Chapter 20
Aberration Theory

Lecture Notes for Modern Optics based on
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Introduction to aberrations

Ga*u*ssian optics or paraxial approximation takes only the first order terms from the sine and cosine expansion :

$$\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} + \cdots \xrightarrow{\textit{First order}} \sin\phi \cong \phi$$

$$\cos\phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} + \cdots \xrightarrow{\textit{First order}} \cos\phi \cong 1$$

By including higher order terms we experience larger departeures from the prefect image known as **aberration**. **Third - order aberration theory** rises from <u>inclusion of the term with third order</u> in expansion of sine. The resulting aberrations are known as or <u>Seidel aberrations</u>.

Seidel aberrations for monochromatic light:

1) Spherical aberration

2) Coma

3) Astigmatism

4) Curvature of field

5) Distortion

For polychromatic light there is an additional aberration:

Chromatic aberration that rises from wavelength dependence of the imaging properties of an optical system or <u>wavelength dpendence of the index of refraction</u> or <u>dispersion</u>.

Ray and wave aberrations

Spherical wavefront: resulting from the Gaussian or paraxial approximation.

Actual wavefront: a surface perpendicular to the sufficiently large number of the rays traced by accurarte formulas.

Ray aberrations: are defined based on deviations of actual rays from the ideal Gausssian rays.

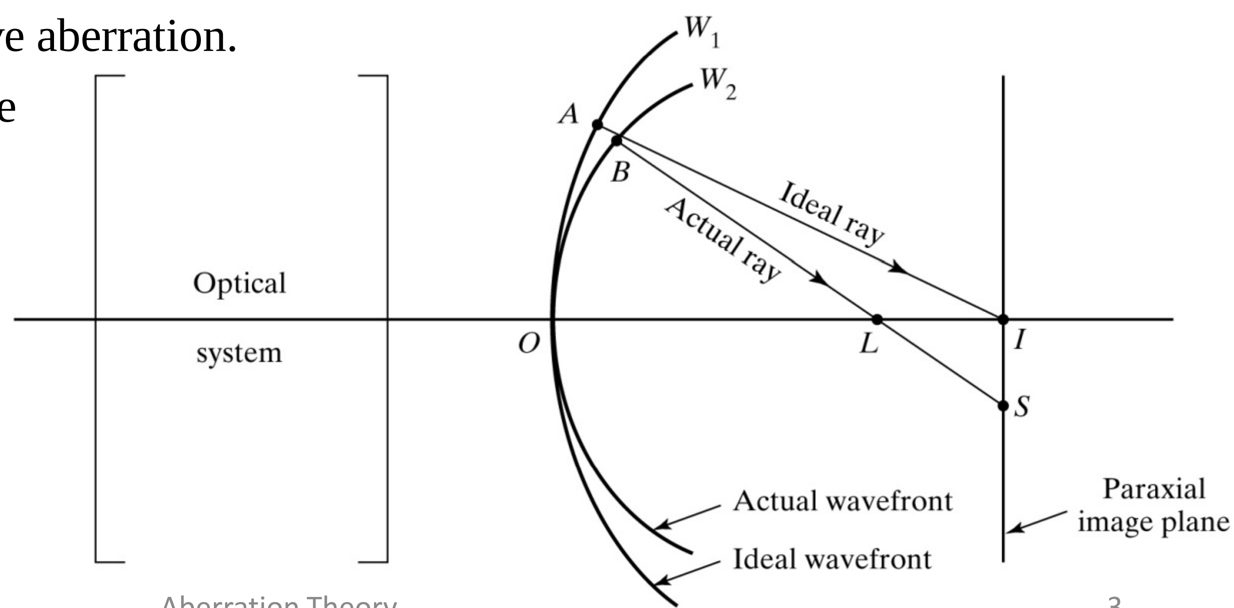
Longitudinal aberration: LI the 'miss' along the optical axis.

Transverse or lateral aberration: IS the 'miss' on the image plane.

Wave aberrations: are defined based on deviations of the deformed wavefront from the ideal Gaussian wavefront at various heights from the optical axis.

In this example AB is the wave aberration.

Goal of optical design: reduce the ray and wave aberrations to their unavoidable limit by diffraction.



Calculating the lateral aberration

Goal: know the variation in AB as the perpendicular distance from the optical axis, y, changes.

da: the incremental wave aberration expressed as optical path difference in the image space

$$da = n_2\left(\alpha dy\right)$$

$\dfrac{da}{dy} = \alpha n_2$ is the local curvature of the ideal wavefronat at P.

The lateral ray aberration b_y and b_x along the plane perpendicular to the z axis is given by:

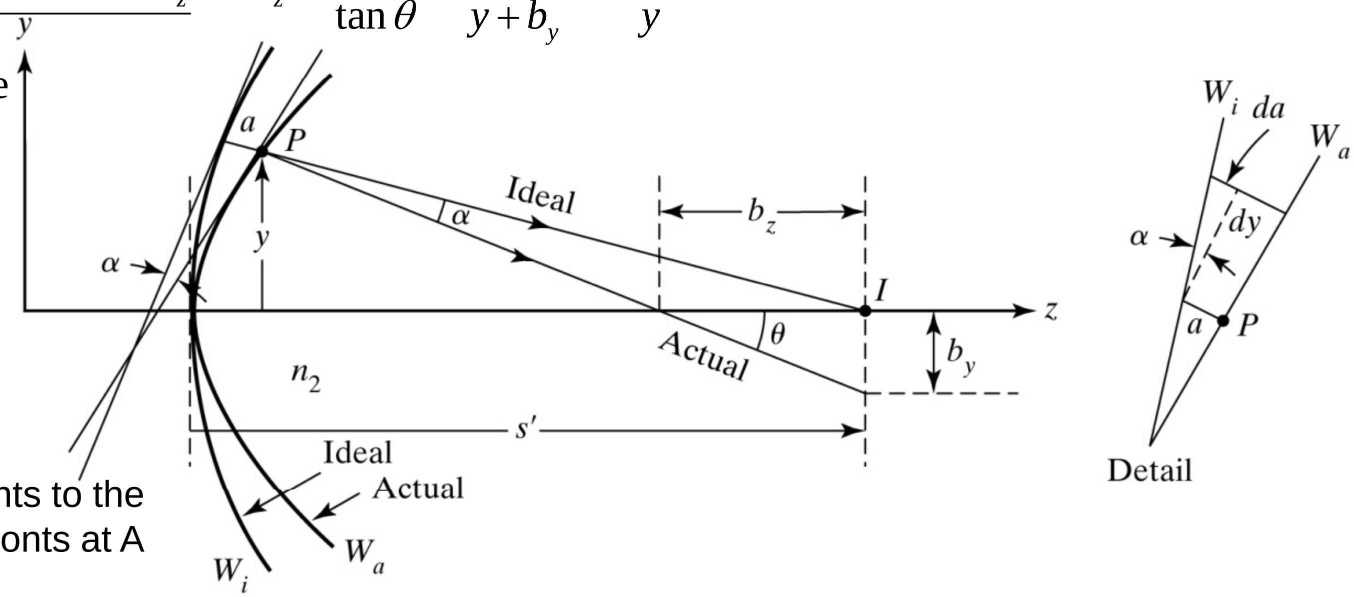
$$b_y = \alpha s' = \frac{s'da}{n_2 dy} \text{ and } b_x = \alpha s' = \frac{s'da}{n_2 dx}$$ where s' is the paraxial image distance from the wavefront.

And the longitudinal ray aberration b_z is: $b_z = \dfrac{b_y}{\tan\theta} = \dfrac{s'b_y}{y+b_y} \cong \dfrac{s'b_y}{y}$

Aberrations in terms of the ideal image parameters.

$$\boxed{\begin{aligned} b_x &= \frac{s'da}{n_2 dx} \\ b_y &= \frac{s'da}{n_2 dy} \\ b_z &= \frac{s'^2\, da}{n_2 y dy} \end{aligned}}$$

Tangents to the wavefronts at A and B



Third order treatment of refraction at a spherical interface: axial object points

To the first order approximation the optical path lengths of PQI and POI are identical acording to the Fermat's principle. Beyond the first order approximation the ray path PQI depends on the position of point Q along the spherical surface.

