ME 297
Opto-mechanical Systems Analysis
L3

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SJSU

1. Plane parallel plates

- 1st order lateral displacement
- Focus shift
- Aberrations
- Applications

Plane Parallel plates

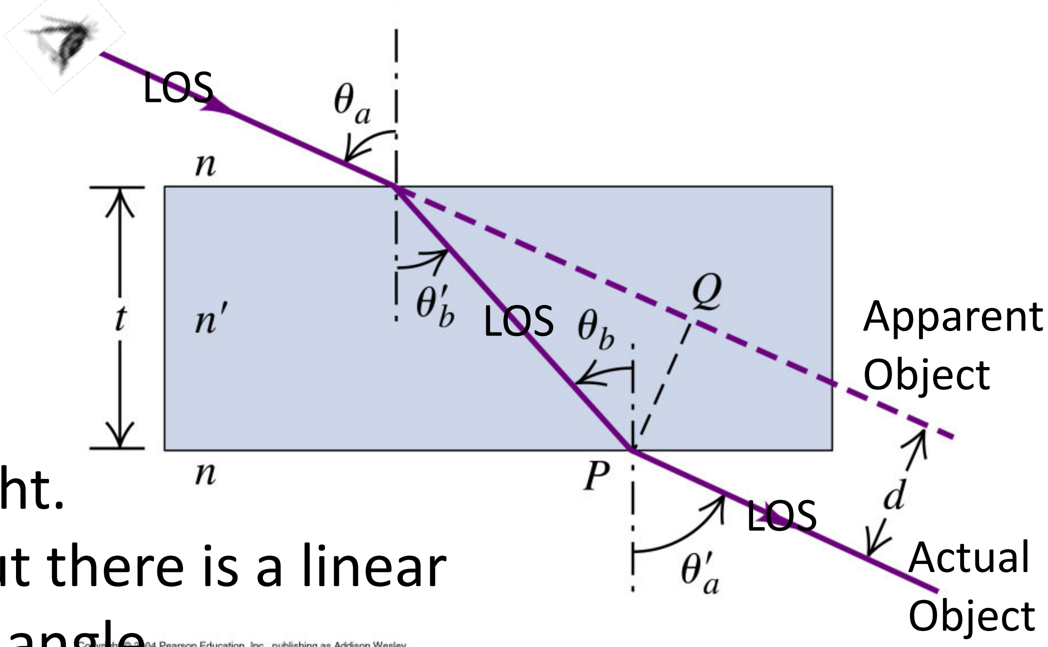
1. For any number of parallel plates:

$$\theta_a = \theta_a^{'} \quad \text{and} \quad \theta_b = \theta_b^{'} \quad \text{and} \quad \theta_c = \theta_c^{'} \quad \text{etc.}$$

2. The lateral displacement d of the emergent beam is given by the relation:

$$d = t\frac{\sin(\theta_a - \theta_b^{'})}{\cos\theta_b^{'}}$$

3. System line of sight follows the actual path of light. No angular change of LOS but there is a linear deviation function of the tilt angle



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Plane Parallel plates and 1st order lateral displacement

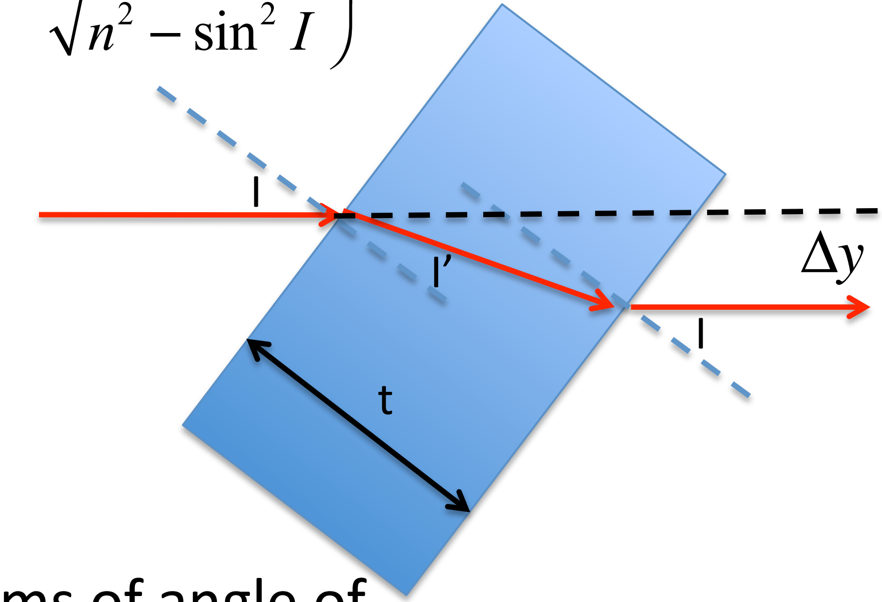
$$\Delta y = t\frac{\sin(I-I')}{\cos I'} = t\frac{\sin I\cos I' - \cos I\sin I'}{\cos I'} = t\left(\sin I - \frac{\cos I}{\cos I'}\frac{\sin I}{n}\right)$$

$$\Delta y = t\sin I\left(1-\frac{\sqrt{1-\sin^2 I}}{n\sqrt{1-\sin^2 I'}}\right) = t\sin I\left(1-\frac{\sqrt{1-\sin^2 I}}{\sqrt{n^2-\sin^2 I}}\right)$$

A power series expansion yeilds:

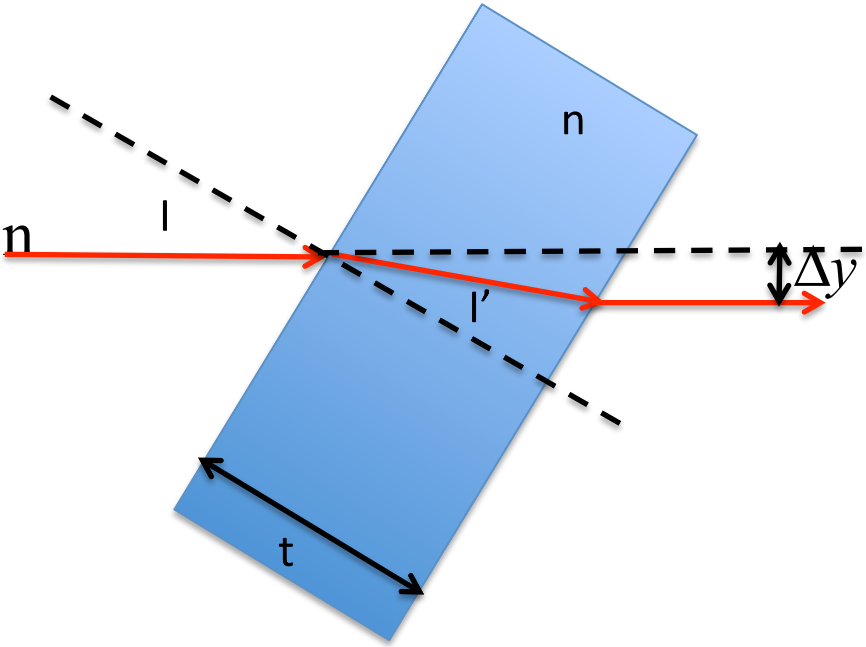
$$\Delta\mathrm{y} = \frac{tI\left(n-1\right)}{n}\left[1+\frac{I^2\left(-n^2+3n+3\right)}{6n^2}+\ldots\right]$$

$$\boxed{\Delta\mathrm{y} = \frac{tI\left(n-1\right)}{n}}$$



Lateral deviation is expressed in terms of angle of incidence, material property and system geometry

Lateral displacement of a ray by a tilted Plane Parallel plate



$$\frac{\sin(I-I')}{\cos I'}=\frac{\Delta y}{t}\simeq(I-I')$$

Using the small angle approximation

& the Snell's law $I=nI'$, we get:

$$\boxed{\Delta y=\frac{n-1}{n}tI}$$

Where Δy is the lateral displacement of the incident beam.

If plane parallel plates are used for **plane waves (parallel beams)**, they are **free of aberration.** But **for converging and diverging beams they introduce aberration**.

