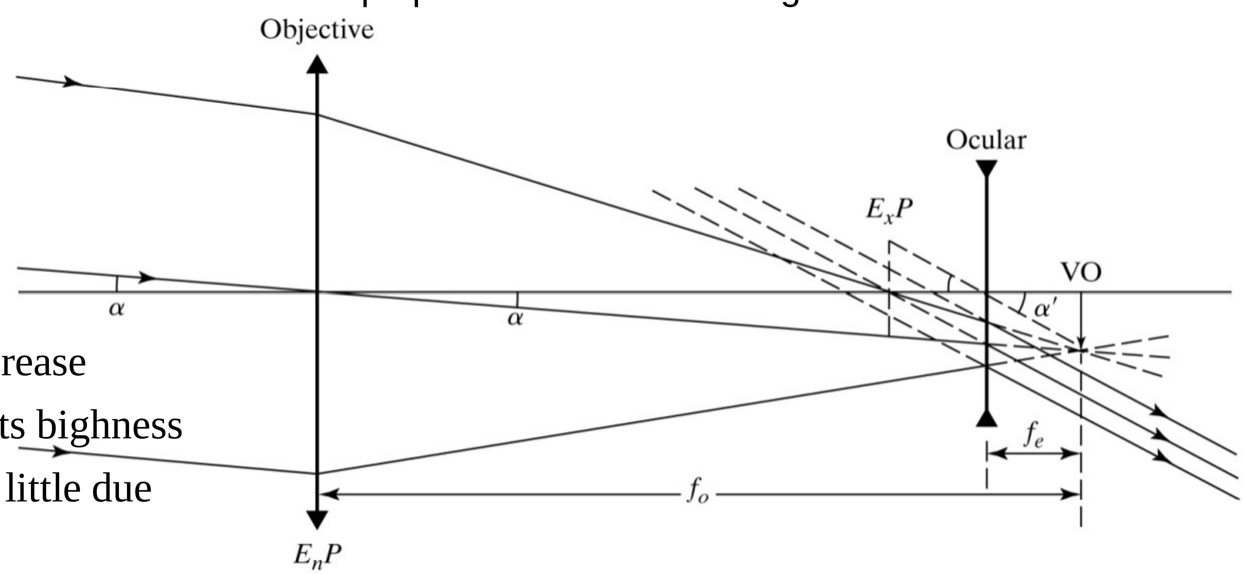
Transverse magnification:

$$\mathrm{m_e} = \frac{h'}{h} = \frac{D_{ExP}}{D_{obj}} = -\frac{f}{x} = -\frac{f_e}{f_o}$$

$$m_e = \frac{1}{M} = \frac{D_{ExP}}{D_{obj}} \rightarrow D_{ExP} = \frac{D_{obj}}{M}$$

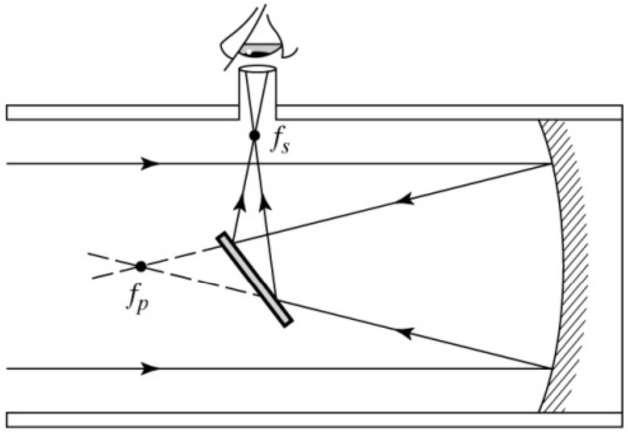
Diameter of the bundle of rays decrease by M but image size increases so its bighness stays the same or even decreases a little due to aberrations and scattering.

Galilean telescope produces an erect image



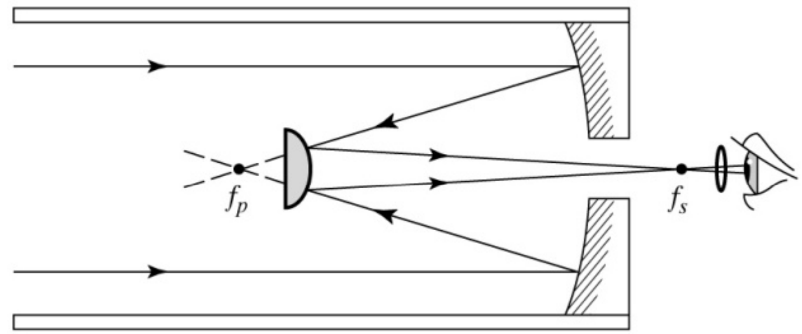
Reflection telescopes
No Chromatic aberration