So aberation is defined as:

$$a(Q) = \underbrace{(PQI - POI)_{opd}}_{\text{Zero if there was no aberration}} = (n_1 l + n_2 l') - (n_1 s + n_2 s')$$

In triangles PQC and CQI the l and l' are exactly:

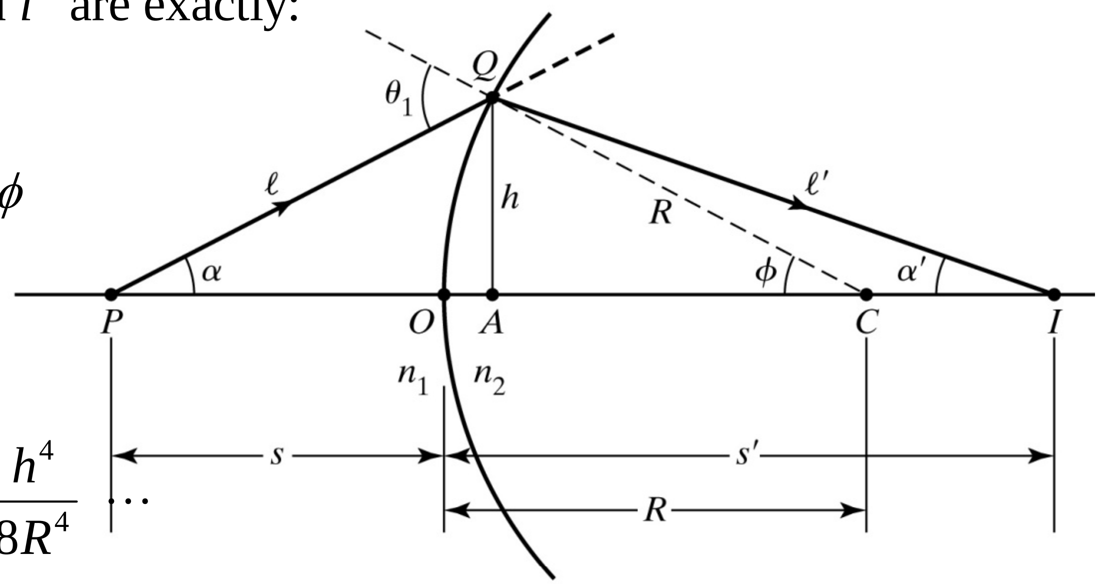
$$l^2 = R^2 + (s+R)^2 - 2R(s+R)\cos\phi$$

$$l'^2 = R^2 + (s'-R)^2 - 2R(s'-R)\cos\phi$$

If we use

$$\cos\phi = \left(1 - \sin^2\phi\right)^{1/2}$$

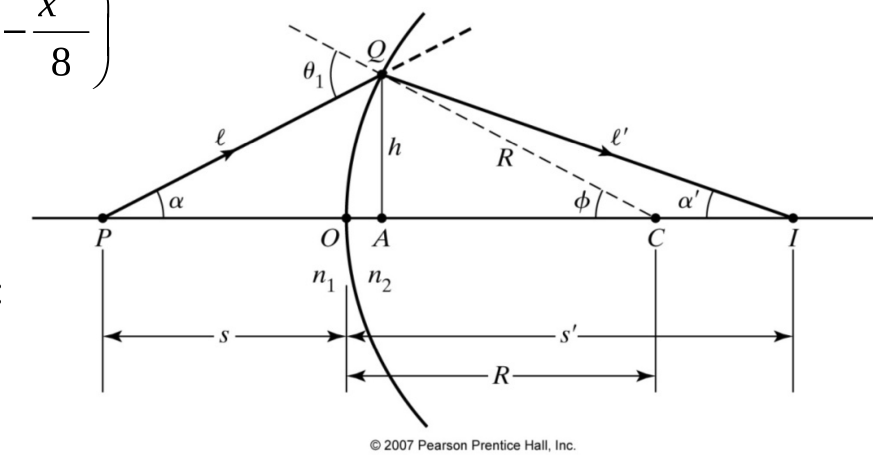
$$\cos\phi = \left[1 - (h/R)^2\right]^{1/2} \cong 1 - \frac{h^2}{2R^2} - \frac{h^4}{8R^4} \cdots$$



Third order treatment of refraction at a spherical interface

$$l = s\left(1+\left[\underbrace{\frac{h^2\left(R+s\right)}{Rs^2}+\frac{h^4\left(R+s\right)}{4R^3s^2}}_{x}\right]\right)^{1/2} \rightarrow l^2 = s\left(1+\frac{x}{2}-\frac{x^2}{8}\right)$$

$$l' = s'\left(1+\left[\underbrace{\frac{h^2\left(R-s'\right)}{Rs'^2}+\frac{h^4\left(R-s'\right)}{4R^3s'^2}}_{x'}\right]\right)^{1/2} \rightarrow l^2 = s\left(1+\frac{x'}{2}-\frac{x'^2}{8}\right)$$



We used $\left(1+x\right)^{1/2} = 1+\frac{x}{2}-\frac{x^2}{8}+\cdots$

When all the terms higher than h^4 are discarded we get:

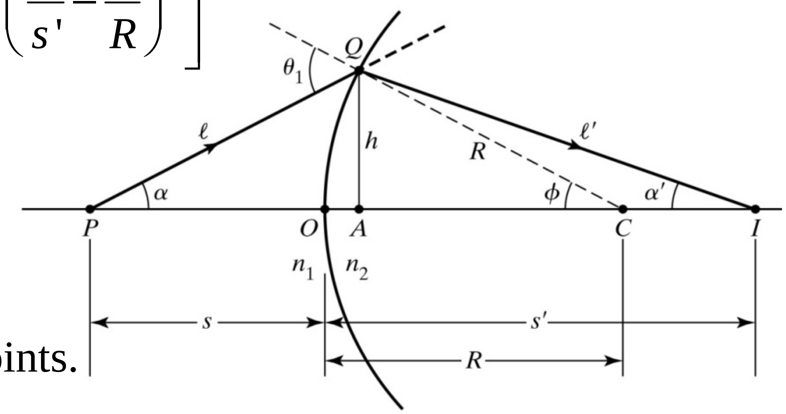
$$l = s\left[1+\frac{h^2\left(R+s\right)}{2Rs^2}+\frac{h^4\left(R+s\right)}{8R^3s^2}-\frac{h^4\left(R+s\right)^2}{8R^2s^4}\right]$$

$$l' = s'\left[1+\frac{h^2\left(R-s'\right)}{2Rs'^2}+\frac{h^4\left(R-s'\right)}{8R^3s'^2}-\frac{h^4\left(R-s'\right)^2}{8R^2s'^4}\right]$$

$$a\left(Q\right) = \left(n_1 s\left[1+\frac{h^2\left(R+s\right)}{2Rs^2}+\frac{h^4\left(R+s\right)}{8R^3s^2}-\frac{h^4\left(R+s\right)^2}{8R^2s^4}\right]+n_2 s'\left[1+\frac{h^2\left(R-s'\right)}{2Rs'^2}+\frac{h^4\left(R-s'\right)}{8R^3s'^2}-\frac{h^4\left(R-s'\right)^2}{8R^2s'^4}\right]\right)$$
$$-\left(n_1 s+n_2 s'\right)$$

Third order treatment of refraction at a spherical interface

$$a(Q)=\frac{h^2}{2}\underbrace{\left[\left(\frac{n_1}{s}+\frac{n_2}{s'}\right)-\left(\frac{n_2-n_1}{R}\right)\right]}_{\text{This is zero according to the Fermat's Principle}}-\frac{h^4}{8}\left[\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2+\frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right]$$



s is the object distance and s' is the ideal image point.

$$a(Q)=-\frac{h^4}{8}\underbrace{\left[\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2+\frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right]}_{\text{Independent of h or system aperture}}$$ axial object points.

For paraxial optics h is small enough so that the aberration $a(Q)$ can be ignored.

Conclusion: according to the prediction of the third-order theory **wave aberration** for axial object points is proportional to the **fourth power of the system aperture**.

$\boxed{a(Q)=ch^4}$ were $c=\frac{1}{8}\left(\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2+\frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right)$ function of system characteristics.

$a(Q)$ is the optical path difference between the actual and ideal rays must correspond to the **wave aberration AB** also known as **spherical aberration** and it is clearly a function of the distance from the optical axis at which the ray intersects the wavefront.

We will use this approach to get the off-axis imaging.

Origin of the other third order aberrations; off-axis object point

Spherical aberration for the axial pencil of rays is proportional to y. Axis of symmetry is OCI.

Spherical aberration for the off-axial (oblique) pencil of rays is proportional to y'.

