ME 297
L5 Optical component movement

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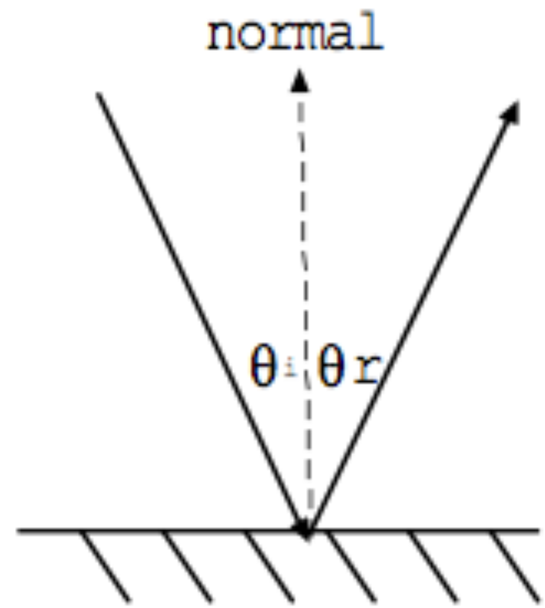
SJSU

Ref. Dr. Jim Burge's Notes

Reflection from a plane mirror

Reflected ray in plane with incident ray and surface normal



Law of reflection

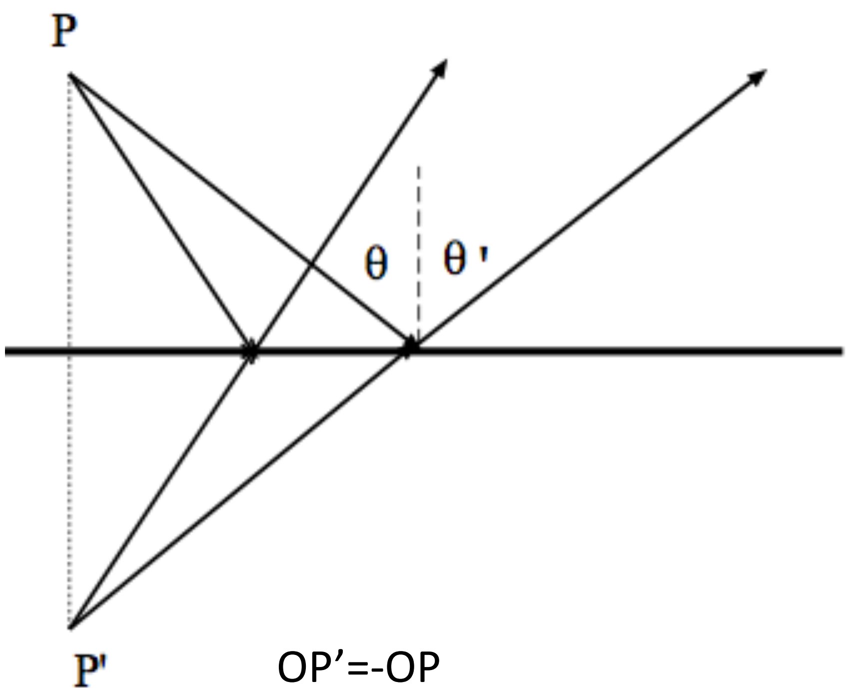
$$\theta_i = \theta_r$$

Vector form of the law of reflection

$$\mathbf{k}_r = \mathbf{k}_i - 2\left(\mathbf{k}_i \bullet \mathbf{n}\right)\mathbf{n}$$

Image formation by a plane mirror

Point object



OP’=-OP
o=-i

Extended object

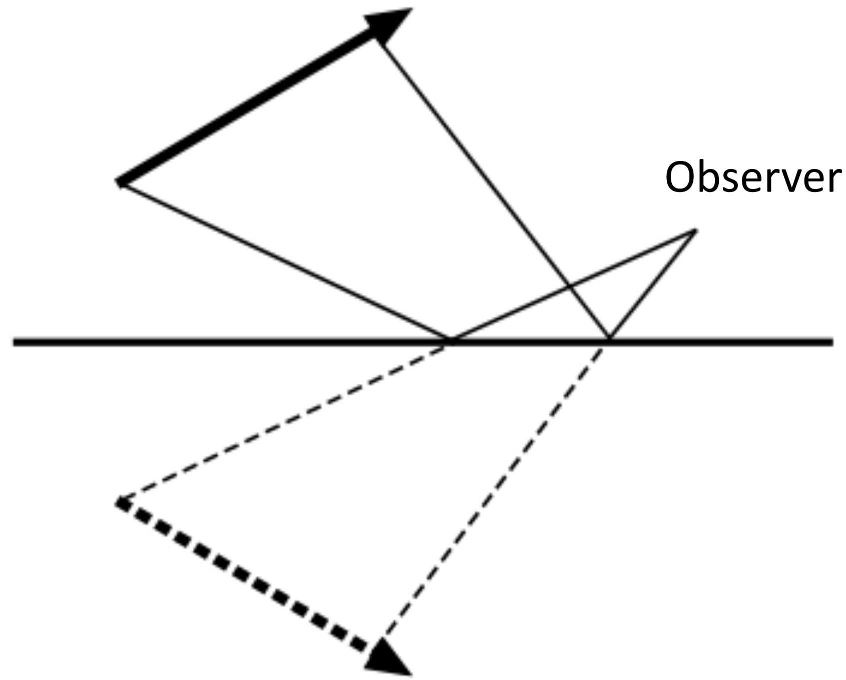
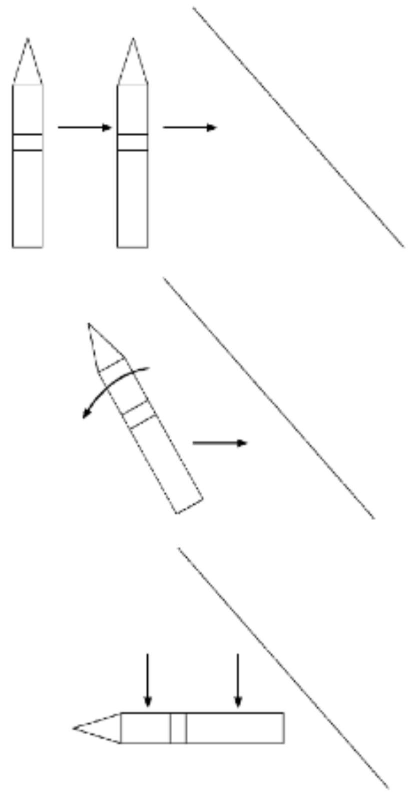
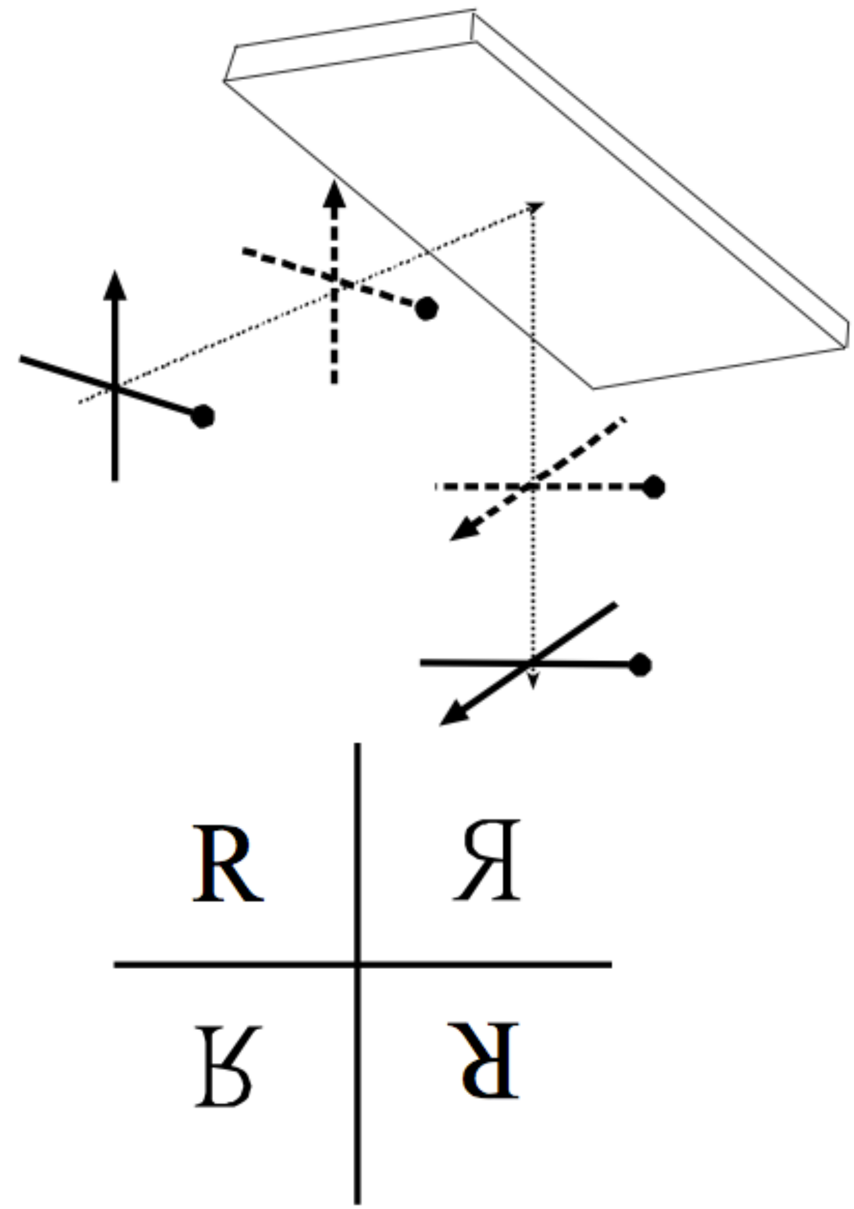