Newtonian telescope



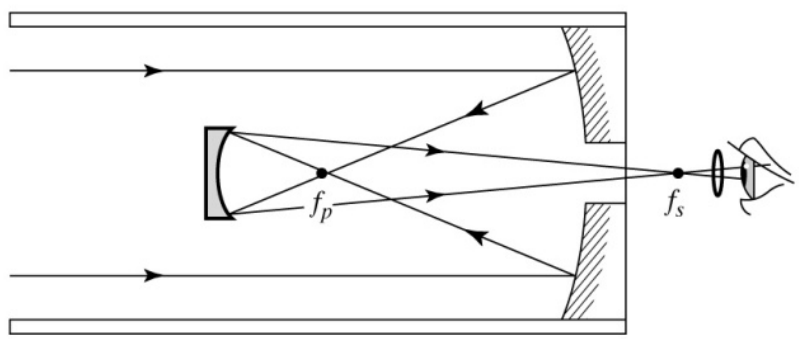
(a)

Cassegrian telescope



(b)

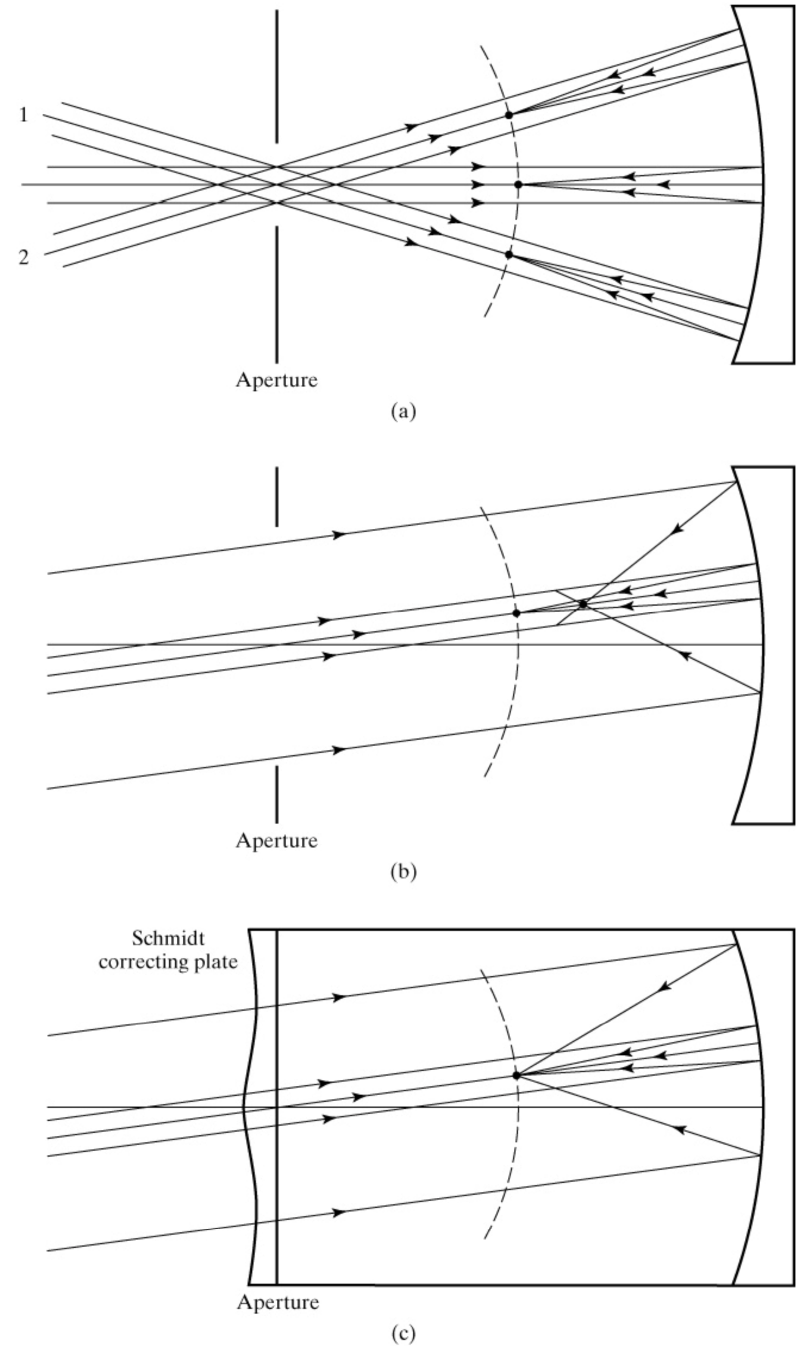