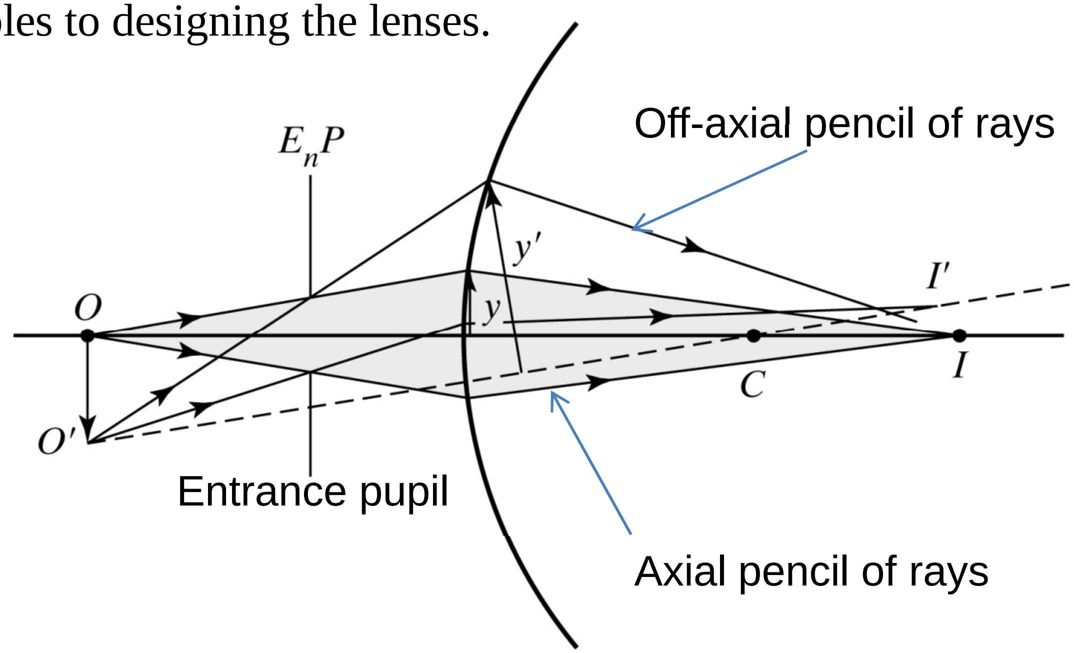
For the oblique pencil the axis of symmetry is $O'CI'$ in the absence of E_nP.

Note: for the same object distance $y' > y$ and since for the

$$\left.\begin{array}{l}\text{aberration for the axial rays } a \propto y^4 \\ \text{aberration for the off-axial rays } a' \propto y'^4\end{array}\right\} \rightarrow$$

oblique pencil of rays is far more susceptible to aberration.

We will see application of these principles to designing the lenses.



Off-axis object point

Consider off-axis pencil of rays from point P.

The aberration function for the point Q on the wavefront: $a'(Q) = (PQP' - PBP')_{opd} = c(BQ)^4 = c\rho'^4$

The aberration function for the point O on the wavefront: $a'(O) = (POP' - PBP')_{opd} = c(BO)^4 = cb^4$

The off-axis aberration function: $a(Q) = a'(Q) - a'(O) = c\rho'^4 - cb^4 = c(\rho'^4 - b^4)$

In $\Delta BOQ \rightarrow \rho'^2 = r^2 + b^2 + 2rb\cos\theta$ In ΔOBC and $SCP' \rightarrow OB = b \propto h' \rightarrow b = kh'$

Replace ρ'^2 and b in $a(Q)$ and regroup all the terms

$$\boxed{a(Q) = {}_0C_{40}r^4 + {}_1C_{31}h'r^3\cos\theta + {}_2C_{22}h'^2 r^2\cos^2\theta + {}_2C_{20}h'^2 r^2 + {}_3C_{11}h'^3 r\cos\theta}$$

The ${}_iC_{jk}$ coenfficients have indecies that are powers of the terms

h': departure from axial image,

r: aperture of the refracting surface,

$\cos\theta$: indicates the symmetry around the optical axis.

Each term comprises one kind of monochomatic aberration or Seidel aberration as follows:

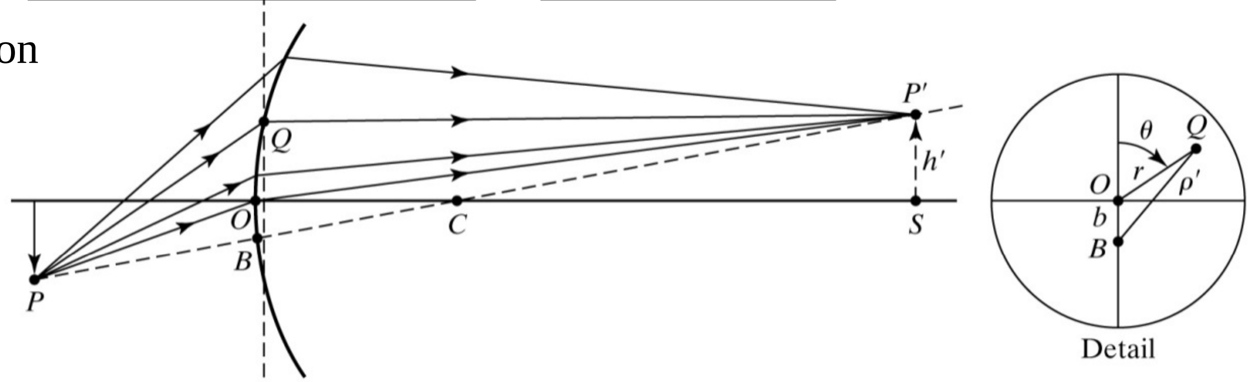
r^4 $\leftarrow$ spherical aberration

$h'r^3\cos\theta$ $\leftarrow$ coma

$h'^2r^2\cos^2\theta$ $\leftarrow$ astigmatism

h'^2r^2 $\leftarrow$ curvature of field

$h'^3r\cos\theta$ $\leftarrow$ distortion



Spherical aberration

Off-axis aberration function

$$\boxed{a(Q) = {}_0C_{40} r^4 + {}_1C_{31} h' r^3 \cos\theta + {}_2C_{22} h'^2 r^2 \cos^2\theta + {}_2C_{20} h'^2 r^2 + {}_3C_{11} h'^3 r \cos\theta}$$

Spherical aberration $\rightarrow {}_0C_{40} r^4$ $\quad r$ is the system aperture.

$h' r^3 \cos\theta \leftarrow$ coma

$h'^2 r^2 \cos^2\theta \leftarrow$ astigmatism

$h'^2 r^2 \leftarrow$ curvature of field

$h'^3 r \cos\theta \leftarrow$ distortion

The only term independent of the h' (departur from axial imaging) so it exists even for paraxial and axial points.

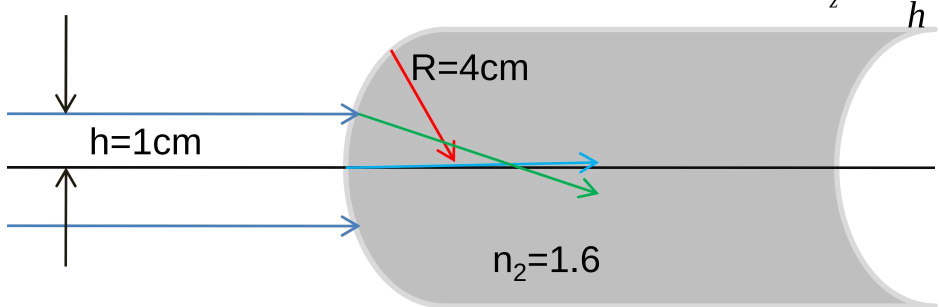
The rays refracted from the extremities of the lens generate two types of spherical aberrations:

$$\left.\begin{aligned} b_y &= \frac{s'}{n_2}\frac{da}{dy} = \frac{s'}{n_2}\frac{da}{dr} \\ a(Q) &= {}_0C_{40} r^4 \rightarrow \frac{da}{dr} = 4\,{}_0C_{40} r^3 \end{aligned}\right\} \boxed{b_y = \frac{4\,{}_0C_{40} s'}{n_2} r^3}$$

$$b_z = \frac{s' b_y}{y} = \frac{s' b_y}{r} \qquad \rightarrow \boxed{b_z = \frac{4\,{}_0C_{40} s'^2}{n_2} r^2}$$

Example

Axially collimated light enters a glass rod through its end. A convex spherical surface of radius 4 cm. The glass rod has a refractive index of 1.6. Determine the longitudinal and lateral spherical ray aberrations for light entering at an aperture height of h=1 cm.