Plane Parallel plates and focus shift

Using the Snell's law and the small angle approximation:

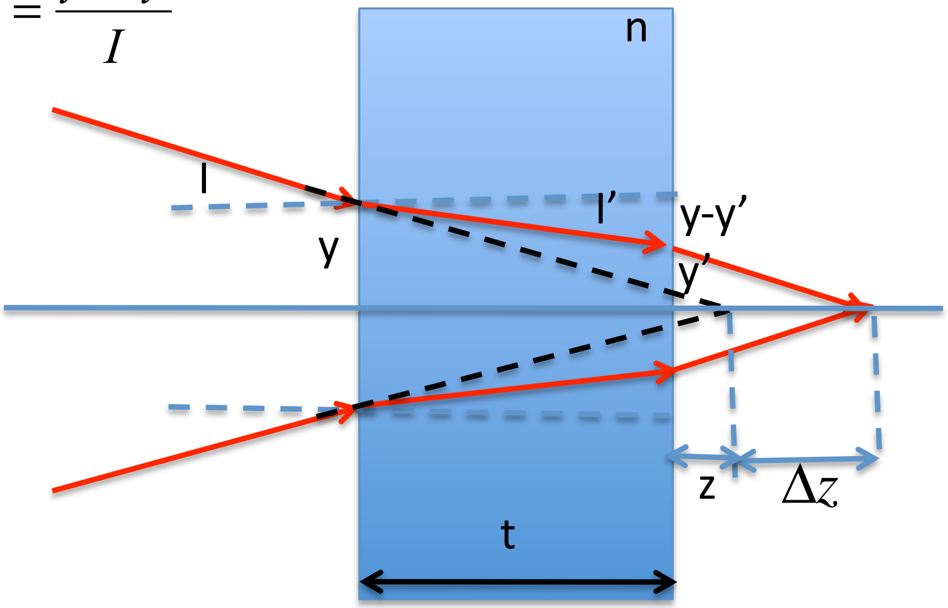
$$\sin I = n \sin I' \quad \& \quad \sin I \simeq \tan I \simeq I \rightarrow I = nI'$$

$$\left.\begin{array}{l} I \simeq \tan I = \dfrac{y}{z+t} \rightarrow z = \dfrac{y}{I} - t \\ I \simeq \tan I = \dfrac{y'}{z+\Delta z} \rightarrow z = \dfrac{y'}{I} - \Delta z \end{array}\right\} \Delta z - t = \dfrac{y' - y}{I}$$

$$\Delta z = t - \frac{y - y'}{I}$$

$$I' \simeq \tan I' = \frac{y-y'}{t} \rightarrow y - y' = I't$$

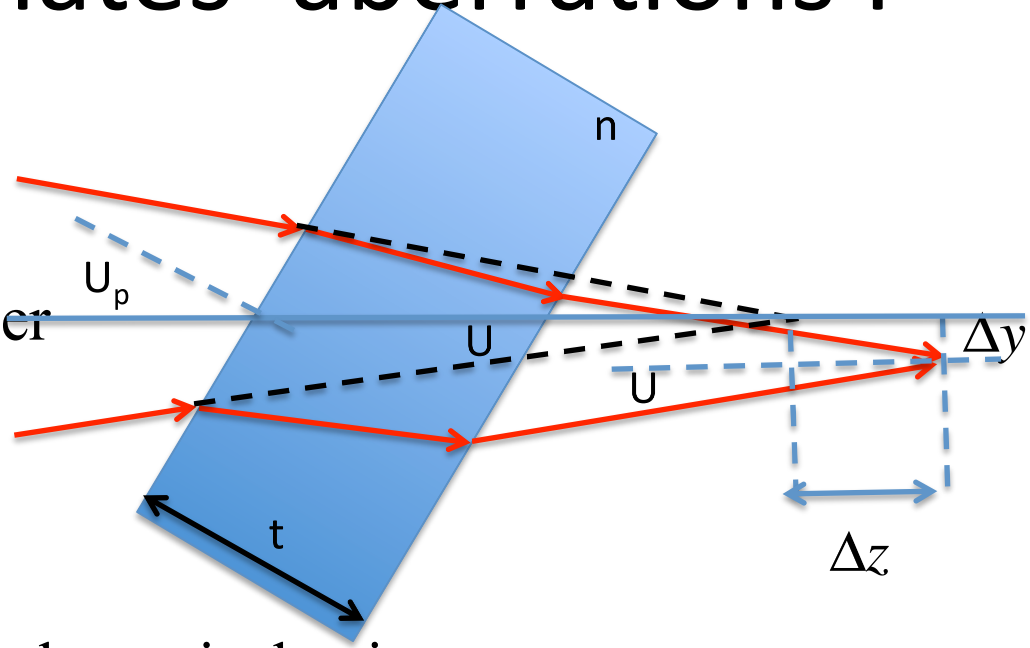
$$\Delta z = t - \frac{I't}{I} = t\left(1 - \frac{1}{n}\right)$$



$\underline{\Delta z = t\left(\frac{n-1}{n}\right)}$ **Shift in focus** or Δz is **independent** of the **orientation** or **tilt**.

Thus **equivalent air thickness** for a parallel plate is: $t - t\left(\frac{n-1}{n}\right) \rightarrow \boxed{t_{eq} = \frac{t}{n}}$

Plane Parallel plates' aberrations I



TO ≡ Third Order

$$V = \frac{n_d - 1}{n_F - n_C} \leftarrow \text{Abbe V number}$$

Angles in capital for exact,

lower case for third order.

U & u: angles of the rays with the optical axis.

U_p & u_p : the tilt angles of the plate.

$$W_{x\lambda} = \Delta y_F - \Delta y_C = \frac{tU_p(n-1)}{n^2 V} \leftarrow \text{Transverse chromatic}$$

$$W_{z\lambda} = \Delta z_F - \Delta z_C = \frac{t(n-1)}{n^2 V} \leftarrow \text{Longitudinal chromatic}$$

Plane Parallel plates' aberrations II

$$L'-l'=\frac{t}{n}\left(1-\frac{n\cos U}{\sqrt{n^2-\sin^2 U}}\right)\leftarrow \text{Spherical aberration (exact)}$$

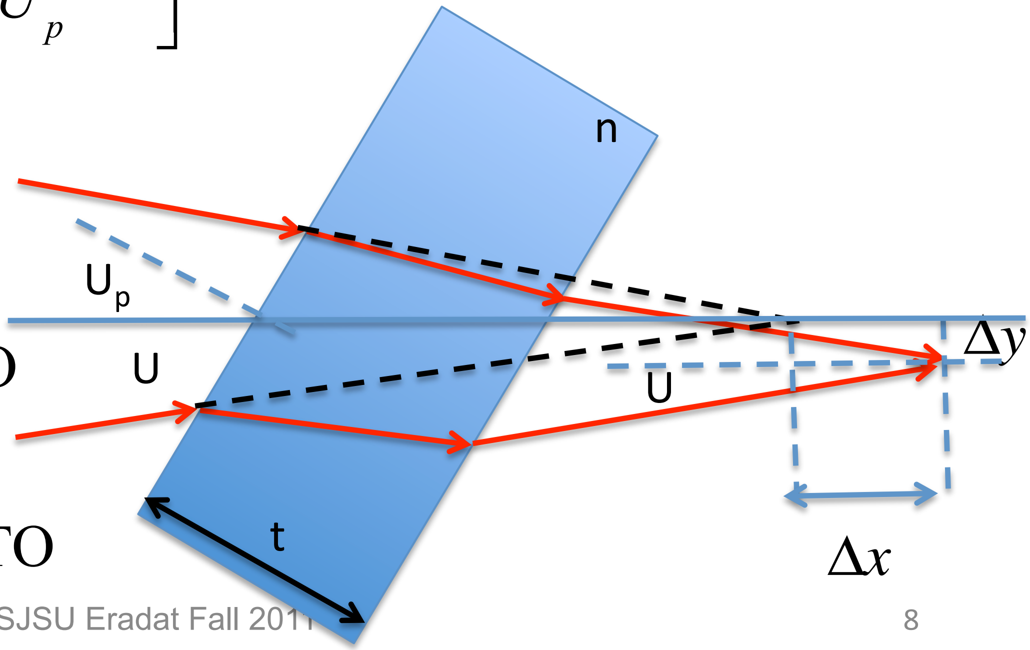
$$L'-l'=\frac{-tu^2(n^2-1)}{2n^2}\leftarrow \text{Spherical aberration TO}$$

$$l_s^{'}-l_t^{'}=\frac{t}{\sqrt{n^2-\sin^2 U_p}}\left[\frac{n^2\cos^2 U_p}{n^2-\sin^2 U_p}-1\right]\leftarrow \text{Astigmatism (exact)}$$

$$\frac{-tu_p^2\left(n^2-1\right)}{n^3}\leftarrow \text{Astigmatism TO}$$

$$\frac{tu^2u_p\left(n^2-1\right)}{2n^3}\leftarrow \text{Sagital coma TO}$$

$$\frac{tu_p\left(n-1\right)}{n^2V}\leftarrow \text{Lateral chromatic TO}$$



Applications of the plane parallel plates and challenges

- Tilted at 45^0 is usually used as beam splitters, however introduce astigmatism as much as t/4 unless $U_p=0$ means only parallel beams. This happens since image on meridional plane shifts backward more than the image on the sagittal plane.
- Tricks to eliminate astigmatism include
  - introduction of another plate at 90^0 with respect to the original one
  - weak cylinder
  - tilted spherical surface
  - wedging the plate.
- Not recommended for the diverging or converging beams.

2. Stops and pupils and other basic principles

- Aperture stop
- Entrance and exit pupils
- Field stop
- Entrance and exit window
- Vignetting

Special rays

- The chief or principal ray is a ray from an object point that passes through the axial point, in the plane of aperture stop (AS).
- Marginal ray: on-axis ray that goes through edge of the aperture stop.

Stops, pupils, windows

Stops, pupils, windows are of great importance for <u>control of light in optical instrumentation</u>.

Not all rays leaving the object participate in image formation.

<u>Aperture</u>: an opening defined by a geometrical boundary that creates spatial limitation for the light beams.

Apertures are used to:

generate sharp <u>boundaries for images</u>

<u>correct aberrations</u> such as spherical, astigmatism and distortion

<u>shield the image from undesirable scattered light</u>.