Gregorian telescope

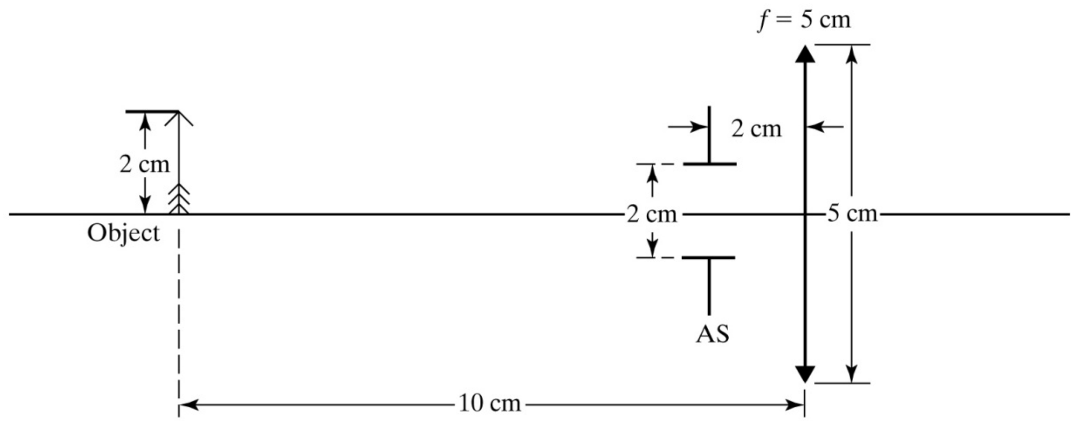


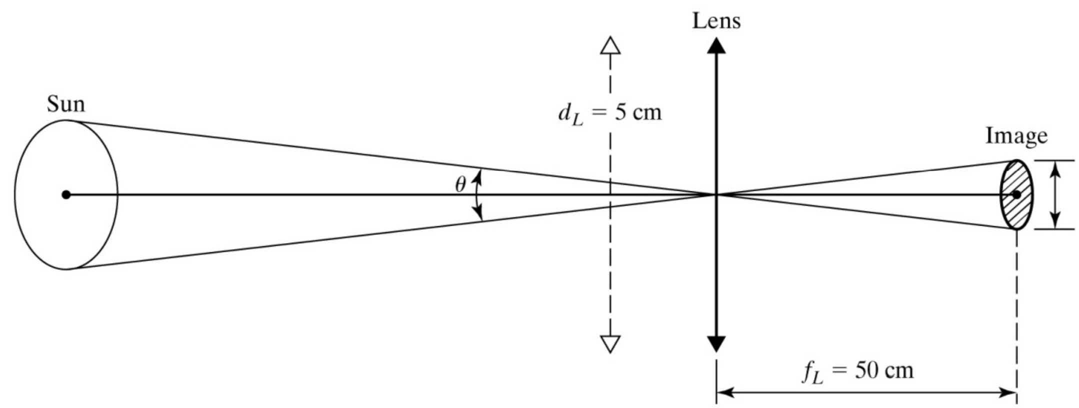
(c)