Spherical aberration for axial object points

$$a(Q) = -\frac{h^4}{8}\left[\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2 + \frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right]$$

if s $\to \infty$ then $a(Q) = -\frac{h^4}{8}\left[\frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right]$

We need da/dh to calculate b_y and b_z

$$\frac{da}{dh} = -\frac{4h^3}{8}\left[\frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right]$$

Next we need s' the image distance for the system without aberration.

$$\frac{n_1}{s}+\frac{n_2}{s'} = \frac{n_2-n_1}{R} \to \frac{1}{\infty}+\frac{1.6}{s'} = \frac{0.6}{4} \to s' = 10.667$$

$$\frac{da}{dh} = -\frac{(1)^3}{2}\left[\frac{1.6}{10.667}\left(\frac{1}{10.667}-\frac{1}{4}\right)^2\right] = -0.001831$$

$$b_y = \frac{s'}{n_2}\frac{da}{dh} = \frac{10.667}{1}(-0.001831) = -0.0122cm$$

$$b_z = \frac{s' b_y}{h} = \frac{10.667}{1}(-0.0122) = -0.130cm$$

Spherical aberration of the thin lenses

EI: longitudinal spherical aberration

For a positive lens E falls to the left of I, for a negative lens E falls to the right of I

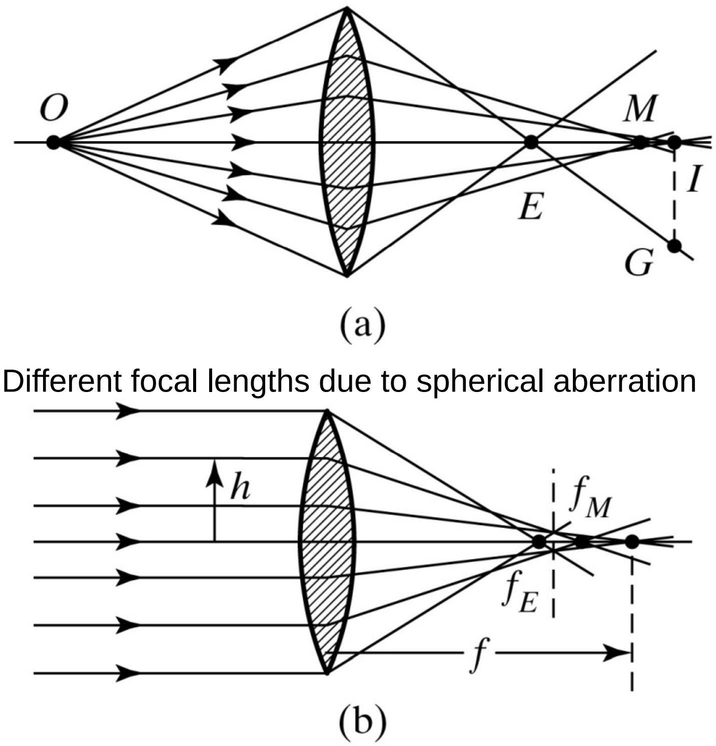
IG: transverse spherical aberration

At point M, somewhere between E and I we have the best focus.

Image of a point at the best focus point

is called the circle of least confusion.

Different image distances due to spherical aberration



Coddington shape factor of a lens

f is defined for the paraxial rays in a thin lens. As $h \to \infty$ then $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$

It is possible to achieve a given f with a different combinations of r_1 and r_2.

We define the Coddington shape factor σ as a measure of bending of a lens.

$\sigma = \dfrac{r_2 + r_1}{r_2 - r_1}$ with the usual sign convention for radii: convex +, concave $-$.

Example: $n = 1.50$ and $f = 10cm$

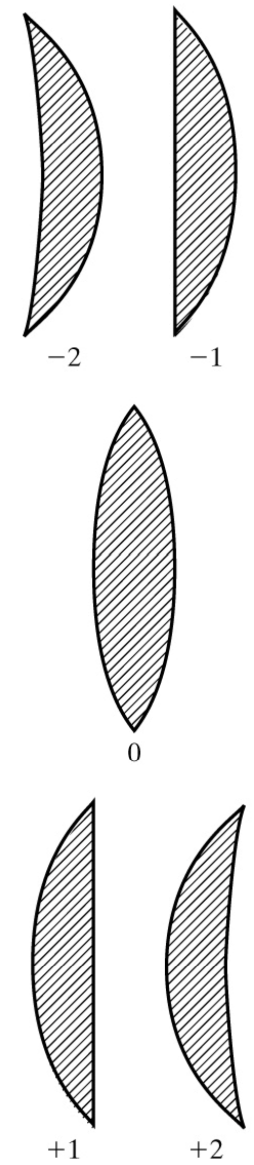
$\sigma = -2 \to r_1 = 10cm,\ r_2 = 3.33cm,$ meniscus

$\sigma = -1 \to r_1 = \infty,\ r_2 = 5cm,$ planoconvex

$\sigma = 0 \to r_1 = 10cm,\ r_2 = -10cm,$ equiconvex

$\sigma = +1 \to r_1 = 5cm,\ r_2 = \infty,$ planoconvex

$\sigma = +2 \to r_1 = 3.33cm,\ r_2 = 10cm,$ meniscus



Minimum spherical aberration condition for bending factor

Spherical aberration of a single spherical refracting surface

$$a(Q) = -\frac{h^4}{8}\left[\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2 + \frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right]$$

A thin lens is a combination of two such surfaces.

Each surface has a contribution to the total aberration.

$s'_h - s'_p$: total longitudinal spherical aberration

s'_h : image distance for a ray at elevation h

s'_p : image distance for a paraxial ray

Spherical aberration: $\frac{1}{s'_h} - \frac{1}{s'_p} = \frac{h^2}{8f^3}\frac{1}{n(n-1)}\left[\frac{n+2}{n-1}\sigma^2 + 4(n+1)p\sigma + (3n+2)(n-1)p^2 + \frac{n^3}{n-1}\right]$

Where $p = \frac{s'-s}{s'+s}$. The minimum spherical aberration results when the bending factor is:

$$\sigma = -\frac{2(n^2-1)}{n+2}p$$

For $s \to \infty$ and $n = 1.50$ we get bending factor $\sigma \cong 0.7$.

This is close to σ of a planoconvex lens σ=+1 with convex side facing the parallel incident rays.

In general there is a possibility of cancelling spherical aberration by using two surfaces that have equal refraction with opposite signs since:

the positive and negative lenses produce spherical aberration of opposite signs.

Coma (resembles comet)

Off-axis aberration function: $\boxed{a(Q) = {}_0C_{40}r^4 + {}_1C_{31}h'r^3\cos\theta + {}_2C_{22}h'^2 r^2\cos^2\theta + {}_2C_{20}h'^2 r^2 + {}_3C_{11}h'^3 r\cos\theta}$

Coma represented by: ${}_1C_{31}h'r^3\cos\theta$

Coma is an off-axial aberration.

$\left.\begin{array}{l} h' \neq 0 \text{ and} \\ \cos\theta \neq \text{constant} \end{array}\right\}$ image is not symmetrical about the optical axis.

Coma rapidly increases with system aperture (r^3).

Zone: a thin annular region of a lens centered at optical axis.

Comatic circle: is created by all the rays arriving from a distant object and passing through a zone.

Radius of the comatic circles increas with radius of the generating zone (figure a).

Figure b: fromation of different comatic circles.

Each zone produces a different magnification.

h_e: magnification due to extreme rays.

h_c: magnification due to central rays.

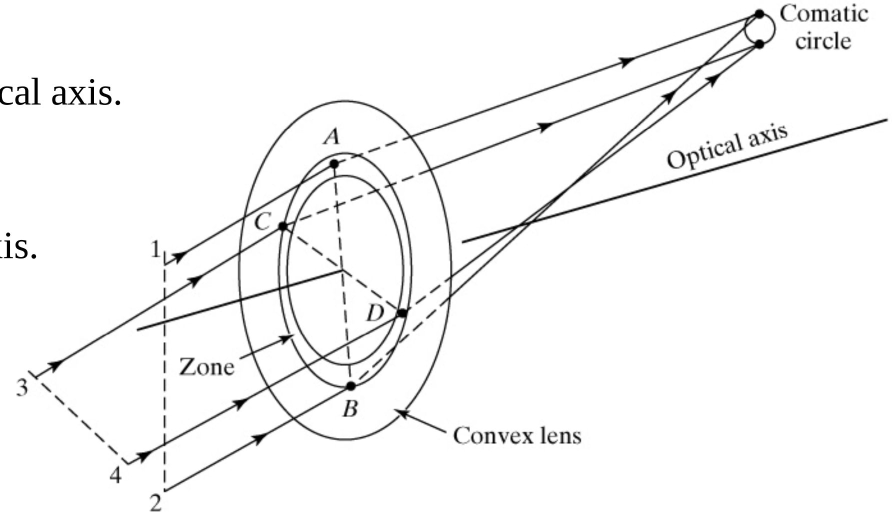
Coma may occur in two forms:

a positive quantity $(h_e > h_c)$

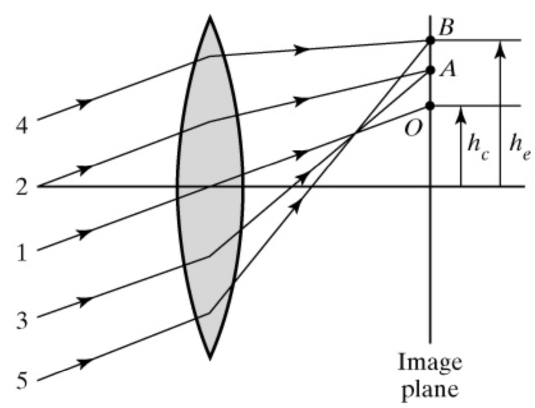
a negative quantity $(h_e < h_c)$

Maximum extent of a comatic image: $3R_e$

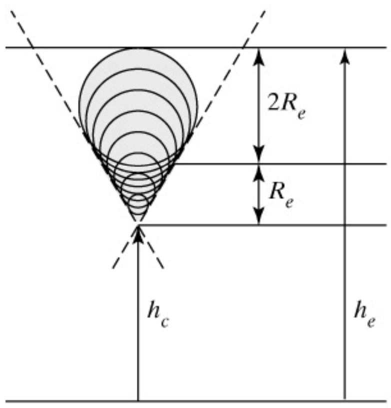
R_e is the radius of the extreme comatic circle.



(a)



(b)



(c)

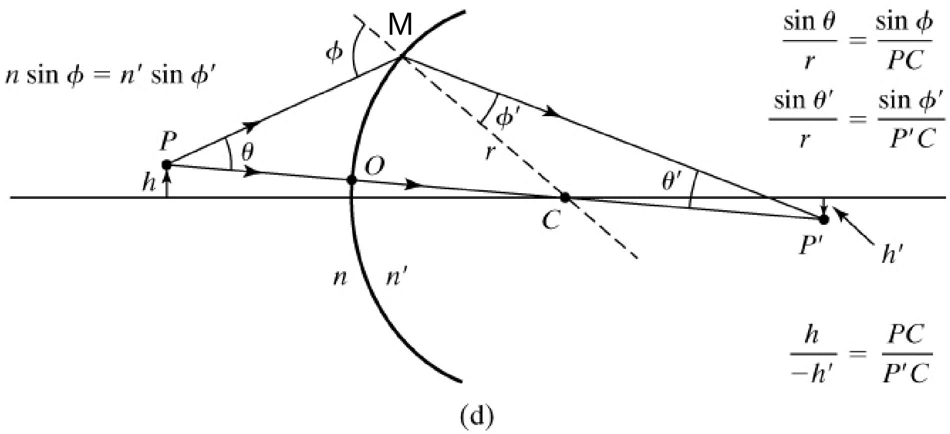
Minimizing coma

For samll objecs near axis, any ray refracted at a spherical surface must satisfy the *Abbe* sine condition:

Snell's law: $n\sin\phi = n'\sin\phi'$

From the law of sins in ΔPCM : $\dfrac{\sin\theta}{r} = \dfrac{\sin(\pi-\phi)}{PC} = \dfrac{\sin\phi}{PC}$

From the law of sins in $\Delta P'CM$: $\dfrac{\sin\theta'}{r} = \dfrac{\sin\phi'}{P'C}$



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$$\frac{\sin\theta'}{r} = \frac{\sin\phi'}{P'C} \xrightarrow{\text{using Snell's law}} \frac{\sin\theta'}{r} = \frac{n\sin\phi}{n'P'C}$$

$$\xrightarrow[\text{in } \Delta PCM]{\text{law of sins}} \frac{\sin\theta'}{r} = \frac{n}{n'P'C}\frac{PC\sin\theta}{r}$$

Also we have $-\dfrac{h}{h'} = \dfrac{PC}{P'C}$

$$\frac{\sin\theta'}{1} = -\frac{nh\sin\theta}{n'h'} \to \xrightarrow[\text{sine condition}]{Abbe's} \boxed{nh\sin\theta + n'h'\sin\theta' = 0}$$

We can rewrite the condition: $\boxed{m = \dfrac{h'}{h} = -\dfrac{n\sin\theta}{n'\sin\theta'}}$

To prevent coma all magnifications must be independent of θ and that is only possible if $\boxed{\dfrac{\sin\theta}{\sin\theta'} = \text{constant}}$

The proper Coddington shape factor for absence of coma: $\boxed{\sigma = \left(\dfrac{2n^2 - n - 1}{n+1}\right)\left(\dfrac{s-s'}{s+s'}\right)}$

Example: $n = 1.50$, object at infinity, we get σ=0.8 very close to the value of minimum spherical aberration 0.7.

Thus we can minimize both spherical and coma aberration simultaneously in one design called *aplanatic* optics.

Astigmatism and curvature of field

Off-axis aberration function:

$$a(Q) = \underbrace{{}_0C_{40}r^4 + {}_1C_{31}h'r^3\cos\theta}_{\textit{Aplanatic}\text{ optics corrects spherical and coma}} +$$

$$+h'^2 r^2 \left(\underbrace{{}_2C_{22}\cos^2\theta}_{\text{Astigmatism}} + \underbrace{{}_2C_{20}}_{\text{Curvature of field}} \right) + {}_3C_{11}h'^3 r\cos\theta$$

Astigmatism and curvature of field have same dependencies to:

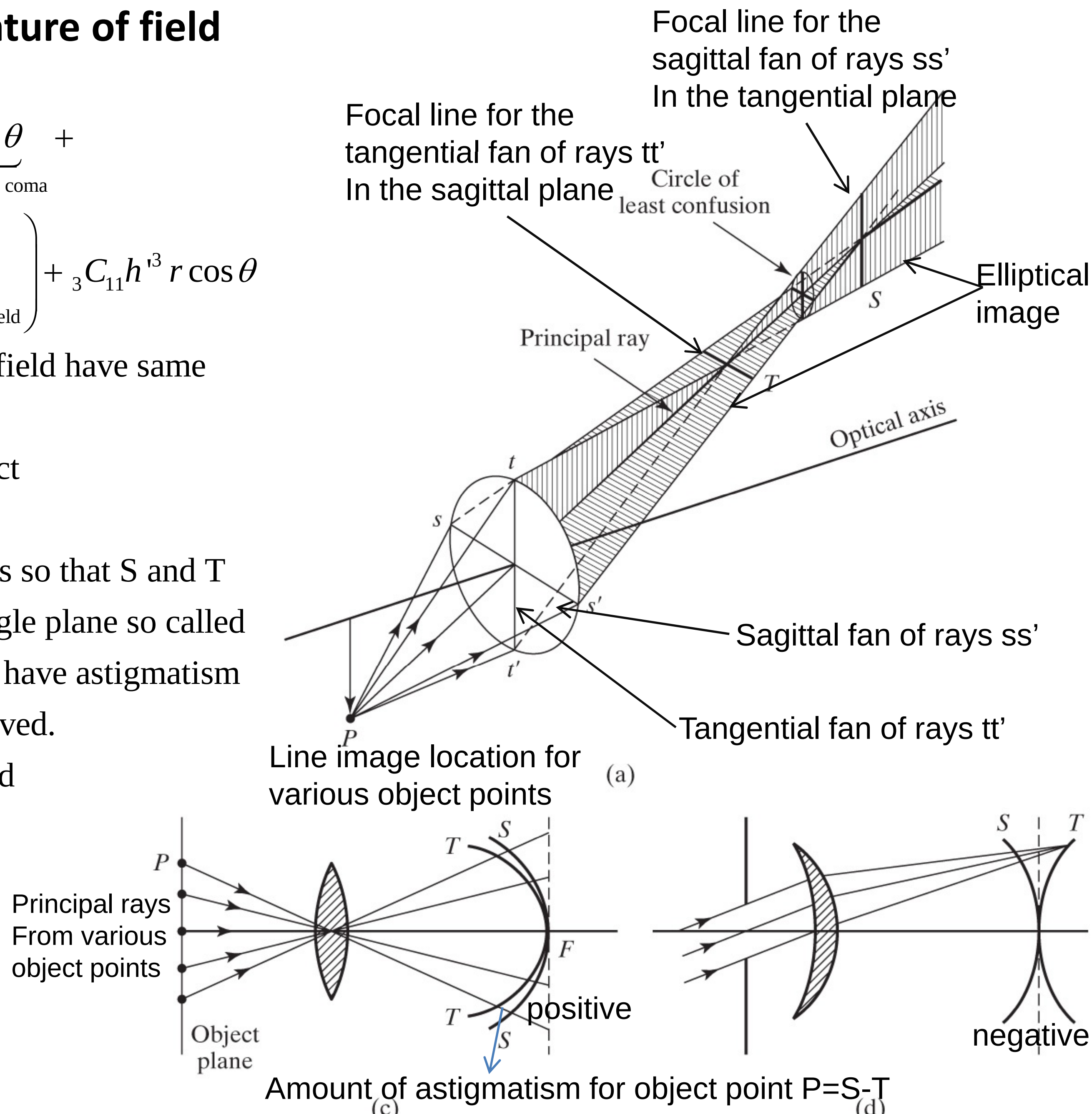
a) off-axis distance of the object

b) aperture of the system

If we use combination of lenses so that S and T image planes coinside on a single plane so called **Petzval surface**, we no longer have astigmatism but the image plane is now curved.