Image orientation

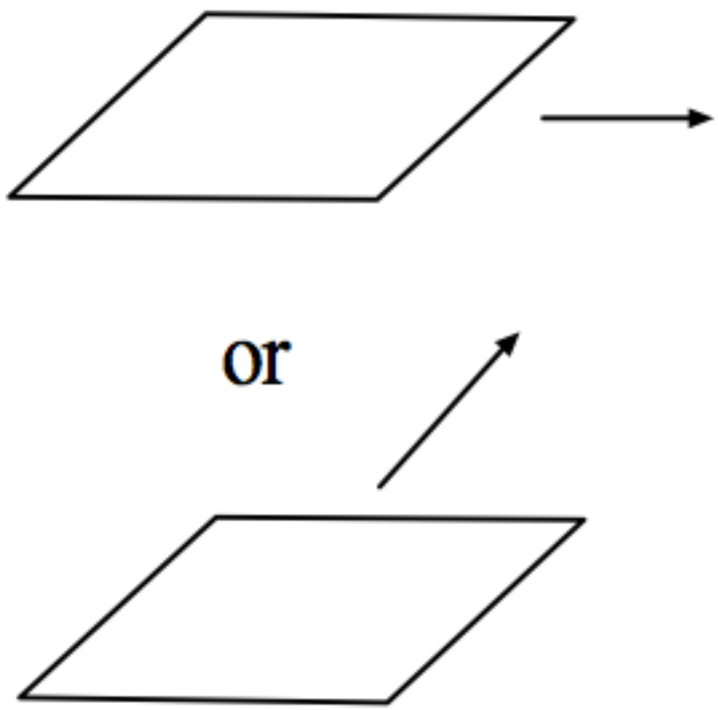
Bouncing from the mirror surface



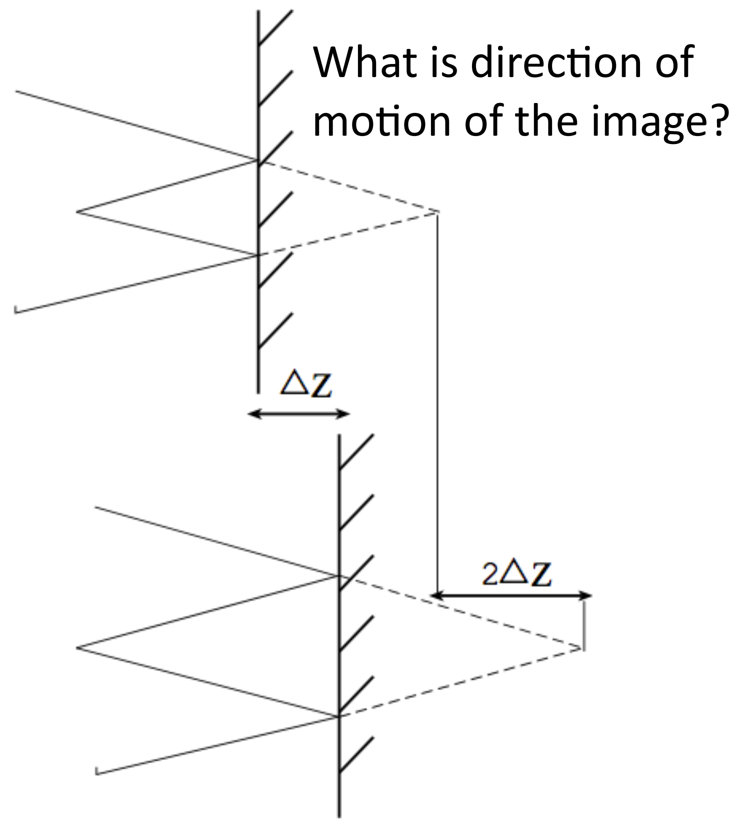


Motion of a plane mirror
xyz translation

For 2 DOFs (xyz translation) nothing changes

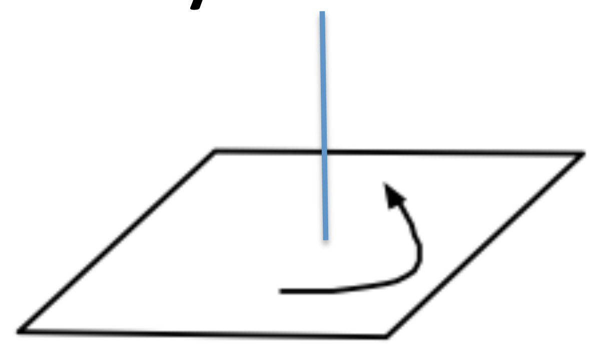


Motion along the optical axis:
Image moves twice as much as the mirror



Motion of a plane mirror II rotation around xyz

Axis of rotation perpendicular to the mirror surface (z): nothing changes



Axis of rotation on the mirror (x,y) or perpendicular to the optical axis:

Mirror tilt by an angle: LOS is rotated twice as much as the tilt angle

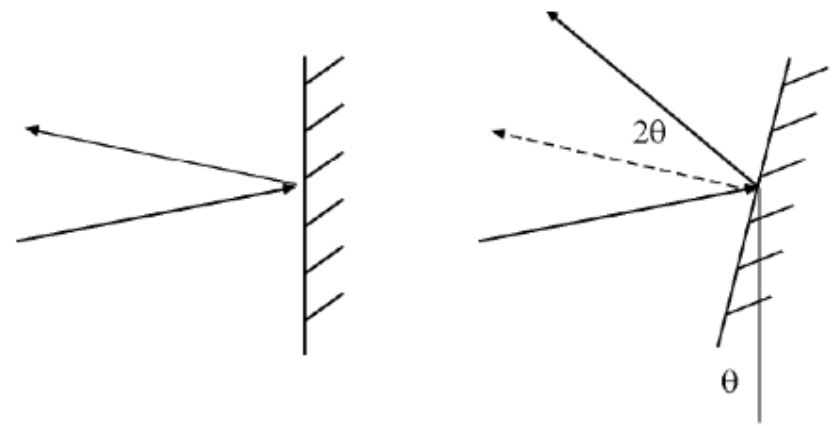


Image motion when mirror is tilted by an angle

Image moves on a circle formed by the line connecting the object to the axis of rotation on the same direction as the rotation of the mirror.