Effects of aperture in an optical system:

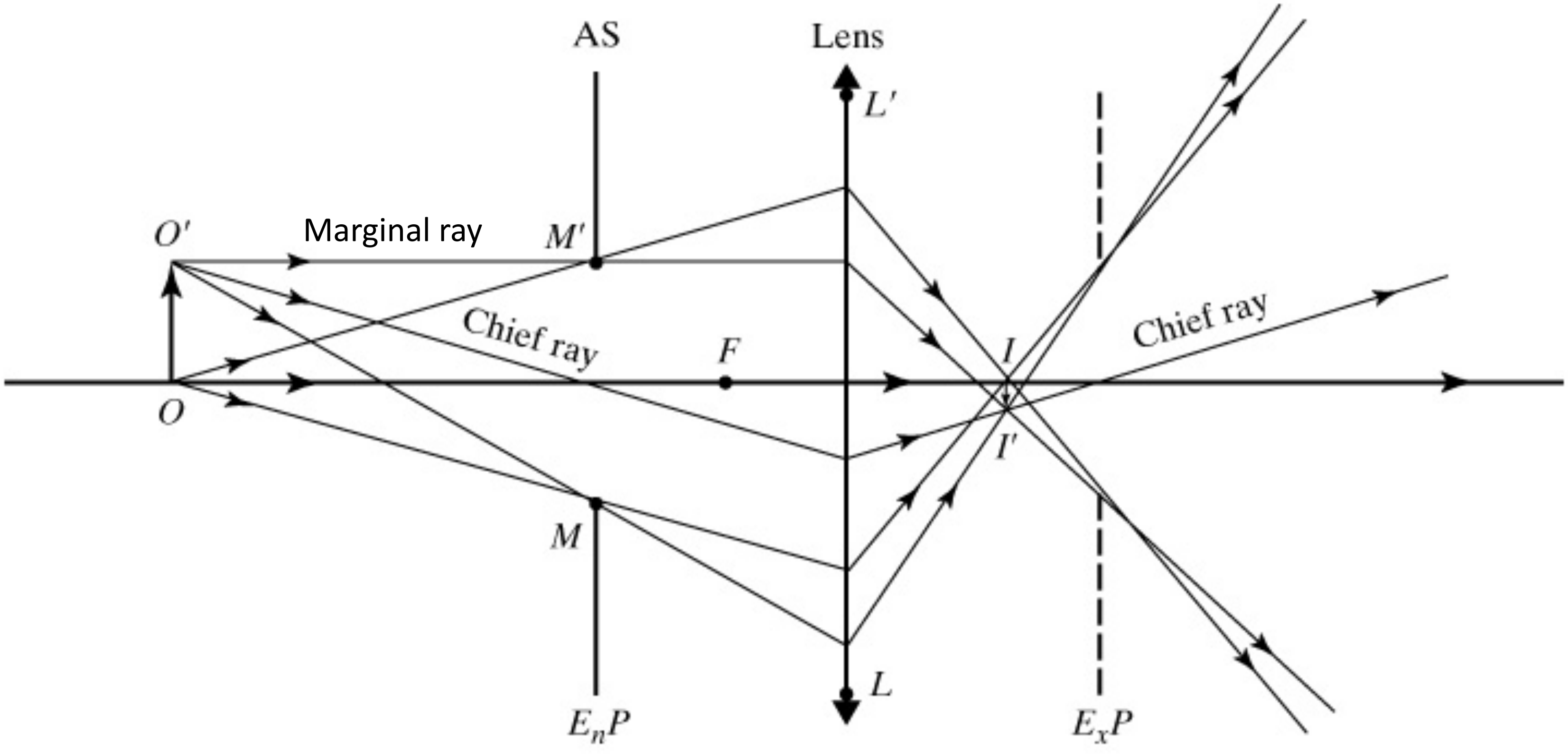
<u>limiting the field of view</u>

<u>controlling the image brightness</u> (irradiance W/m^2)

Aperture stop (AS)

The aperture stop of an optical system is the ***actual physical component*** that limits the size of the maximum cone of rays-from an object point to an image point –that can be processed by the entire optical system.

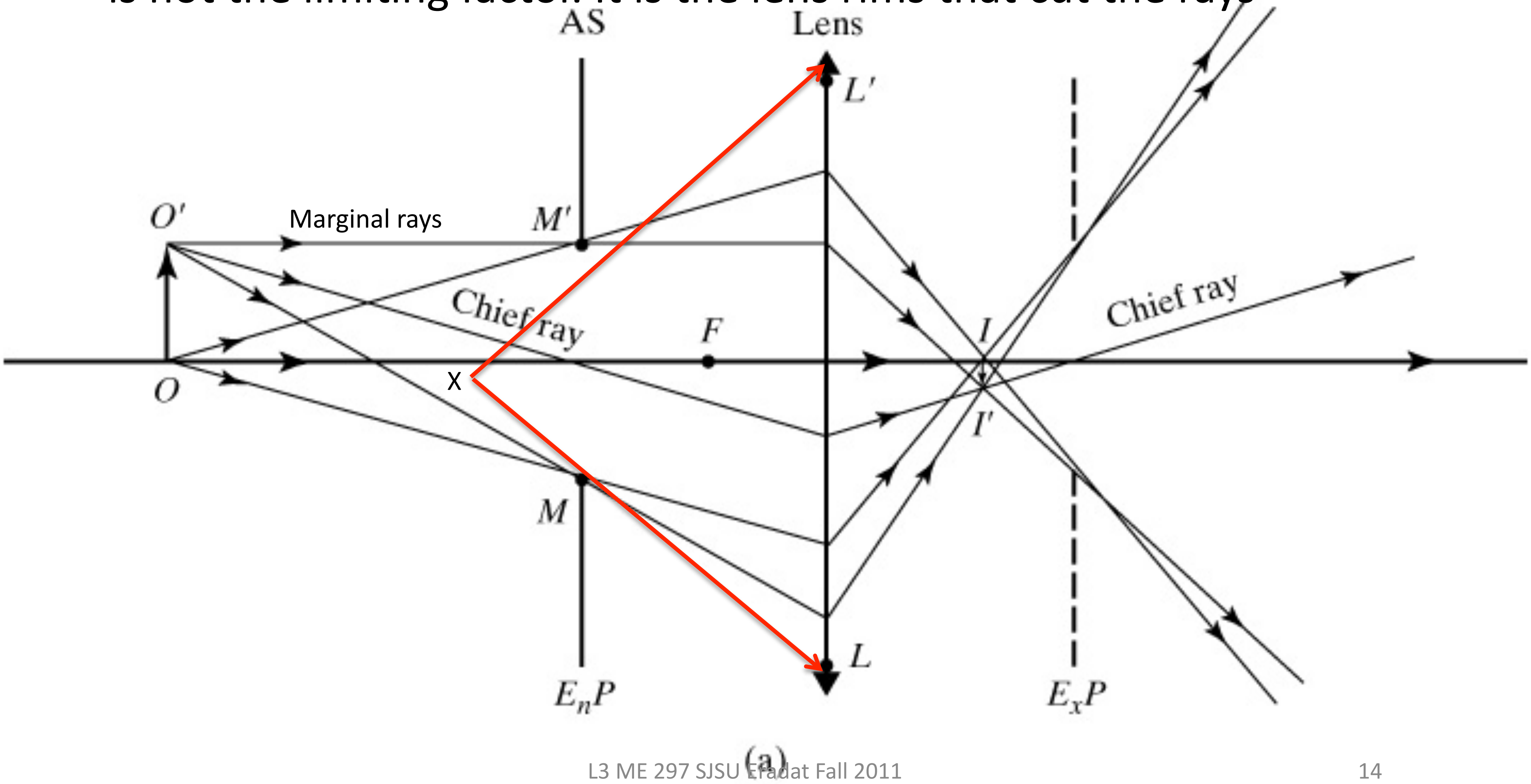
Example: diaphragm of a camera or iris of the human eye.



(a)

Aperture stop (AS)

<u>Location of the AS depends on the object point. The limiting component not always the same.</u> **Example:** for object point X, AS is not the limiting factor. It is the lens rims that cut the rays