This kind of aberration is called **curvature of (image) field**.

The sharp image forms on a curved surface.



Astigmatism and curvature of field

For two thin lenses the <u>Petzval surface is flat if</u>

$$n_1 f_1 + n_2 f_2 = 0$$

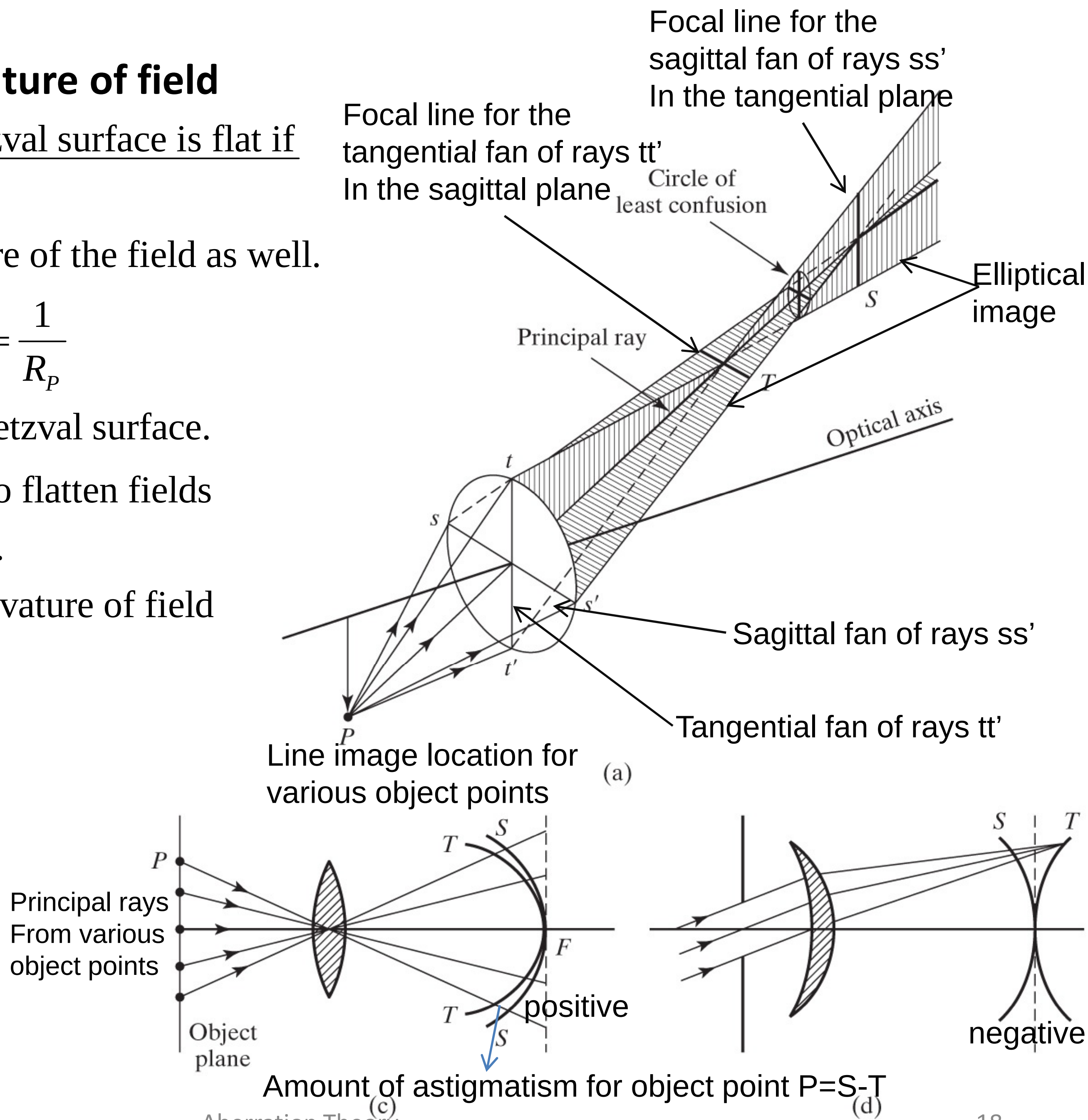
This eliminates the curvature of the field as well.

For k, thin lenses: $$\sum_{i=1}^{k} \frac{1}{n_i f_i} = \frac{1}{R_P}$$

where R_p is the radius of Petzval surface.

We can also use apertures to flatten fields like in a simple box camera.

For actual flattening the curvature of field we need 5th order analysis.



Distortion

$$a(Q) = \underbrace{{}_0C_{40}r^4 + {}_1C_{31}h' r^3 \cos\theta}_{\textit{Aplanatic}\text{ optics corrects spherical and coma}} + h'^2 r^2 \left(\underbrace{{}_2C_{22}\cos^2\theta}_{\text{Astigmatism}} + \underbrace{{}_2C_{20}}_{\text{Curvature of field}} \right) + \underbrace{{}_3C_{11}h'^3 r\cos\theta}_{\textit{Distortion}}$$

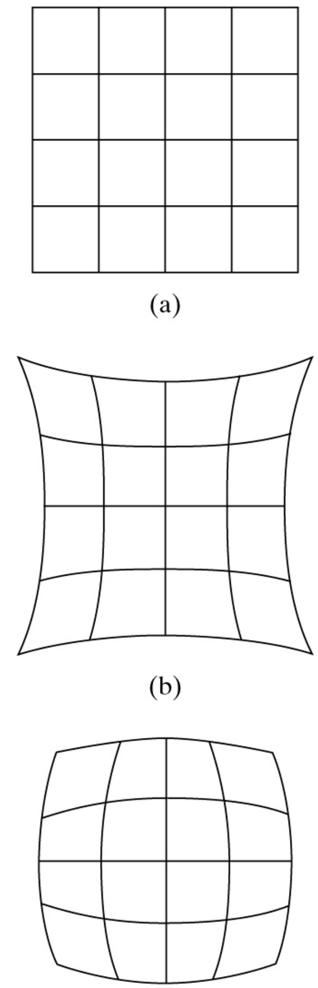
Distortion exists even if all the other monochromatic Seidel aberations have been eleiminated.

It is caused by variations of the lateral magnifications for the object points at different distance from the optical axis.

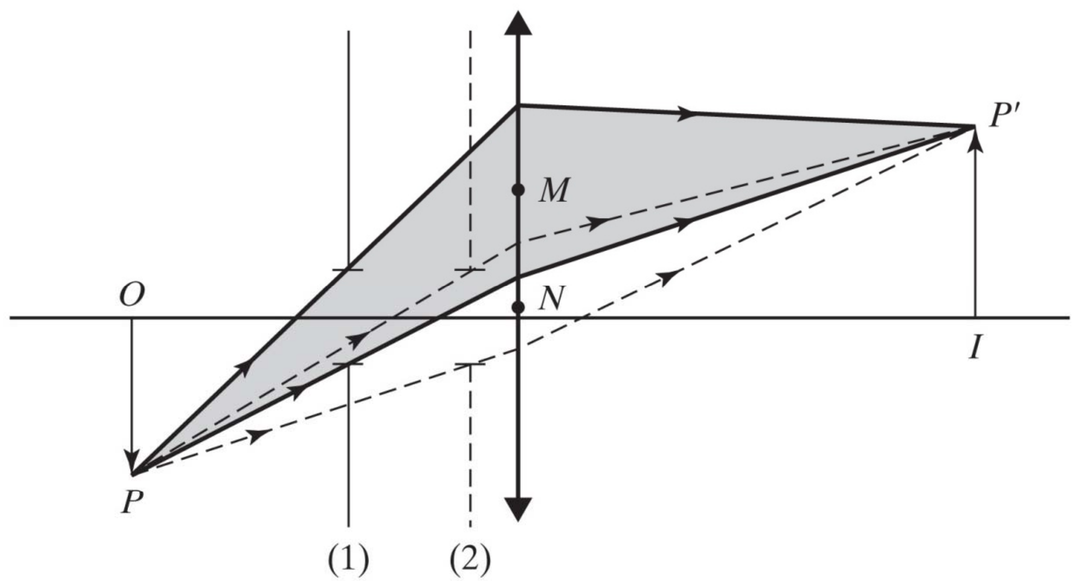
Pincushion distortion: if magnification increases with distance from the axis.