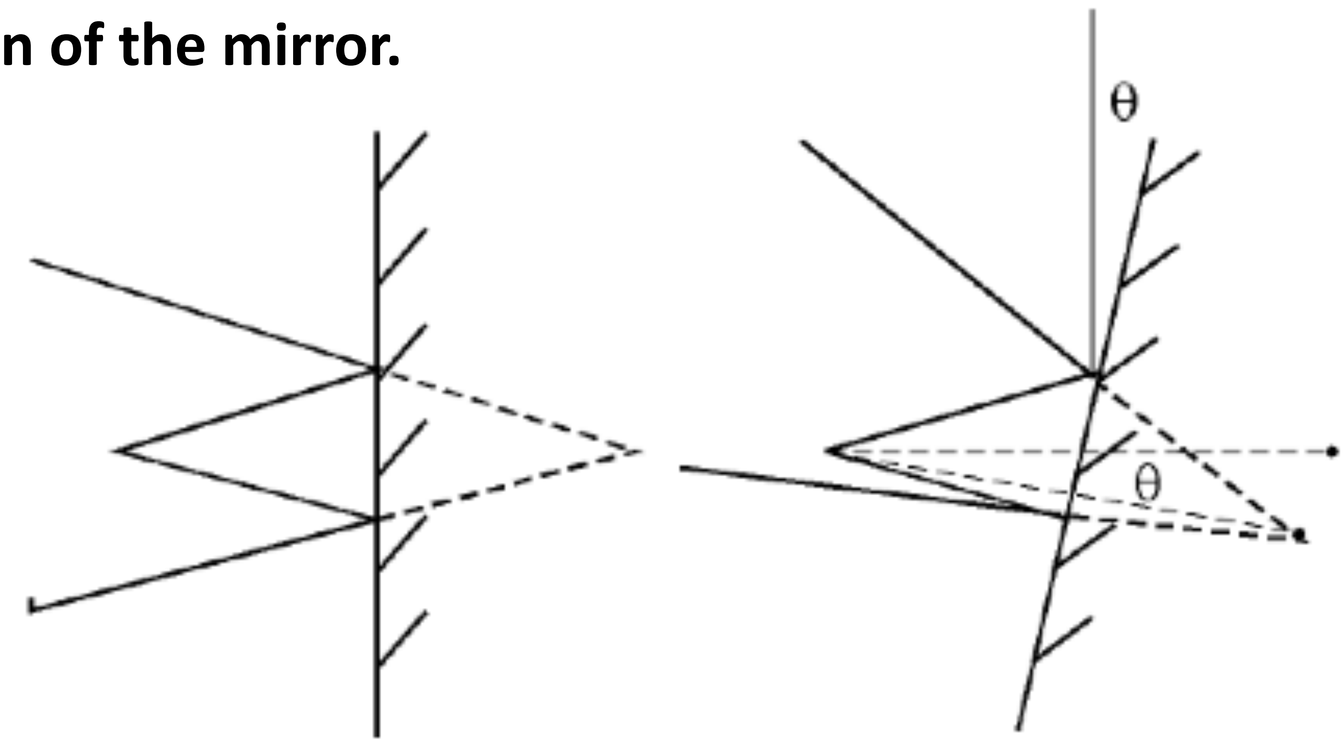


Image motion due to lateral motion of the object for a thin lens (paraxial)

Lateral motion: for a simple thin lens if the object moves by dy_o how much the image moves?

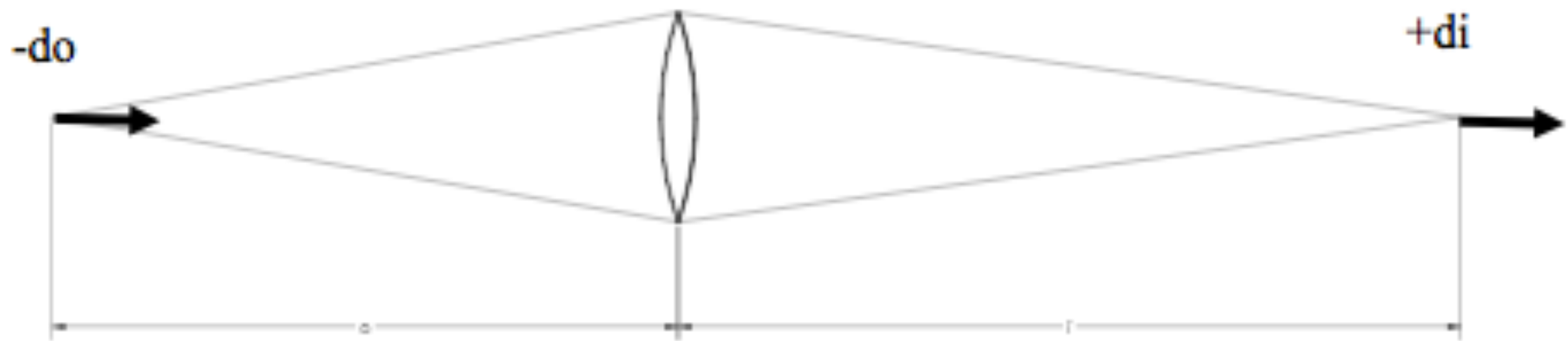
image is rotated 180°, maintains 'handedness'

$$y_i = my_o \rightarrow dy_i = mdy_o \rightarrow \frac{dy_i}{dy_o} = m = -\frac{s_i}{s_o}$$

For lateral motion, simply scales by magnification

Image motion due to longitudinal motion of the object for a thin lens (paraxial)

Longitudinal motion: for a simple thin lens if the object moves by ds_o how much the image moves?



$$\frac{1}{f}=\frac{1}{o}+\frac{1}{i}$$

$$\frac{-di}{i^2}+\frac{-do}{o^2}=0$$

$$\frac{di}{do}=-\frac{i^2}{o^2}=-m^2$$

This is often called the axial magnification

(Object and image always move in the same direction)

Image motion due to lateral motion of the lens for a thin lens (paraxial)

$$\frac{i}{o} = -m$$

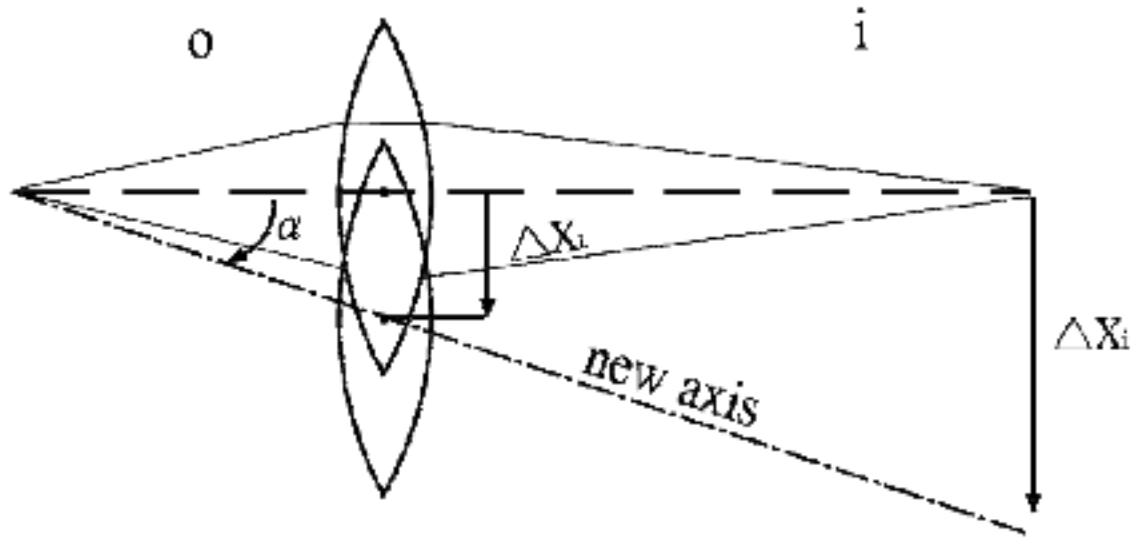
New Axis , angle $\alpha = \dfrac{\Delta X_L}{o}$

Image moves $\alpha(o+i) = \Delta X_i$

$$\Delta X_i = \Delta X_L \frac{o+i}{o}$$

$$\Delta X_i = \Delta X_L (1-m)$$

For object at infinity, $\Delta X_i = \Delta X_L$



Note: it is common to use m=0 for object at infinity and m=infinite for image at infinity in simplifying the paraxial optics formulas but in reality the lateral magnification is not defined for any of the cases of object or image at infinity. Angular magnification is a more correct term in these cases.