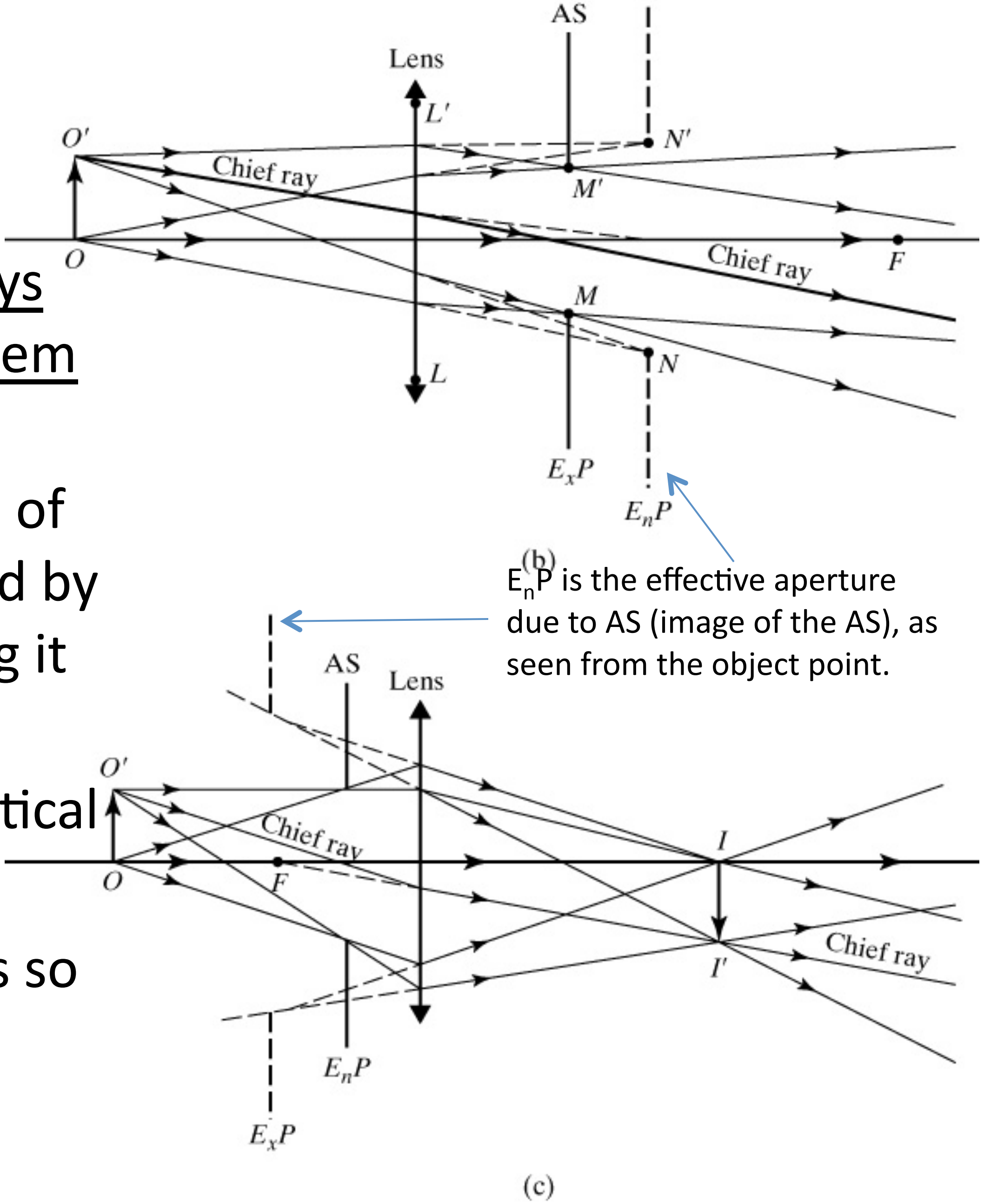
Entrance Pupil E_nP

Entrance pupil is the <u>limiting aperture (opening) that light rays see looking into the optical system from any object point</u>.

The entrance pupil is the image of the stop in object space (formed by the imaging elements preceding it or on the left of stop).

Sometimes AS and E_nP are identical but in this figure they differ.

AS and E_nP are conjugate points so E_nP is image of AS.

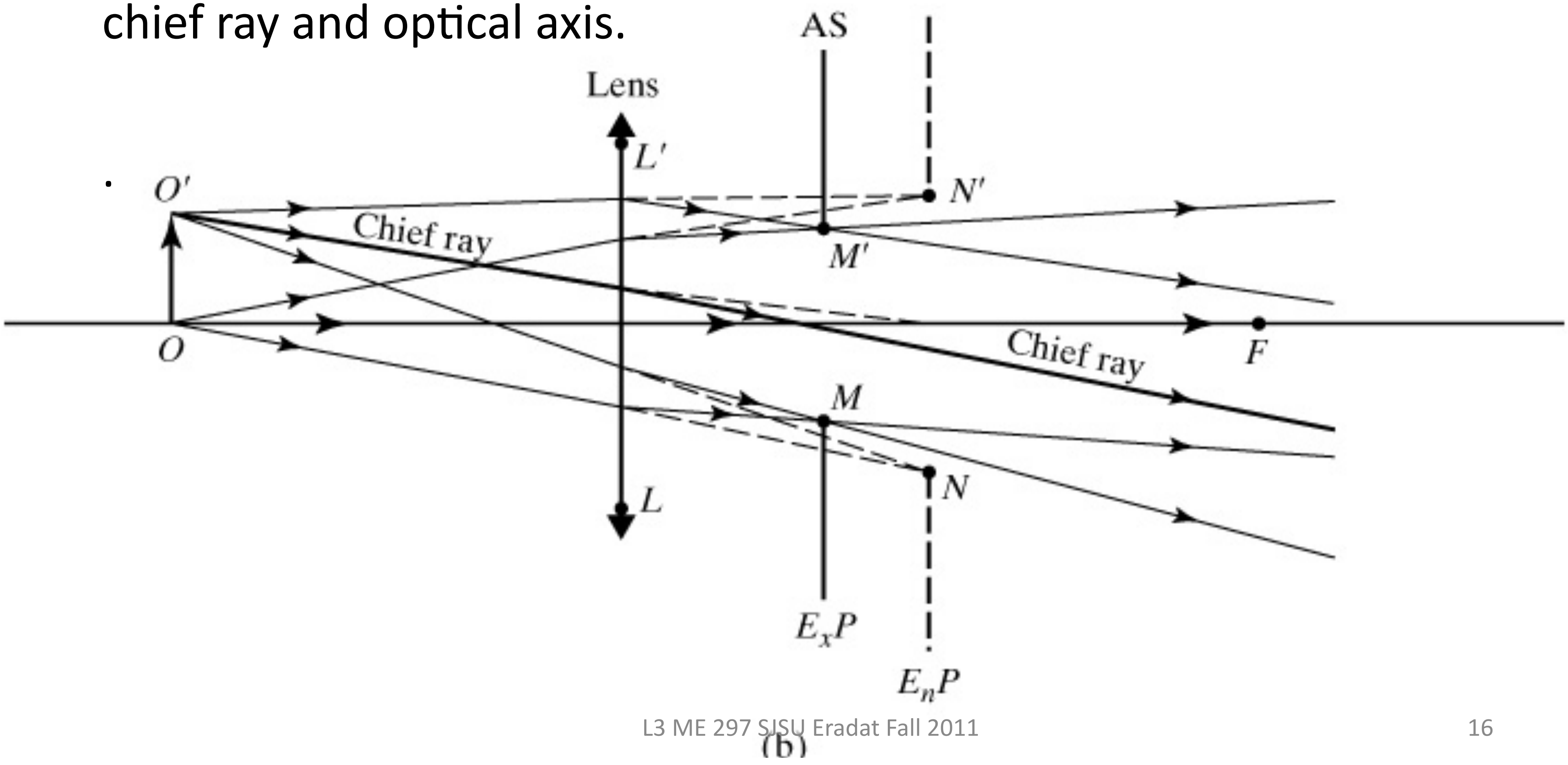


E_nP is the effective aperture due to AS (image of the AS), as seen from the object point.

Location and size of the entrance pupil

Location of the entrance pupil plane E_nP is intersection of the extension of the chief ray and the optical axis.

Size of the E_nP is marginal ray height of pupil image in object space. Extend the marginal ray and height of the ray at intersection of the chief ray and optical axis.



Exit pupil E_XP

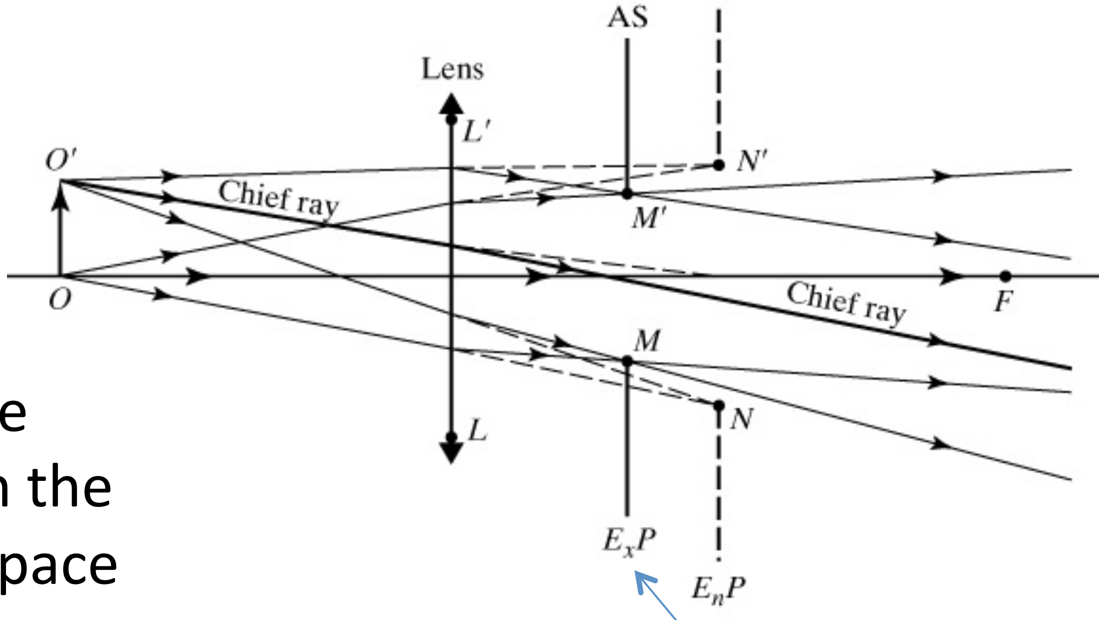
image space. E_XP limits the output beam size.