Barrel distortion: if magnification decreases with distance from the axis.

The image is sharp but distorted. Can be treated by using stops and apertures at approprite locations between the lens and object or lens and image.



(a) (b) (c)



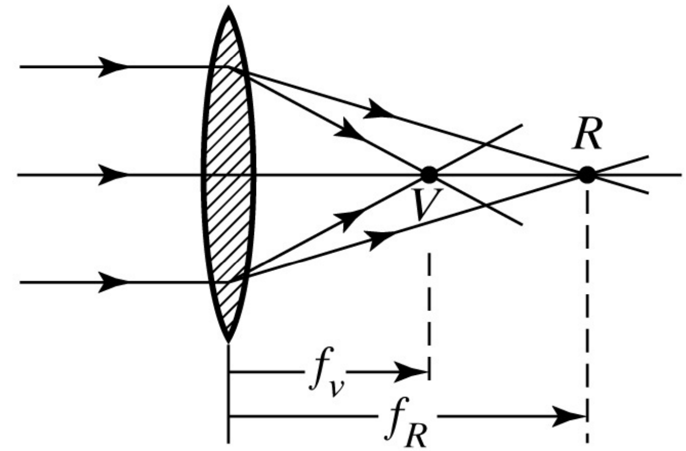
(a)

Chromatic aberration

Chromatic aberration is not a Seidel aberration.
It is caused by variation of refractive index with wavelength or dispersion.

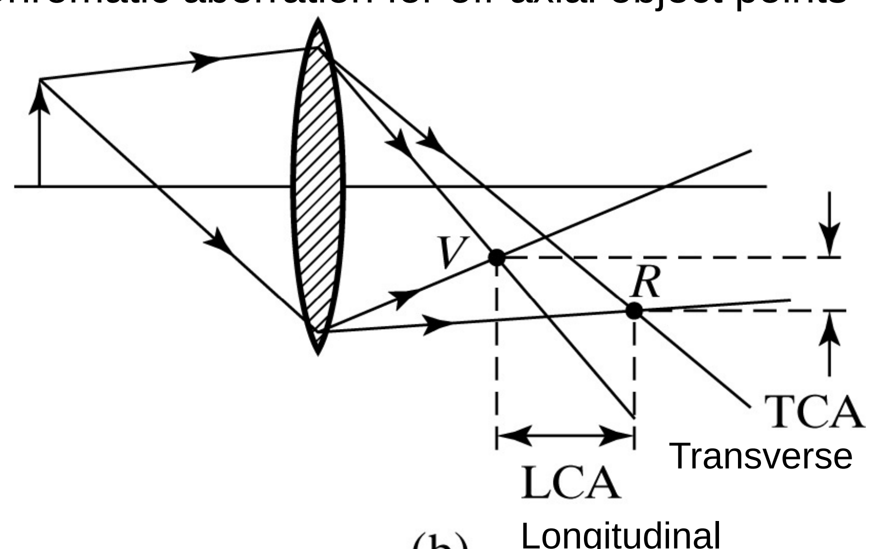
f, focal length of a lens depends on n and n depends on wavelength so $f \rightarrow f(\lambda)$

Chromatic aberration for axial object points



(a)

Chromatic aberration for off-axial object points



(b)

Eliminating chromatic aberration

We can eleiminate chromatic aberration by using refractive elements of opposite power.

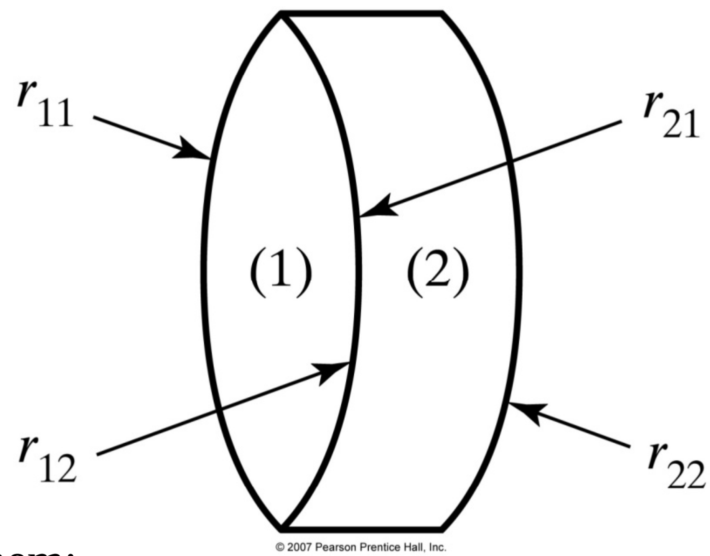
Goal: finding the proper radii of curvature for an achromatic doublet.

Fraunhofer spectral lines:

$\lambda_F = 486.1nm$ (hydrogen); $\lambda_D = 587.6nm$ (sodium); $\lambda_C = 656.3nm$;

Dispersive constant of a glass defined as: $V \equiv \dfrac{1}{\Delta} = \dfrac{n_D - 1}{n_F - n_C}$ where Δ is dispersive power.

Assume variations of n with λ is: $\dfrac{\partial n}{\partial \lambda} \cong \dfrac{n_F - n_C}{\lambda_F - \lambda_C}$



Power of the two lenses for the sodium yellow line:

$$P_{1D} = \frac{1}{f_{1D}} = (n_{1D} - 1)\left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right) = (n_{1D} - 1) K_1$$

$$P_{2D} = \frac{1}{f_{2D}} = (n_{2D} - 1)\left(\frac{1}{r_{21}} - \frac{1}{r_{22}}\right) = (n_{2D} - 1) K_2$$

Total power of two thin lenses with distance L between them:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2} \rightarrow P = P_1 + P_2 - L P_1 P_2$$

Total power of two thin lenses cemented: $P = P_1 + P_2 = (n_1 - 1) K_1 + (n_2 - 1) K_2$

Eliminating chromatic aberration

Total power of two thin lenses cemented: $P = (n_1 - 1)K_1 + (n_2 - 1)K_2$

If the power of the combination is independent of wavelength, λ, to achieve that $(\partial P / \partial \lambda)_D = 0$

$$\frac{\partial P}{\partial \lambda} = K_1 \frac{\partial n_1}{\partial \lambda} + K_2 \frac{\partial n_2}{\partial \lambda} = 0 \quad with \quad \frac{\partial n}{\partial \lambda} \cong \frac{n_F - n_C}{\lambda_F - \lambda_C}$$

$$K_1 \frac{\partial n_{1D}}{\partial \lambda} = K_1 \left(\frac{n_{1F} - n_{1C}}{\lambda_F - \lambda_C} \right) \left(\frac{n_{1D} - 1}{n_{1D} - 1} \right) = \frac{P_{1D}}{(\lambda_F - \lambda_C) V_1}; \; K_2 \frac{\partial n_{2D}}{\partial \lambda} = K_2 \left(\frac{n_{2F} - n_{2C}}{\lambda_F - \lambda_C} \right) \left(\frac{n_{2D} - 1}{n_{2D} - 1} \right) = \frac{P_{2D}}{(\lambda_F - \lambda_C) V_2}$$

$$\frac{\partial P}{\partial \lambda} = \frac{P_{1D}}{(\lambda_F - \lambda_C) V_1} + \frac{P_{2D}}{(\lambda_F - \lambda_C) V_2} = 0 \rightarrow V_2 P_{1D} + V_1 P_{1D} = 0$$

The powers of individual elements are:

$$\begin{cases} V_2 P_{1D} + V_1 P_{1D} = 0 \\ P = P_{1D} + P_{2D} \end{cases} \rightarrow \begin{cases} P_{1D} = P_D \dfrac{-V_1}{V_2 - V_1} \\ P_{2D} = P_D \dfrac{V_2}{V_2 - V_1} \end{cases} \rightarrow \begin{cases} K_1 = \dfrac{P_{1D}}{n_{1D} - 1} = \left(\dfrac{1}{r_{11}} - \dfrac{1}{r_{12}} \right) \\ K_2 = \dfrac{P_{2D}}{n_{2D} - 1} = \left(\dfrac{1}{r_{21}} - \dfrac{1}{r_{22}} \right) \end{cases}$$