Image motion due to axial motion of the lens for a thin lens (paraxial)

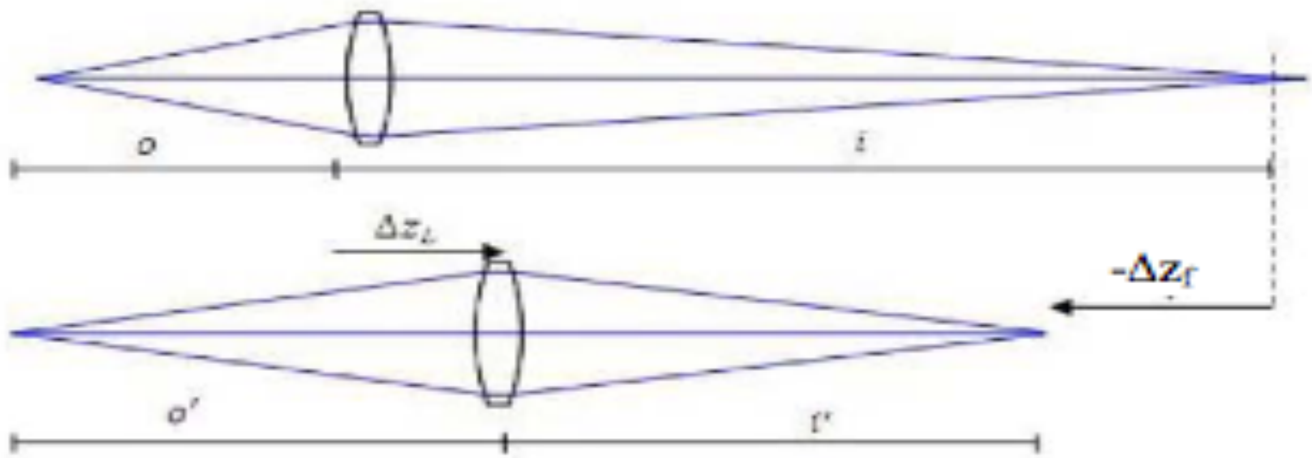
Absolute image motion = Lens motion + (Image motion relative to lens)

$$o' = o + \Delta z$$

$$i' = i - \Delta Z_L + \Delta Z_f$$

$$\Delta o = o - o' = -\Delta Z_L$$

$$\Delta i = i - i' = \Delta Z_L = \Delta Z_f$$



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

$$\frac{-\partial i}{i^2} + \frac{-\partial o}{o^2} = 0$$

$$\frac{\Delta i}{\Delta o} = -\frac{i^2}{o^2} = -m^2 \rightarrow \underline{\Delta Z_f = \Delta Z_L\left(1 - m^2\right)}$$

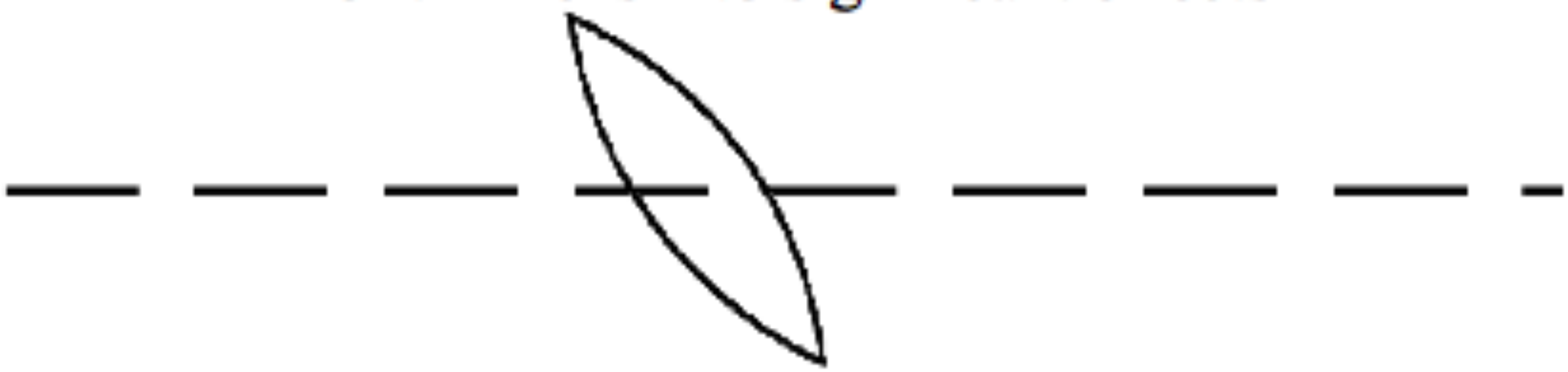
$$o \rightarrow \infty;\ m = 0 \rightarrow \Delta Z_f = \Delta Z_L$$

$$m = -1 \rightarrow \frac{\Delta Z_f}{\Delta Z_L} = 0$$ → For 1:1 conjugate system the focus is a stationary point

Tilt of an optical element about its center

- Tilt a lens

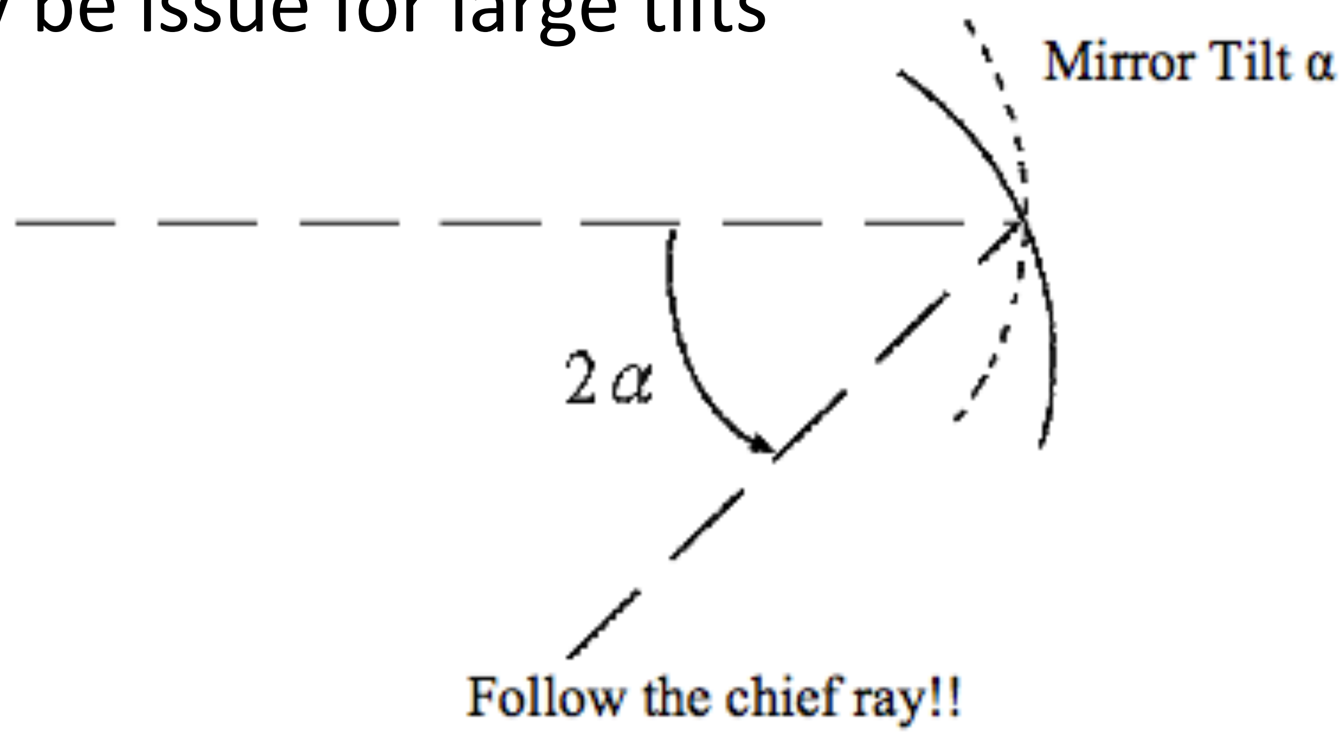
For thin lens- No significant effects



(Large tilt cause aberrations)