Exit pupil is the image of the aperture stop (as seen from the image point) in the image space (formed by the imaging elements following it on the right of the stop).

E_XP is located where the extension of the chief ray crosses the optical axis.

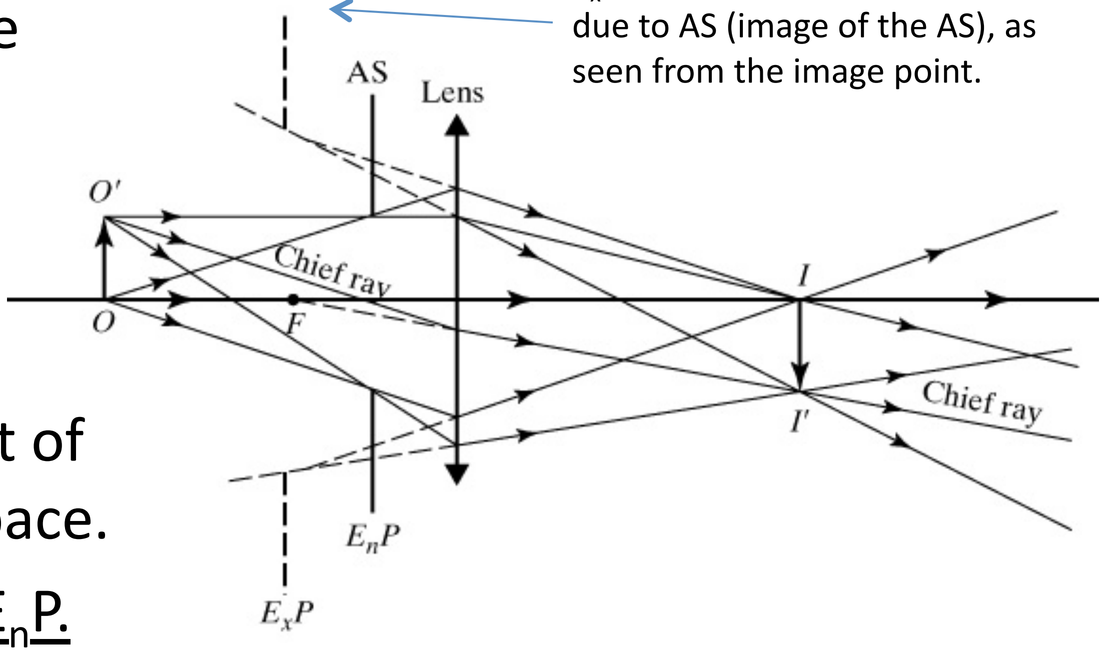
Sized by marginal ray height of pupil image in the image space.

E_XP is conjugate of AS and E_nP.



(b)

E_xP is the effective aperture due to AS (image of the AS), as seen from the image point.



(c)

Field stops, Entrance Window, Exit window

Field stop (FS): the aperture that controls the field of view to **eliminate poor quality image points** due to aberration or vignetting.

Practical criteria to determine field stop: as seen from the center of the entrance pupil, the field stop or its image subtends the smallest angle.

Entrance window (E_nW): is the image of the field stop by all optical elements preceding it (to the left of it). It outlines the lateral dimensions of the object being imaged. Conjugate of FS.

Exit window (E_xW): is the image of the field stop by all optical elements following it (to the right of it). This is like a window limiting outside view as seen from inside of a room.

Field stops, Entrance Window, Exit window

Field of view in object space: angle subtended by entrance window at the center of the entrance pupil

Field of view in image space: angle subtended by exit window at the center of the exit pupil

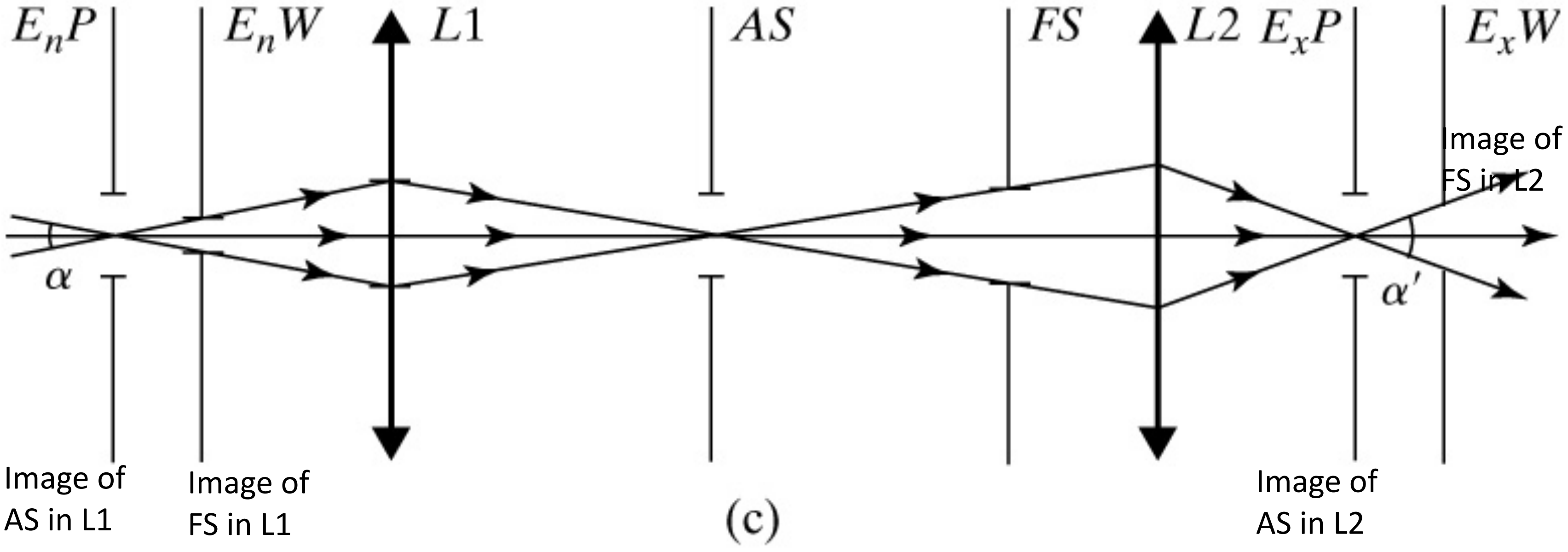


Image of FS in L2

Image of AS in L1

Image of FS in L1

Image of AS in L2

Summary: Stops, Pupils, Windows

- Brightness
  - Aperture stop AS: The real element in an optical system that limits the size of the cone of rays accepted by the system from an axial object point.
  - Entrance pupil EnP: The image of the aperture stop formed by the elements (if any) that precede it.
  - Exit pupil ExP: The image of the aperture stop formed by the elements (if any) that follow it.
- Field of view
  - Field stop FS: The real element in an optical system that limits the angular field of view formed by an optical system.
  - Entrance window EnW: The image of the field stop formed by the elements (if any) that precede it.
  - Exit window ExW: The image of the field stop formed by the elements (if any) that follow it.

3. Camera

- Pinhole camera
- Simple camera
- Camera lens types
- Image-object motion and Newtonian equation
- Aperture, F#, Irradiance
- Aperture size and depth of field
- Requirements for camera lenses

The pinhole camera

Simplest form of camera is a pinhole camera. There is no focusing and every point of image is constructed by the rays that are approximately coming from a point if the pinhole is small enough.

Smaller pinholes cause diffraction.

Optimal pinhole size: 0.5 mm

Optimal film distance: 25 cm

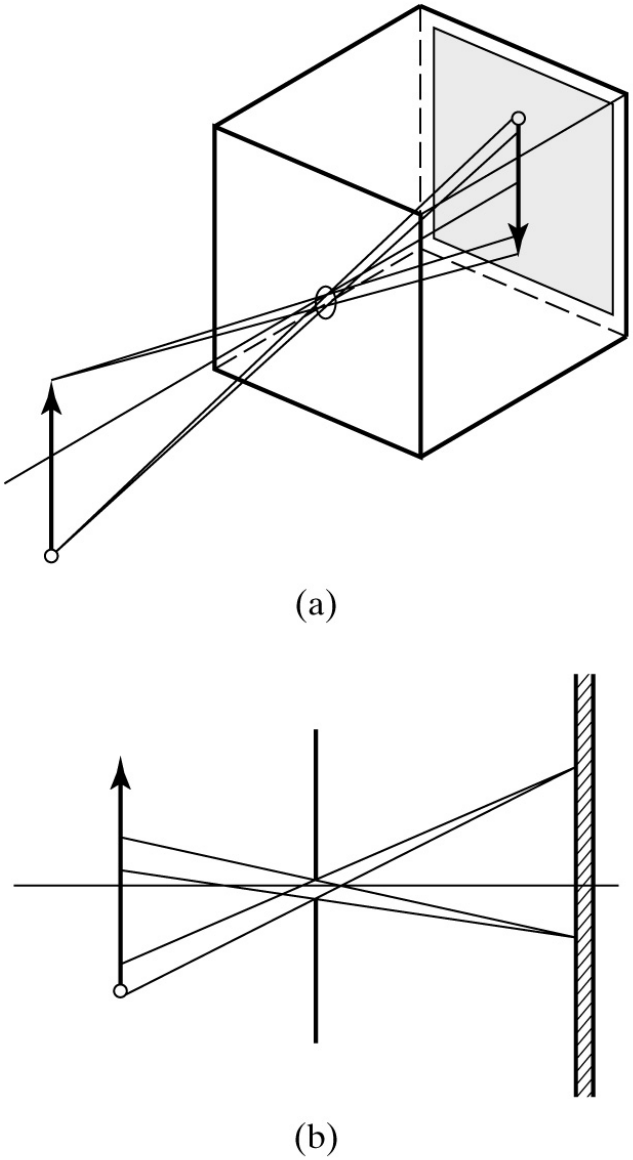
Image properties:

Unlimited depth of field: since there is no focusing element so all the objects appear sharp.