For simplicity we choose the crown glass to be equiconvex: $\boxed{r_{12} = -r_{11}}$

The curvature of the cemented surfaces has to match: $\boxed{r_{21} = r_{12}}$ and $\boxed{r_{22} = \dfrac{r_{12}}{1 - K_2 r_{12}}}$

Example achromatic doublet:

If 520/636 crown glass and 617/366 flint glass are used in design of an achromat of focal lenngth, $f = 15cm,$ find the appropriate radii of curvature and focal length of the each lens and combination for the three Fraunhofer lines.

$$P_D = 1/0.15m = 6.6667; \text{ from the table 20-1: } V \equiv \frac{1}{\Delta} = \frac{n_D - 1}{n_F - n_C} \rightarrow V_1 = 63.59,\ V_2 = 36.60$$

$$P_{1D} = P_D \frac{-V_1}{V_2 - V_1} \rightarrow P_{1D} = 6.6667 \frac{-63.59}{36.60 - 63.59} = 15.707052$$

$$P_{2D} = P_D \frac{V_2}{V_2 - V_1} \rightarrow P_{1D} = 6.6667 \frac{36.60}{36.60 - 63.59} = -9.0403853$$

$$K_1 = \frac{P_{1D}}{n_{1D} - 1} = \frac{15.707052}{1.52015 - 1} = 30.197158 = \left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right) = \frac{2}{r_{11}} \rightarrow \boxed{r_{11} = 6.623139cm}$$

$$K_2 = \frac{P_{2D}}{n_{2D} - 1} = \frac{-9.0403853}{1.61715 - 1} = -14.6486029 = \left(\frac{1}{r_{21}} - \frac{1}{r_{22}}\right)$$

$$\boxed{r_{12} = -r_{11} = -6.623139cm}$$

$$\boxed{r_{21} = r_{12} = -6.623139cm}$$

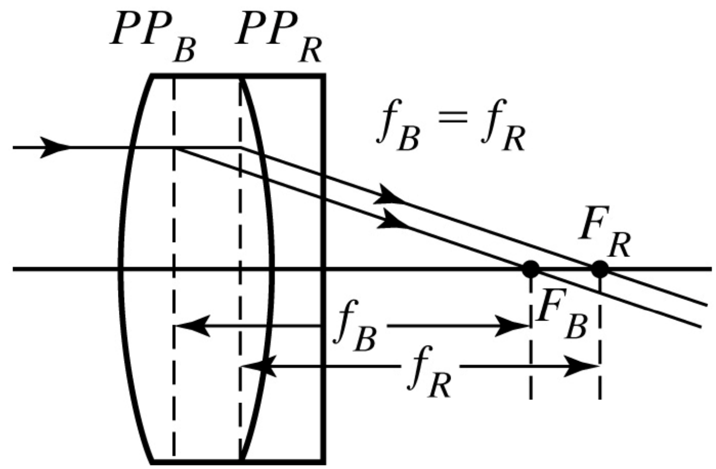
$$r_{22} = \frac{r_{12}}{1 - K_2 r_{12}} = \frac{-6.623139}{1 - (-14.6486029)(-6.623139)} = \boxed{-223.29cm = r_{22}}$$

Optical glasses

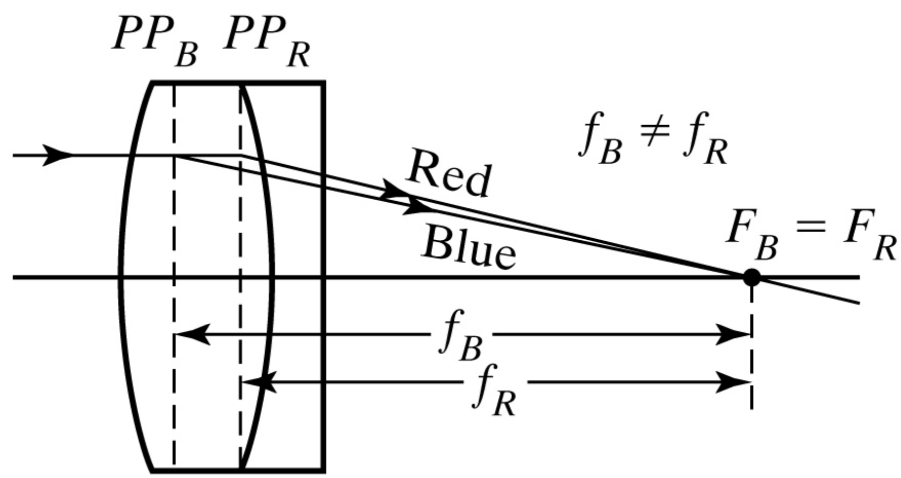
In design process we take the indexes for the Fraunhofer lines from the manufacturer's specification.

TABLE 20-1 SAMPLE OF OPTICAL GLASSES

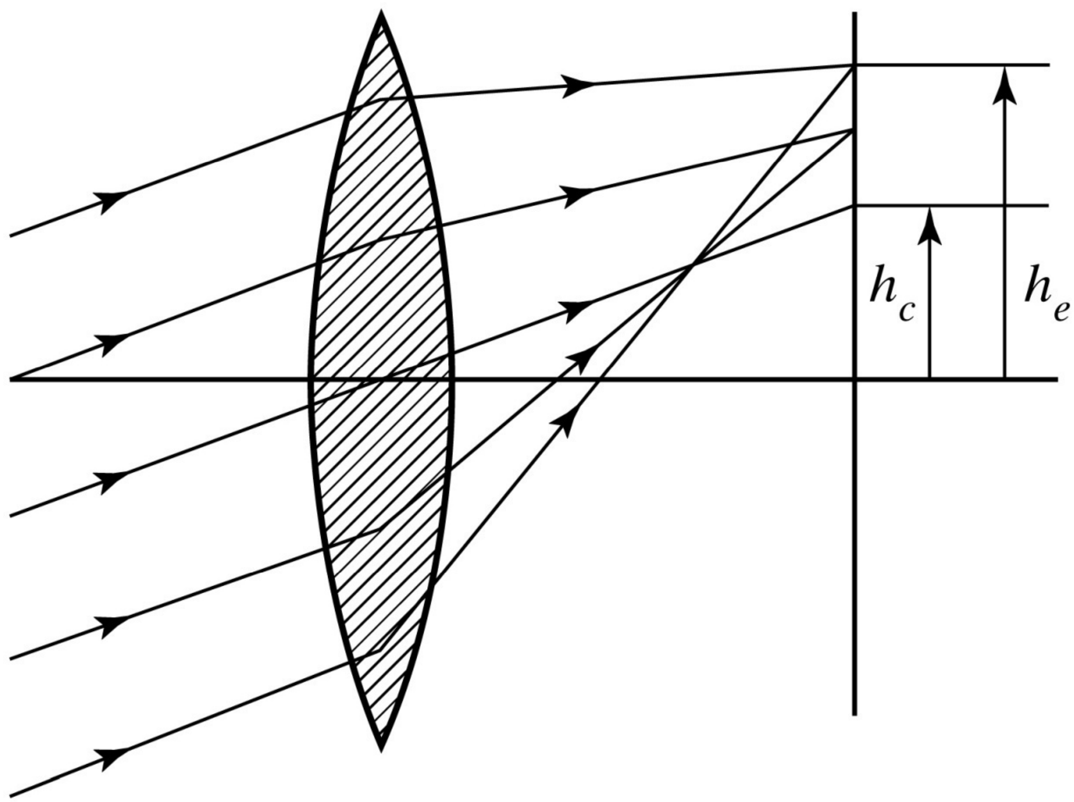
Type	Catalog code	V	n_C	n_D	n_F
	$\frac{n_D-1}{10V}$	$\frac{n_D-1}{n_F-n_C}$	656.3 nm	587.6 nm	486.1 nm
Borosilicate crown	517/645	64.55	1.51461	1.51707	1.52262
Borosilicate crown	520/636	63.59	1.51764	1.52015	1.52582
Light barium crown	573/574	57.43	1.56956	1.57259	1.57953
Dense barium crown	638/555	55.49	1.63461	1.63810	1.64611
Dense flint	617/366	36.60	1.61218	1.61715	1.62904
Flint	620/380	37.97	1.61564	1.62045	1.63198
Dense flint	689/312	31.15	1.68250	1.68893	1.70462
Dense flint	805/255	25.46	1.79608	1.80518	1.82771
Fused silica	458/678	67.83	1.45637	1.45846	1.46313



(a)



(b)


h_c
h_e