Tilt of an optical element about its center

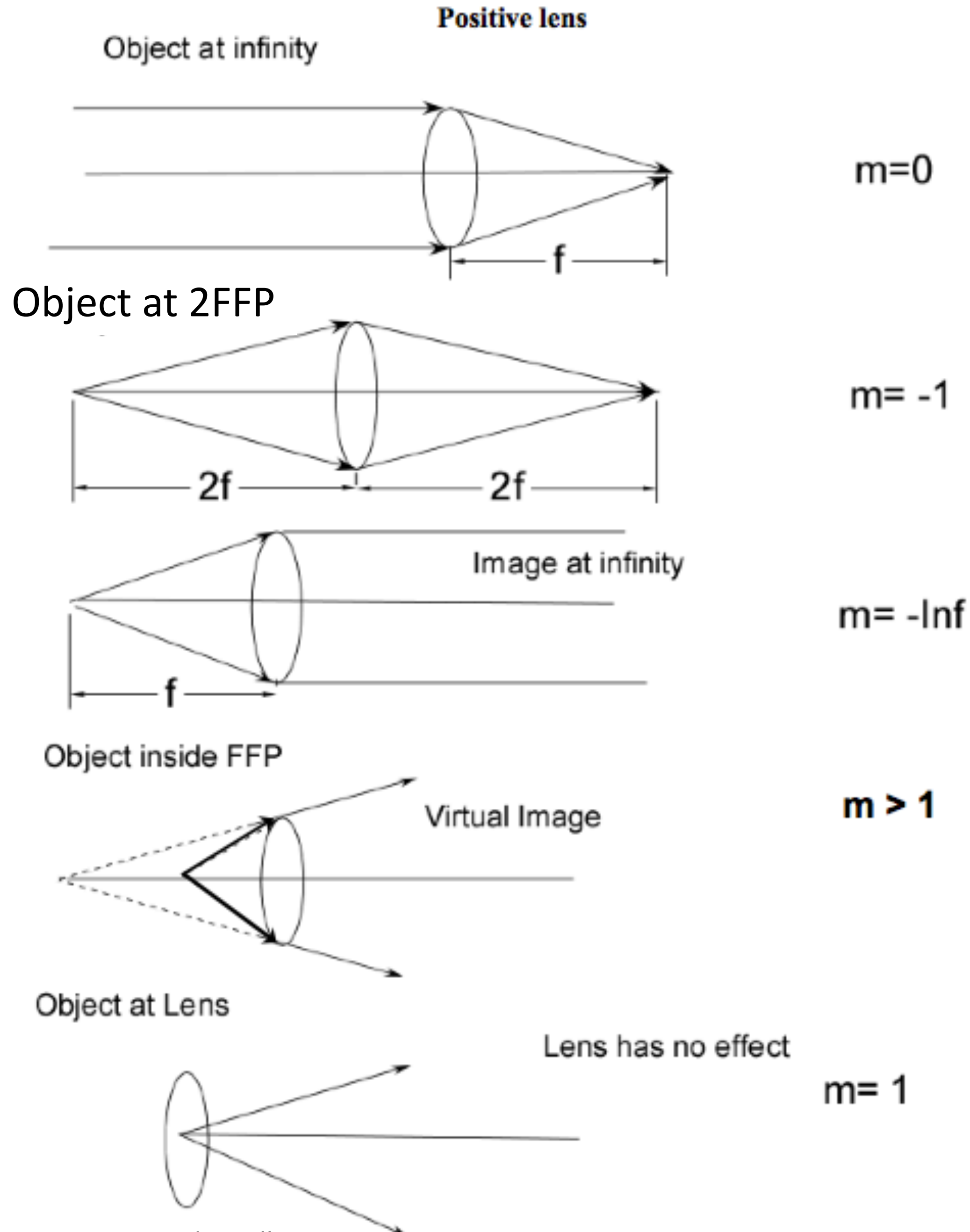
- Tilt a mirror similar to a flat mirror, aberrations may be issue for large tilts



Motion of detector

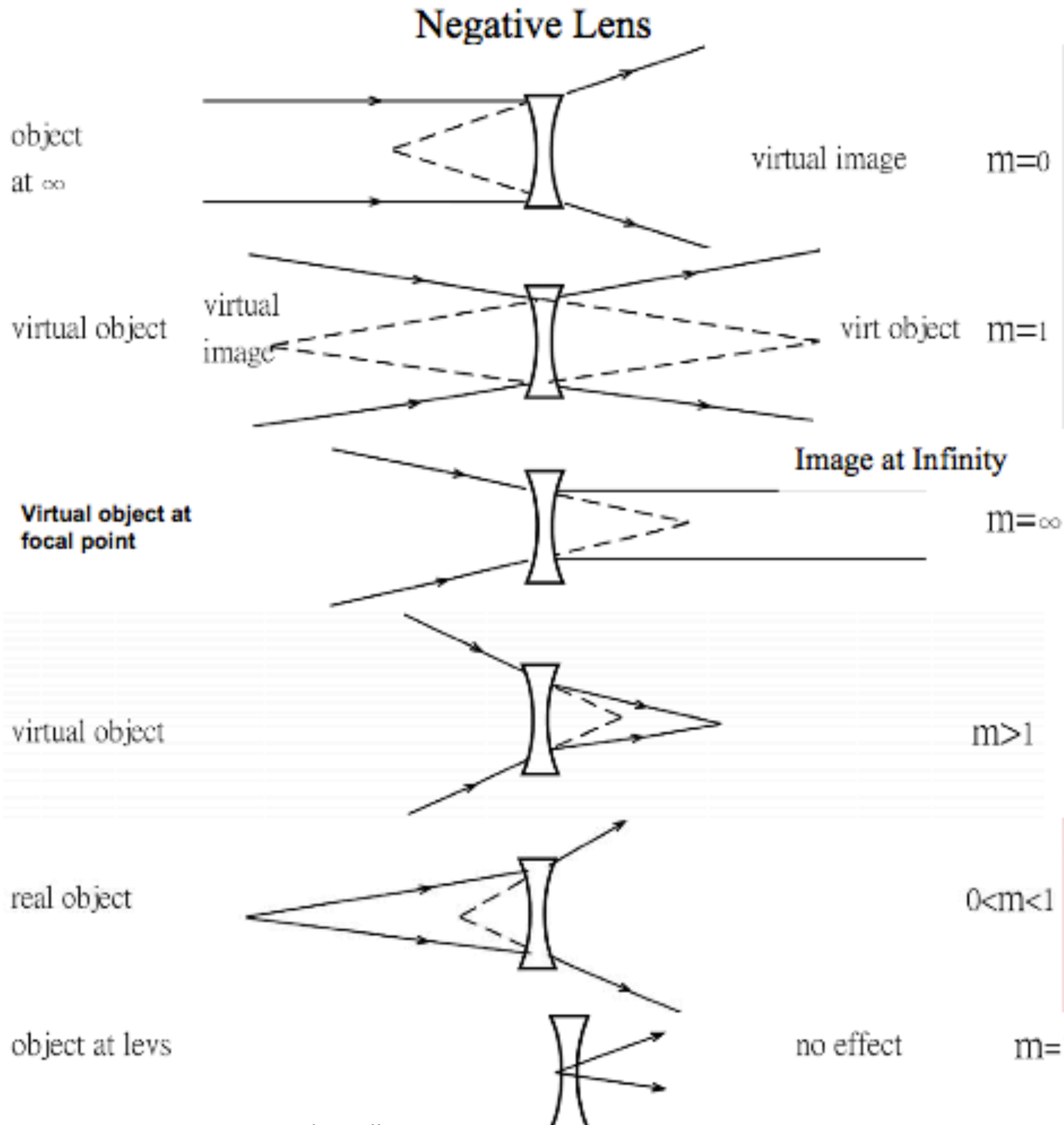
- The "detector" could be film, CCD, fiber end, ...
- What we care about is motion of the image *with respect to the detector. This motion would cause a blurred image, tracking error, or degraded coupling efficiency.*
- *If the image and detector move together, the system performs perfectly.*
- *Motion of the detector has the same (but opposite sign) as motion of the image.*
- *Although pointing performance is defined by image motion on the detector, it is usually not specified in image space where problem occurs, but it is referred back to object space.*
- We must be able to go efficiently back and forth between these two spaces: $\Delta x_i = m\Delta x_o$
- $\Delta x_i = EFL \cdot \Delta\alpha_o$
- *For object at infinity,* $m = 0$
- *Where* $\Delta\alpha_0$ *gives the angle in object space.*