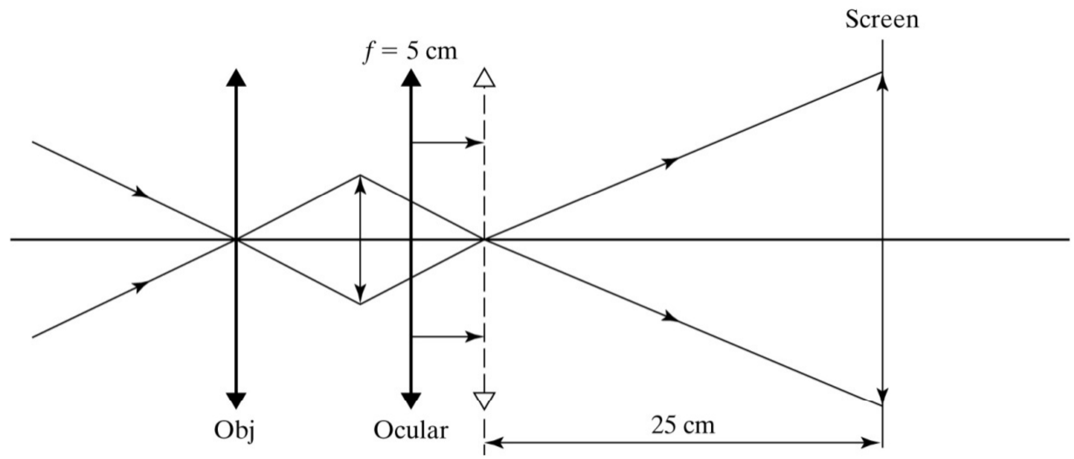
Example 3.6

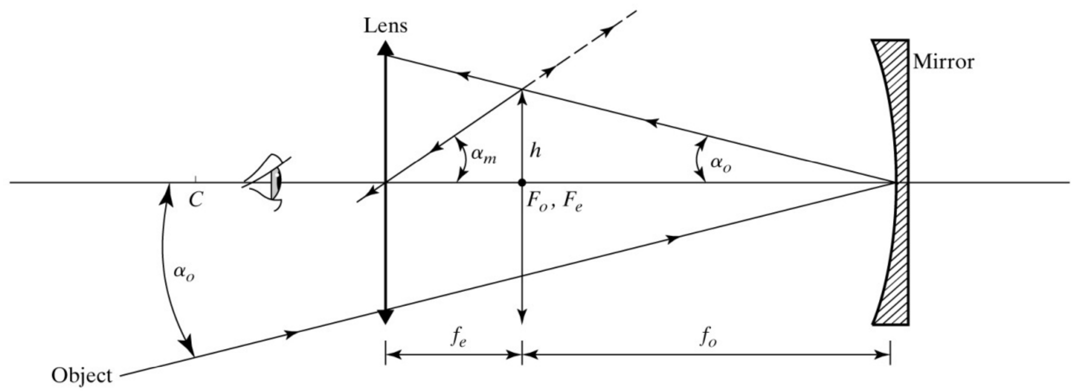
Schmidt telescope

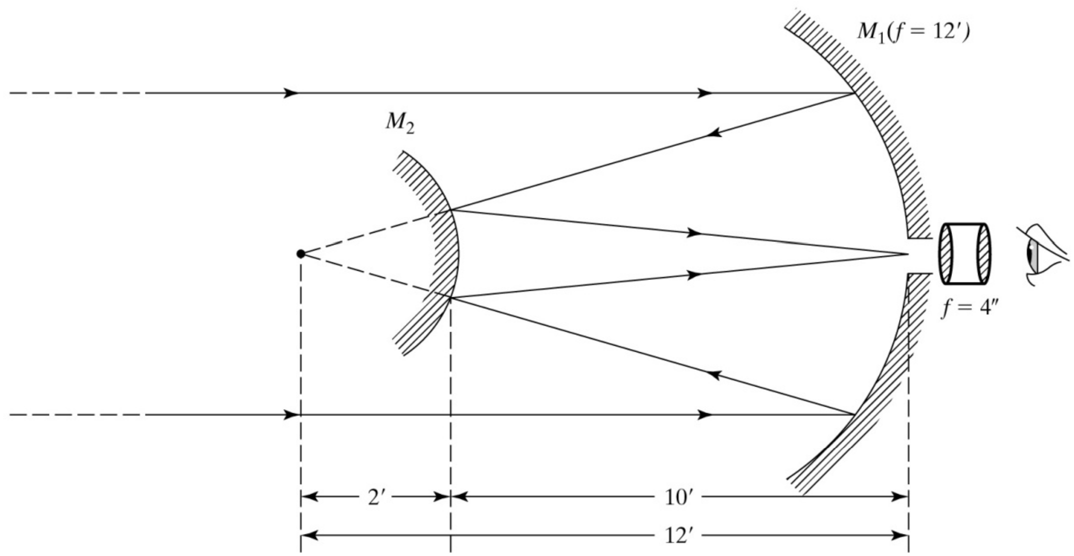



f = 5 cm
2 cm
2 cm
Object
2 cm
5 cm
AS
10 cm


Lens
Sun
d_L = 5 cm
Image
θ
f_L = 50 cm


Screen
f = 5 cm
Obj
Ocular
25 cm


Lens
Mirror
α_m
h
α_o
C
F_o, F_e
α_o
f_e
f_o
Object



$M_1(f = 12')$
M_2
$f = 4''$
2′
10′
12′