Limited image brightness

Limited image sharpness

Imaging by a pinhole camera



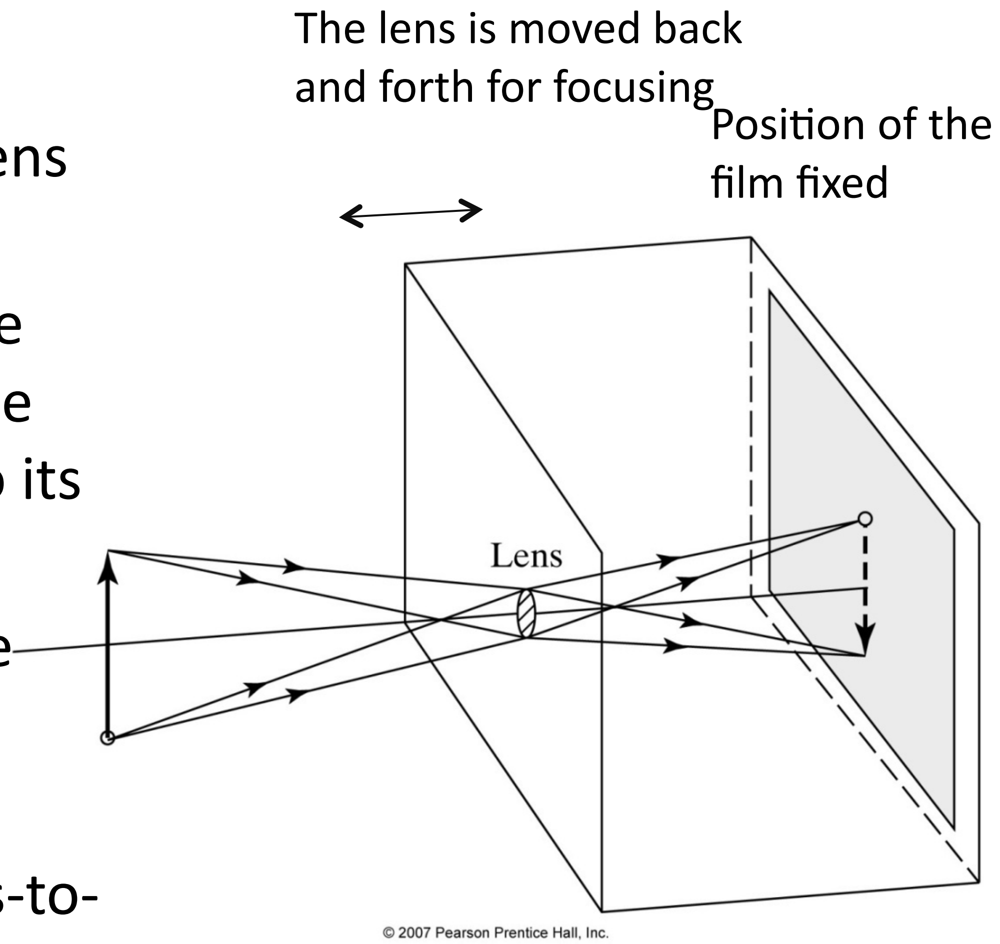
(a)

(b)

The simple camera

By enlarging the aperture in a pinhole camera and placing a lens in it several changes happen:

1. **Increase in brightness** of the image due to focusing all the rays from an object point to its conjugate on the film
2. **Increase in sharpness** of the image due to the focusing power of the lens.
3. **Increased sensitivity** to lens-to-film distance.
4. For object at infinity the film is at the focal point of the lens.



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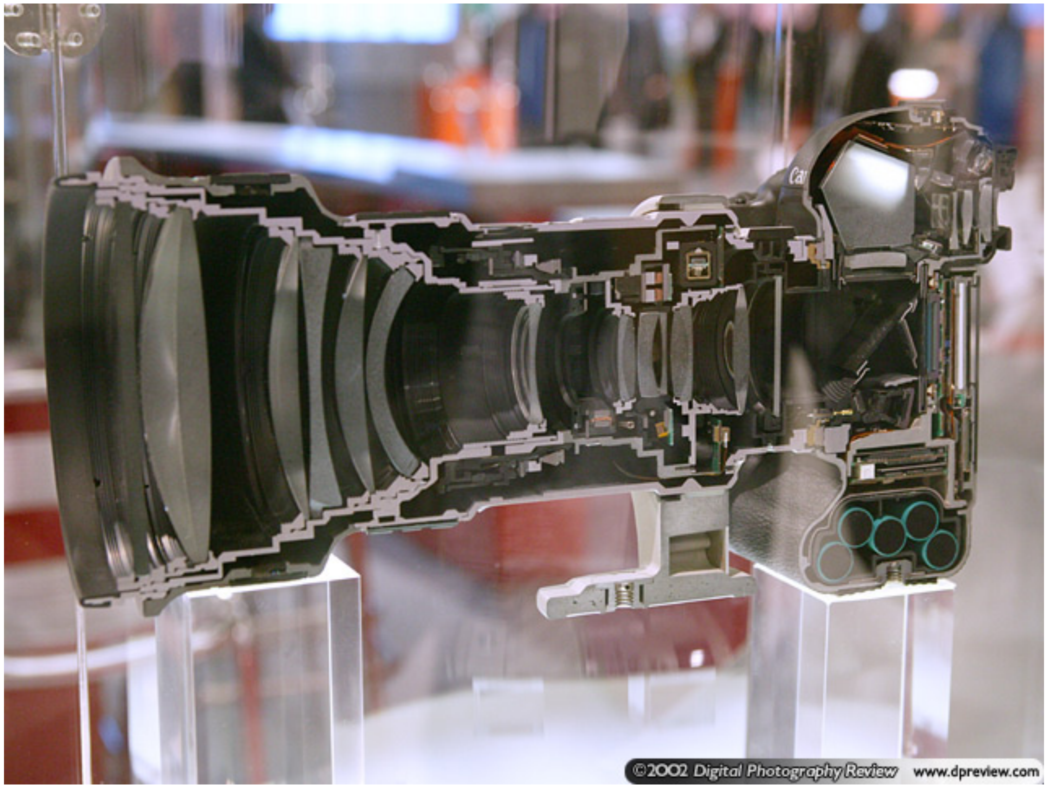
Camera lens types

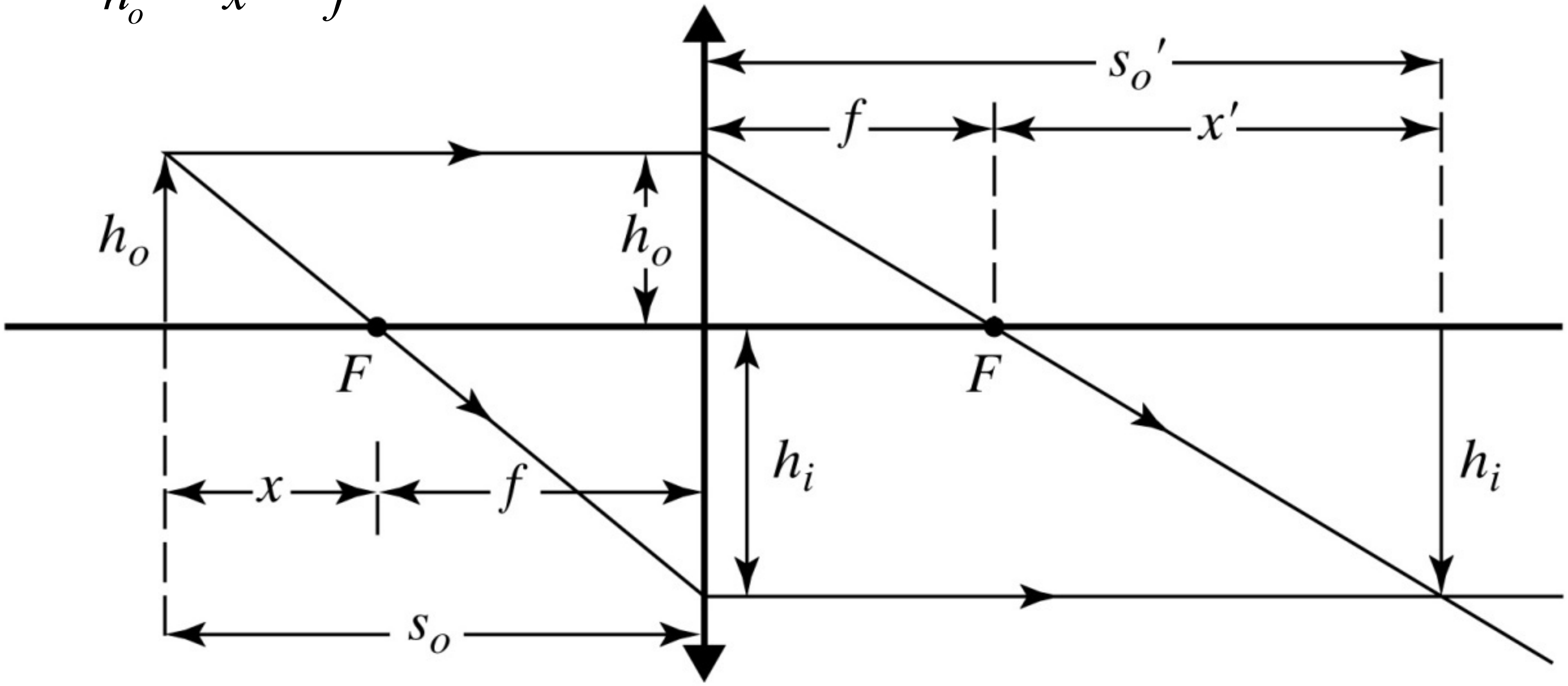
1. **Close-ups**: lenses with short focal length that can handle near objects.
2. **Telephoto**: lenses with long focal length that images far object at the expense of subject area.
3. **Wide-angle**: lenses with short focal length and large field of view.
4. Combination of positive and negative lenses is used to avoid a long camera tube .