Some Rules



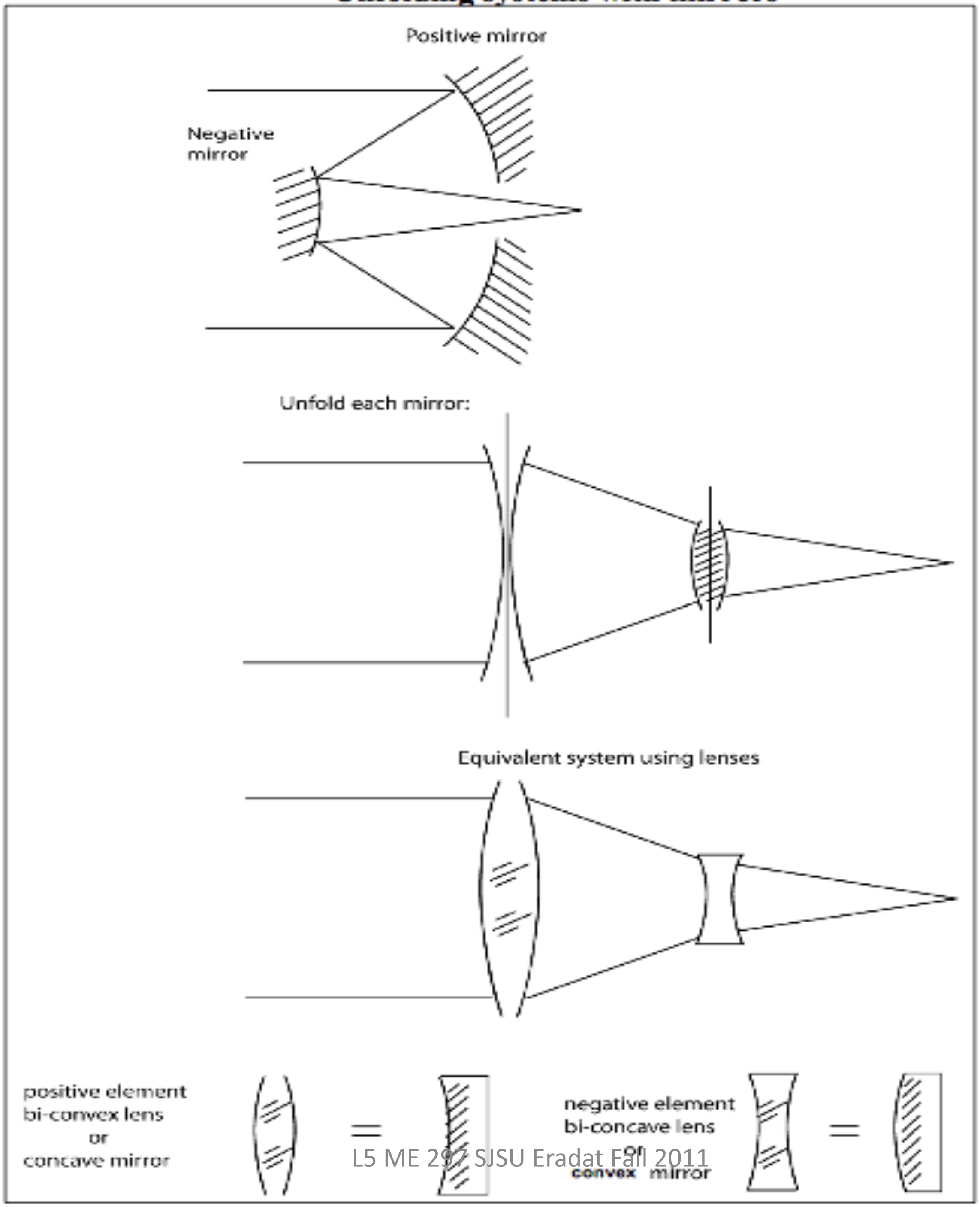
Positive lens
Object at infinity
f
m=0
Object at 2FFP
2f
2f
m= -1
Image at infinity
f
m= -Inf
Object inside FFP
Virtual Image
m > 1
Object at Lens
Lens has no effect
m= 1

More Rules



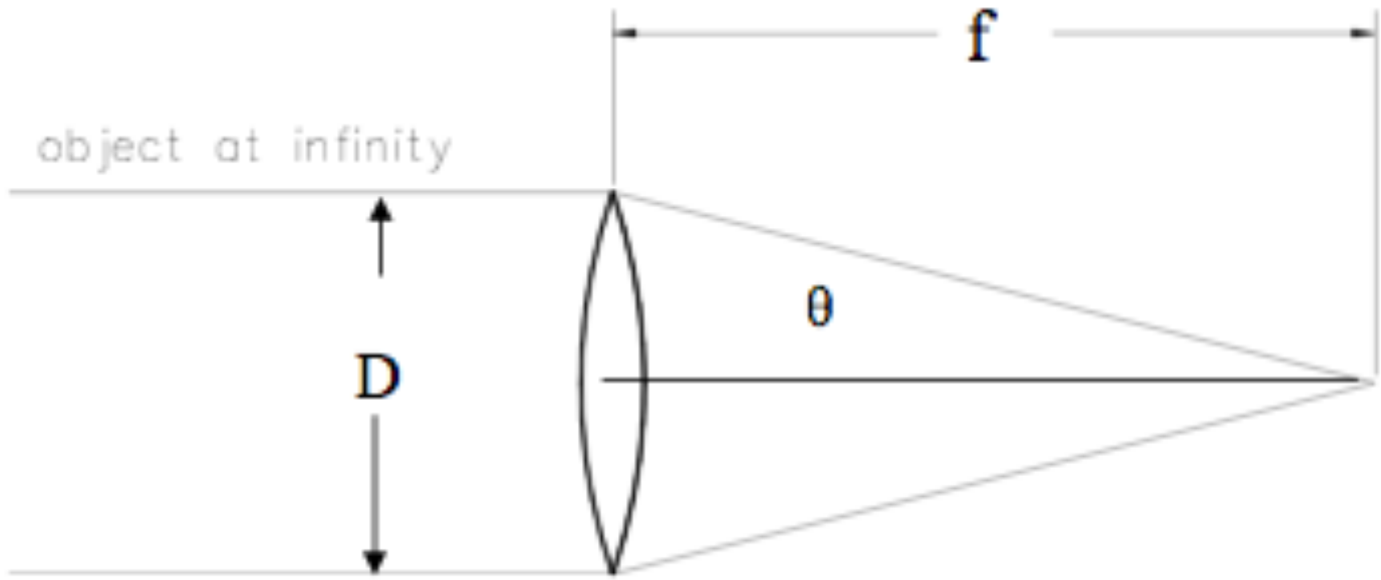
Negative Lens
object
at ∞
virtual image
m=0
virtual object
virtual
image
virt object
m=1
Image at Infinity
Virtual object at
focal point
m=∞
virtual object
m>1
real object
0<m<1
object at levs
no effect
m=1

Unfolding systems with mirrors



Focal ratio

Simple case
stop at lens, object at infinity



f/number : $F\# = \frac{f}{D}$

100 mm focal length, 10 mm diameter lens -- *f*/10

$$NA = n\sin\theta$$

$$\text{Infinity f-number} \rightarrow f\# = \frac{f}{D}$$

$$\text{Image side}\begin{cases} \text{Working f-number} \rightarrow f\#_w = \dfrac{1}{2NA} = \dfrac{1}{2\sin\theta} \\ \text{Working f-number} \rightarrow f\#_w = \dfrac{f}{D}(1-m) \end{cases}$$

Numerical aperture NA
(in medium with refractive index n)

$u = \sin\theta$

$$NA = n\sin\theta \cong \frac{1}{2F\#}$$

Diffraction limit:

Width of Airy function = 2.44 λ F#

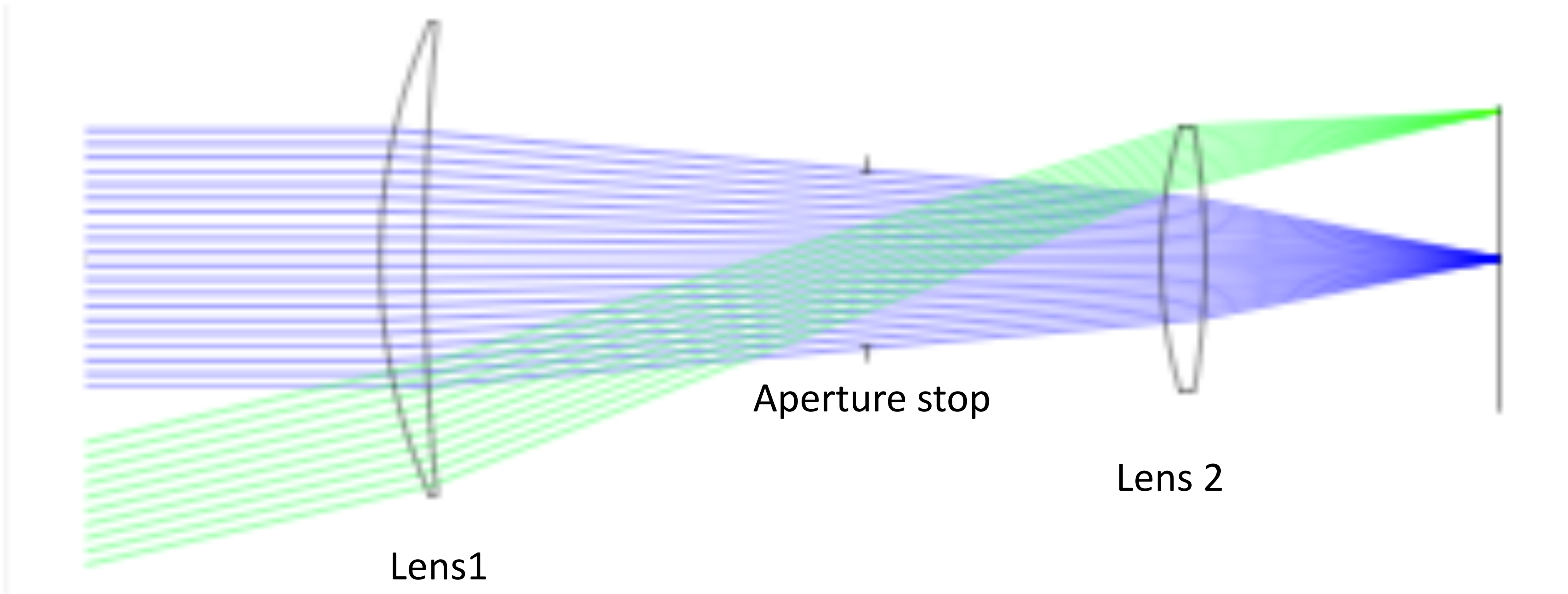
(FWHM = λ F#)

Depth of focus : $\Delta z = \pm 2\,\lambda\,(F\#)^2$

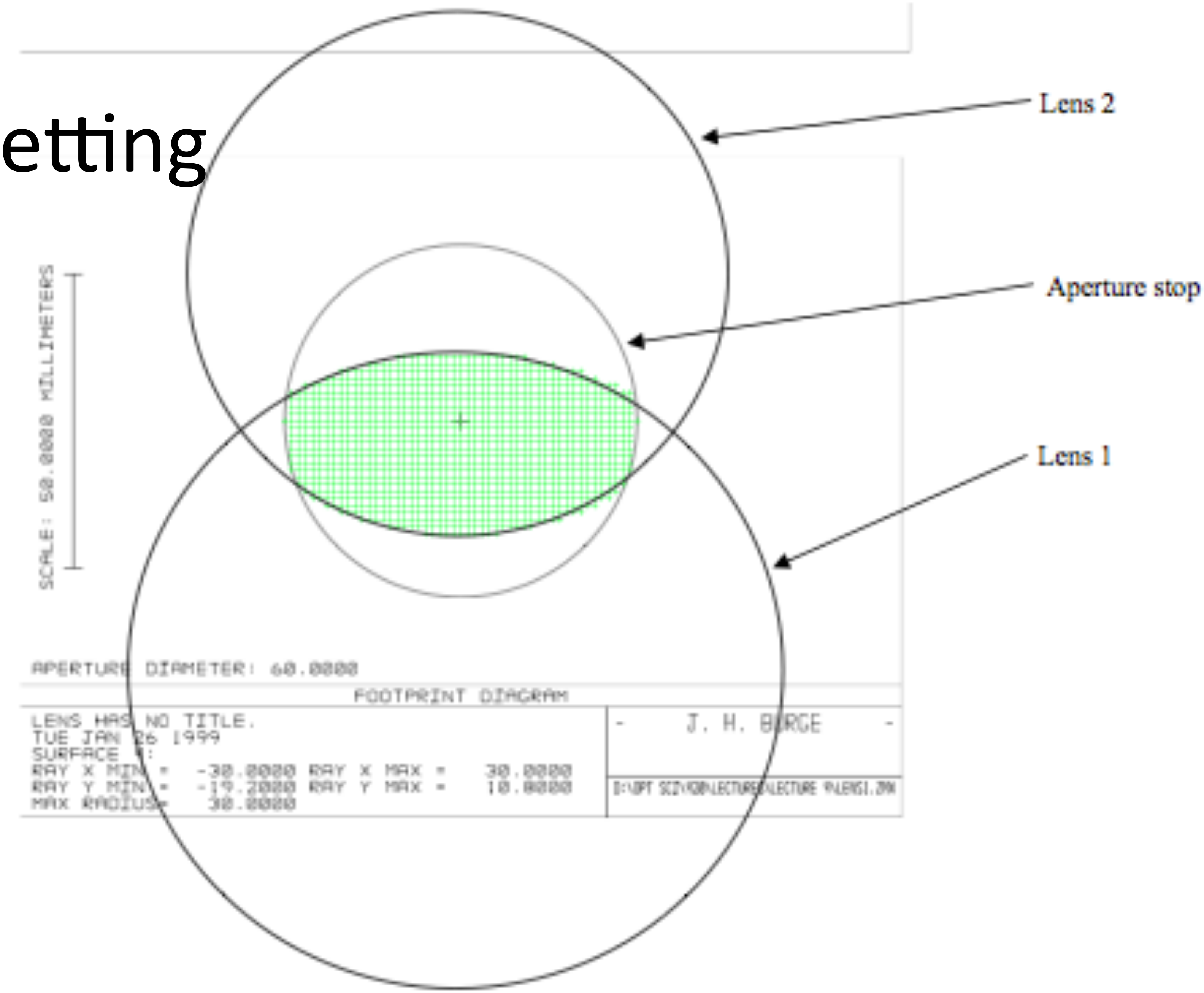
MTF cutoff : $f_c = 1/(\lambda\, F\#)$

Vignetting

- When something other than the aperture defines which rays get through. Leads to loss of light.

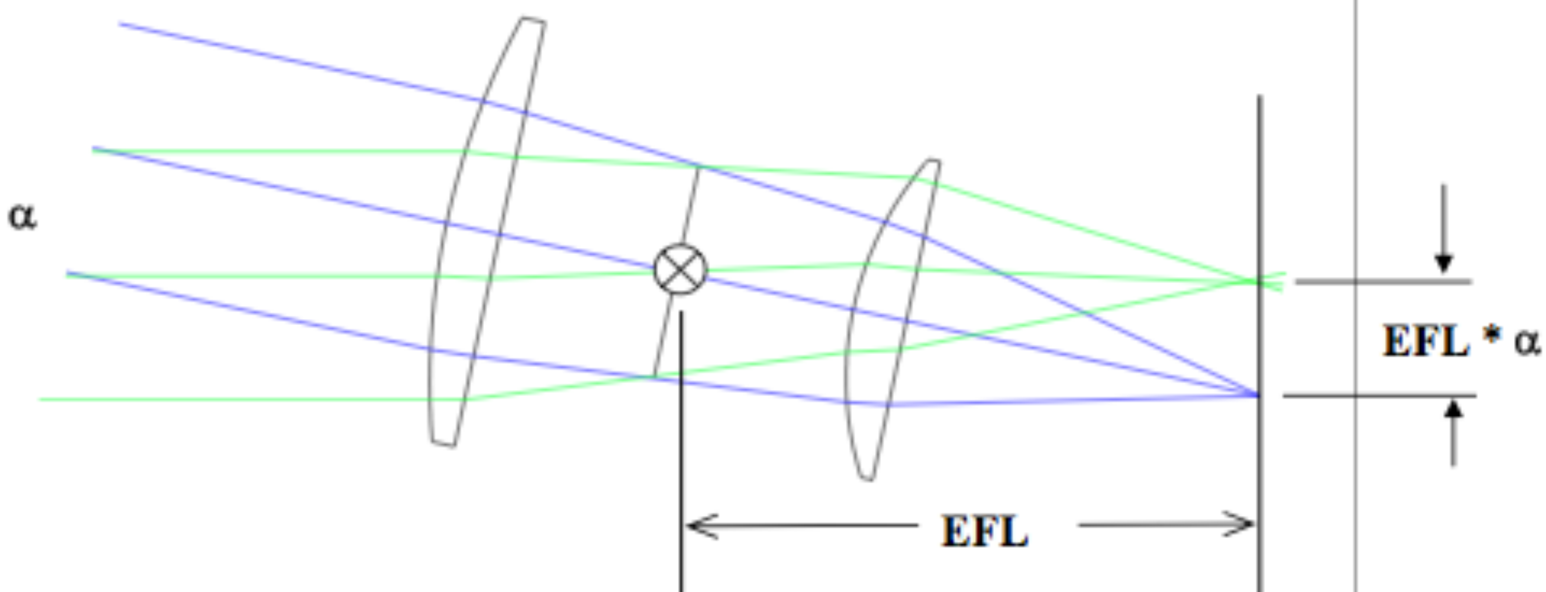


Vignetting


Lens 2
Aperture stop
Lens 1
SCALE: 50.0000 MILLIMETERS
APERTURE DIAMETER: 60.0000
FOOTPRINT DIAGRAM
LENS HAS NO TITLE.
TUE JAN 26 1999
SURFACE
RAY X MIN = -30.0000 RAY X MAX = 30.0000
RAY Y MIN = -19.2000 RAY Y MAX = 10.0000
MAX RADIUS= 30.0000
J. H. BURGE

Nodal point at rear principal plane

- In air, object at infinity, nodal point coincides with rear principal point
- Rotation of lens system about nodal point does not move image



Rotation around the nodal point
Simple proof (for image in air)

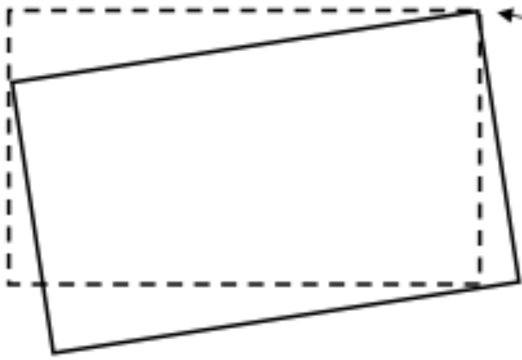
Object at field angle α has image height of EFL x α relative to axis
Lens rotation α about $\mathbf{PP_2}$ moves system axis at focal plane by EFL x α
Lens rotation α causes a fixed object to shift by angle $-\alpha$ relative to axis

The absolute image motion is

image motion relative to lens axis	=	EFL x $-\alpha$
+ motion of lens axis		+ EFL x α
		0, no motion

Only for the case where the system is rotated about the rear principal point.

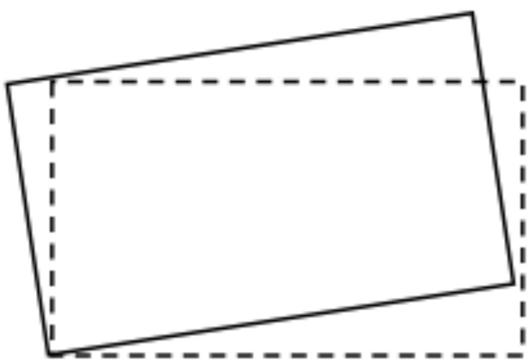
Rigid body rotation



α° rotation about this corner

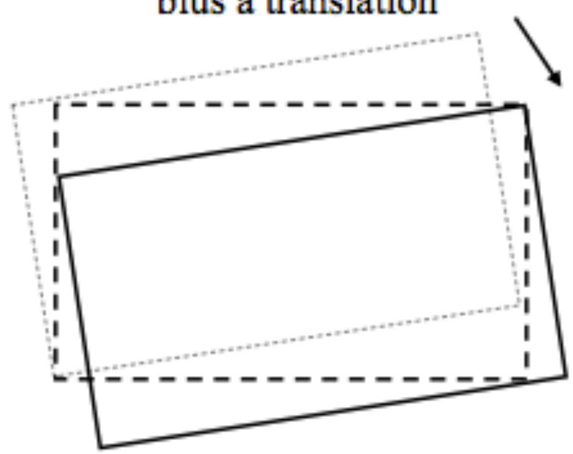
- Rotation about one point on an object is equivalent to rotation about any other point plus a translation.

is equivalent to



α° rotation about this corner

plus a translation



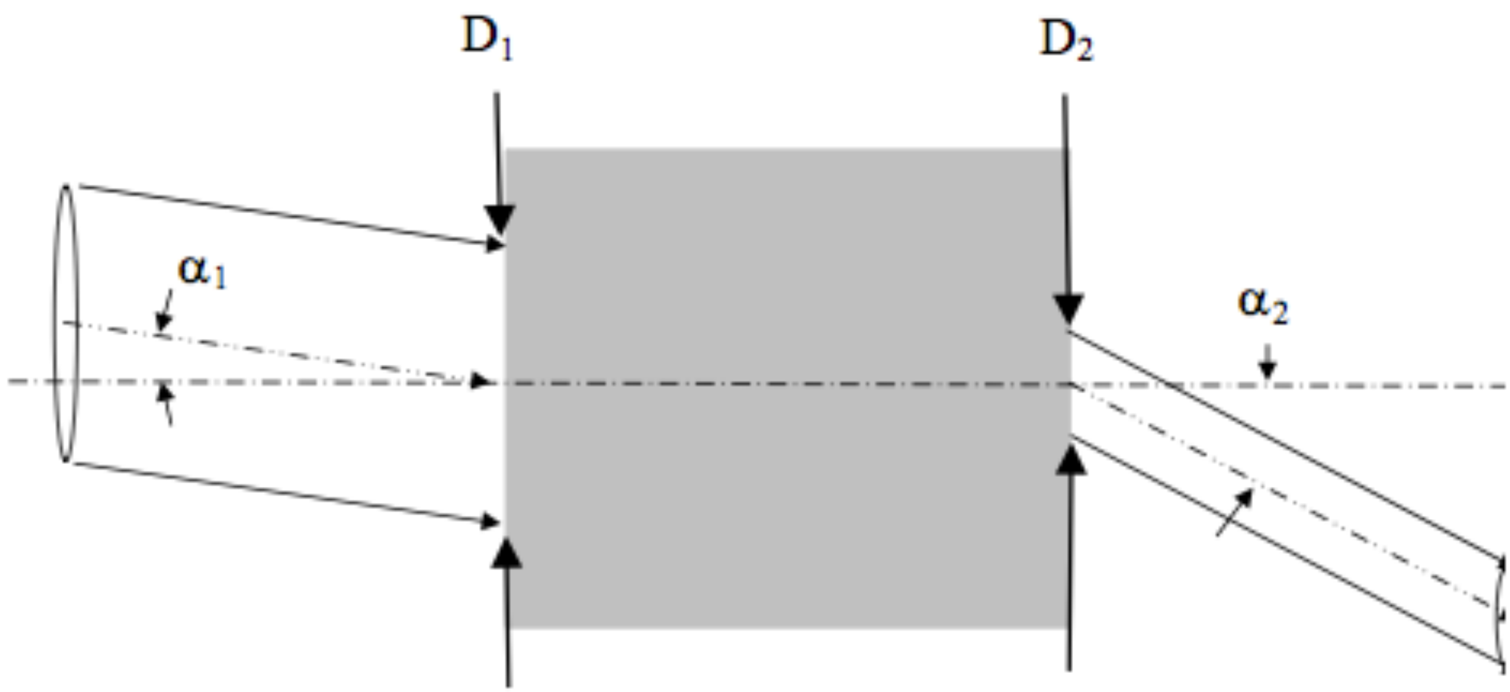
(Calculate the magnitude of the translation using trigonometry)

Rigid body rotation

- Rotation about one point on an object is equivalent to rotation about any other point plus a translation.
- We can choose any point you want to rotate about as long as you keep track of the translation
- To calculate effect of rotating an optical system:
  – Decompose rotation to
    - translation of the nodal point
    - rotation about that point
  – Image motion will be caused only by ***translation of nodal point***

Afocal system

Do not create a real image -- object at infinity, image at infinity



D_1 = Entrance Pupil
D_2 = Exit pupil

Afocal systems

It makes stuff appear larger – magnifying power

$$MP = \frac{\alpha_2}{\alpha_1}$$

LaGrange Invariant requires $D_1\alpha_1 = D_2\alpha_2$

Examples:
Galilean, Keplerian telescope, laser beam projector
Binoculars