Newtonian equation for the thin lens

- The object and image distances are measured from the focal points like the picture.
- The equation is simpler and is used in certain applications like cameras.

$$m = \frac{h_i}{h_o} = \frac{f}{x} = \frac{x'}{f} \rightarrow xx' = f^2$$



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Camera aperture, irradiance and f#

Two elements control the amount of admitted light into the camera:

1) The aperture size

2) Shutter speed

$$\text{Irradiance} = \frac{\text{light power incident at the image plane}}{\text{area of the film or CCD or CMOS imager}} \propto \text{Relative aperture}$$

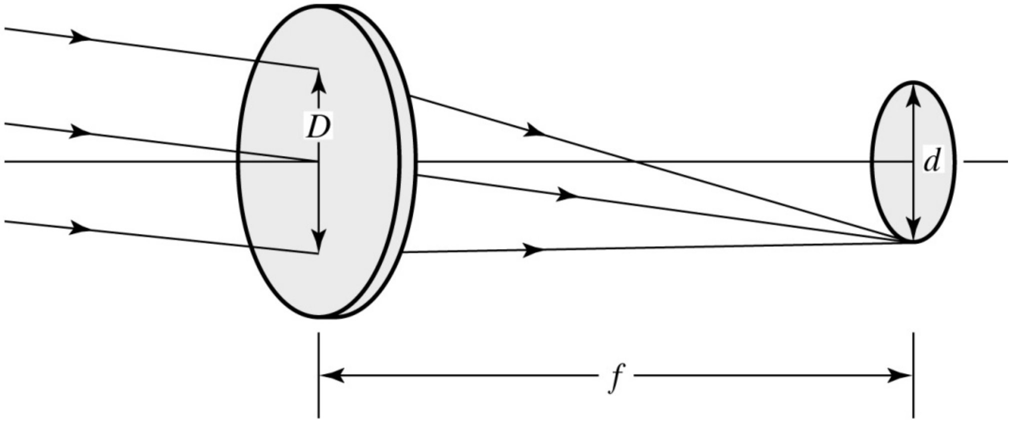
$$\text{Relative aperture of a lens: } E_e \propto \frac{\text{area of aperture}}{\text{area of image}} = \frac{D^2}{d^2}$$

Since image size is proportional to the focal length of the lens $d \propto f$

$$E_e \propto \left(\frac{D}{f}\right)^2$$

We define $f\,\#$ of a lens: $\boxed{f\,\# = \frac{f}{D}}$

Relative aperture: $\boxed{E_e \propto \frac{1}{(f\,\#)^2}}$



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F-number and irradiance

Selectable apertures in cameras usually provide steps that change irradiance, E_e, by a factor of 2, the corresponding $f-number$ changes by a factor of $\sqrt{2}$.

Larger $f-number \rightarrow$ smaller aperture area & smaller exposure, since $f/\# = \dfrac{f}{D}$.

$$\text{Exposure}\left(\frac{J}{m^2}\right) = E_e\left(\frac{J}{m^2 \cdot s}\right) t(s)$$

So for a given film speed or ISO-number variety of $f/\#$ and shutter speed combinations can provide satisfactory exposure.

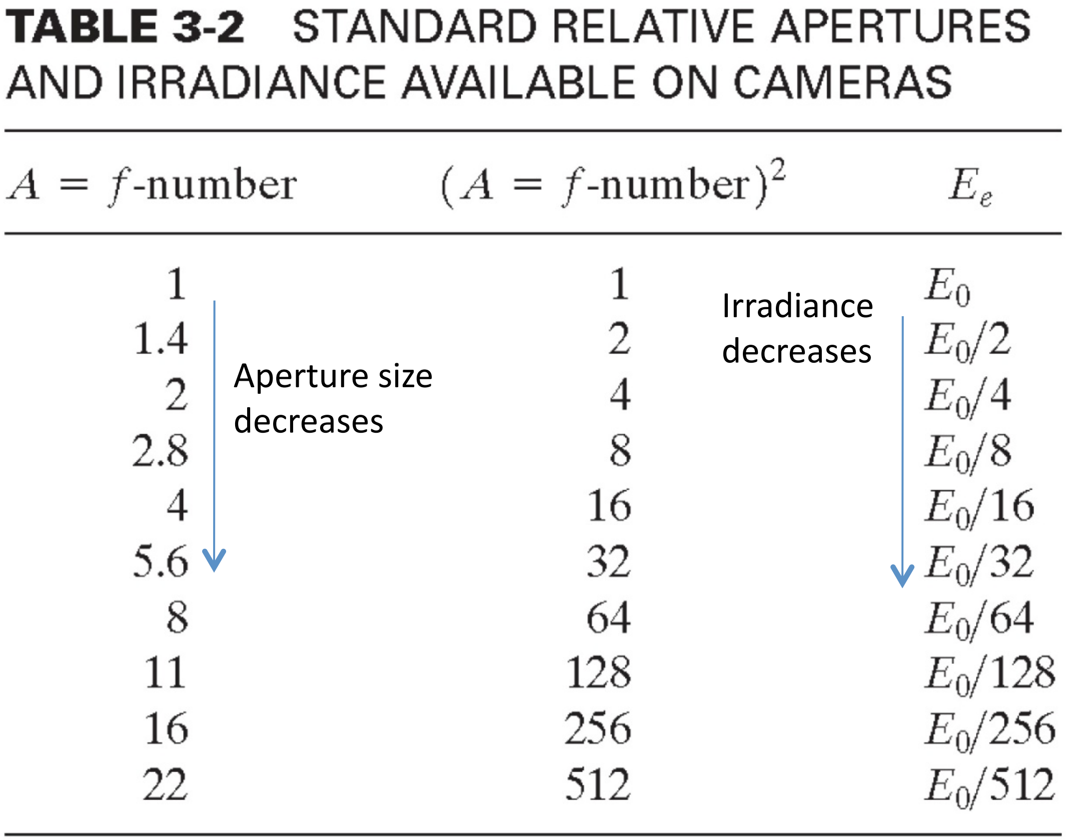


TABLE 3-2 STANDARD RELATIVE APERTURES AND IRRADIANCE AVAILABLE ON CAMERAS

A = f-number	(A = f-number)^2	E_e
1	1	E_0
1.4	2	$E_0/2$
2	4	$E_0/4$
2.8	8	$E_0/8$
4	16	$E_0/16$
5.6	32	$E_0/32$
8	64	$E_0/64$
11	128	$E_0/128$
16	256	$E_0/256$
22	512	$E_0/512$

Aperture size decreases

Irradiance decreases

Aperture size and depth of field: definition

MN : depth of field or distance between the acceptable near-point and far-point.

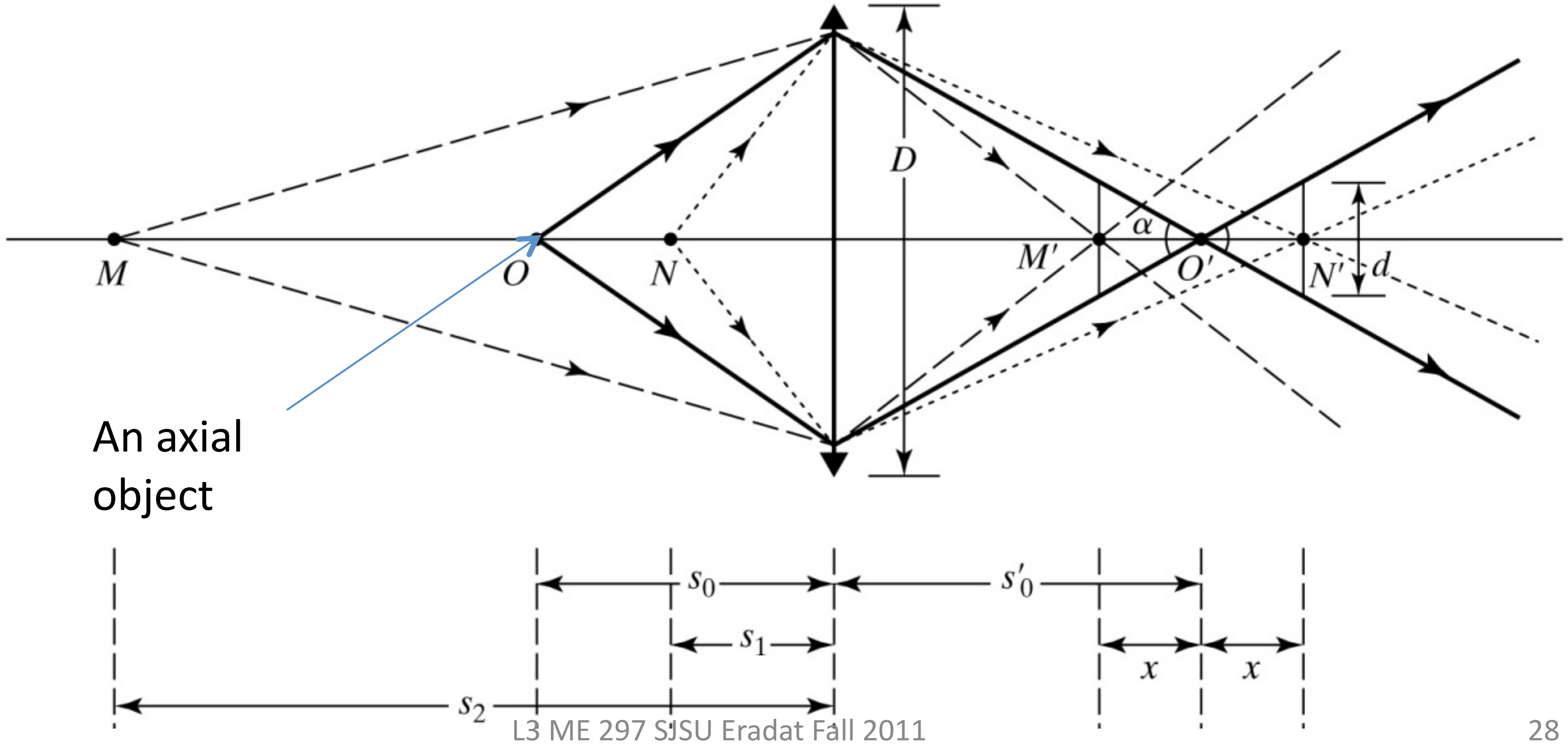
$M'N'$: conjugate of the depth of field in image space

d : or the blurring diameter. This can be the pixel size or film grain size.

x : distance of the images that are acceptable

Near-point of the depth of field is the object distance s_1 for the image at $s'_0 + x$

Far-point of the depth of field is the object distance s_2 for the image at $s'_0 - x$



Aperture size and depth of field II

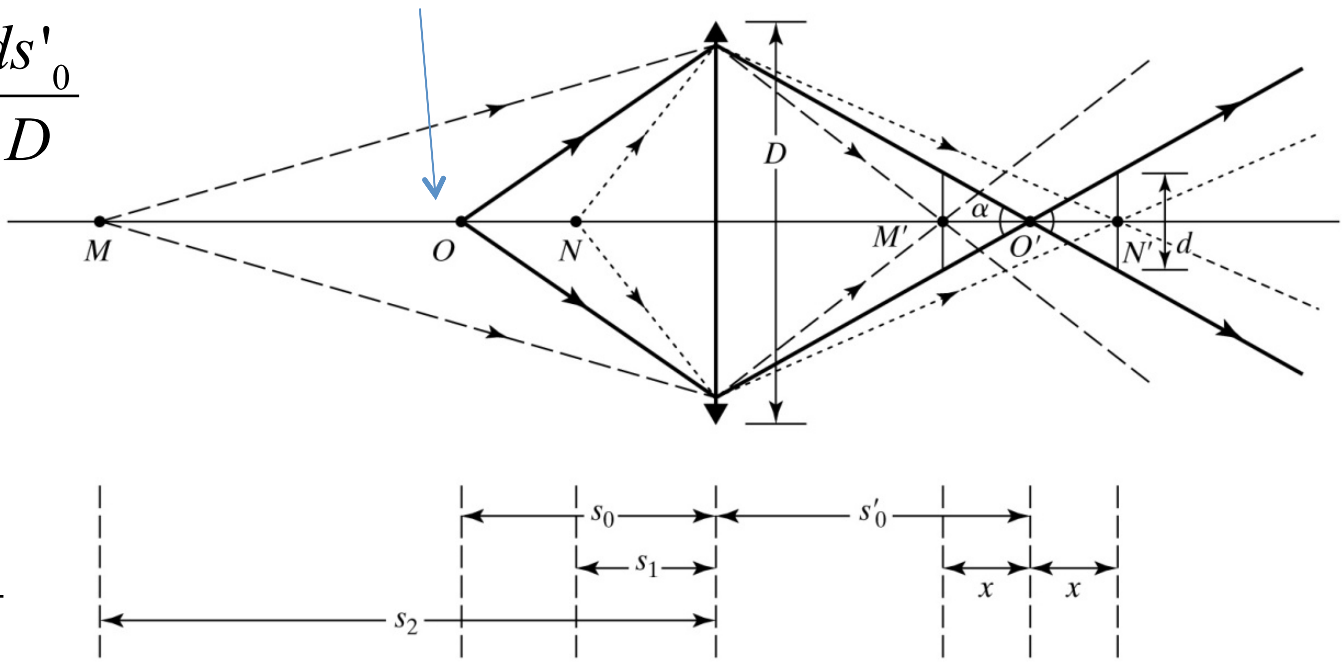
$$\left.\begin{array}{l}\tan\alpha \cong \dfrac{D}{2s'_0} \\[2ex] \tan\alpha \cong \dfrac{d}{2x}\end{array}\right\} x \cong \dfrac{ds'_0}{D}$$

$$s_1 = \frac{s_0 f\left(f + f\#d\right)}{f^2 + f\#ds_0}$$

$$s_2 = \frac{s_0 f\left(f - f\#d\right)}{f^2 - f\#ds_0}$$

$f\# = f\,/\,D$ is the $f\#$

An axial object



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Depth Of the Field: $MN = s_2 - s_1 \rightarrow \boxed{MN = \dfrac{2f\#ds_0\left(s_0 - f\right)f^2}{f^4 - f\#^2 d^2 s_0^2}}$

Aperture size and depth of field derivation (read only if you love derivations)

$$\frac{1}{s_0}+\frac{1}{s'_0}=\frac{1}{f}\rightarrow s'_0=\frac{fs_0}{s_0-f}\text{ with }x\cong\frac{ds'_0}{D}\text{ and }A=\frac{f}{D}=f\#$$

$$\frac{1}{s_1}+\frac{1}{s'_0+x}=\frac{1}{f}\rightarrow \mathrm{s}_1=\frac{f\left(s'_0+x\right)}{s'_0+x-f}=\frac{f\left(s'_0+\frac{ds'_0}{D}\right)}{s'_0+\frac{ds'_0}{D}-f}=\frac{fs'_0\left(1+\frac{Ad}{f}\right)}{s'_0\left(1+\frac{Ad}{f}-\frac{f}{s'_0}\right)}=\frac{f\left(1+\frac{Ad}{f}\right)}{\left(1+\frac{Ad}{f}-\frac{f\left(s_0-f\right)}{fs_0}\right)}$$

$$\mathrm{s}_1=\frac{\left(f+Ad\right)}{\frac{1}{fs_0}\left(\frac{Ads_0}{1}+\frac{f^2}{1}\right)}\rightarrow\boxed{s_1=\frac{s_0f\left(f+Ad\right)}{f^2+Ads_0}}$$

$$\frac{1}{s_2}+\frac{1}{s'_0-x}=\frac{1}{f}\rightarrow \mathrm{s}_2=\frac{f\left(s'_0-x\right)}{s'_0-x-f}=\frac{f\left(s'_0-\frac{ds'_0}{D}\right)}{s'_0-\frac{ds'_0}{D}-f}=\frac{fs'_0\left(1-\frac{Ad}{f}\right)}{s'_0\left(1-\frac{Ad}{f}-\frac{f}{s'_0}\right)}=\frac{f\left(1-\frac{Ad}{f}\right)}{\left(1-\frac{Ad}{f}-\frac{f\left(s_0-f\right)}{fs_0}\right)}$$

$$s_2=\frac{\left(f-Ad\right)}{\frac{1}{fs_0}\left(-\frac{Ads_0}{1}+\frac{f^2}{1}\right)}\rightarrow\boxed{s_2=\frac{s_0f\left(f-Ad\right)}{f^2-Ads_0}}\rightarrow MN=s_2-s_1\rightarrow\boxed{MN=\frac{2Ads_0\left(s_0-f\right)f^2}{f^4-A^2d^2s_0^2}}$$

Requirements on camera lenses

- Large field of view 35^o-65^o for normal lenses and 120^o for wide angle lenses.
- Free from aberration over entire area of the film at focal plane.
- All 5 Seidle aberrations (spherical, coma, curvature of the field, astigmatism, and distortion) plus chromatic must be corrected.
- Computational techniques have relaxed some of these requirements but still designing a good camera lens requires human ingenuity.
- Usually there is more than one solution. The choice depends on compromises and other design concerns.