

ME 297
Opto-mechanical Systems Analysis
L2

Nayer Eradat

Fall 2011

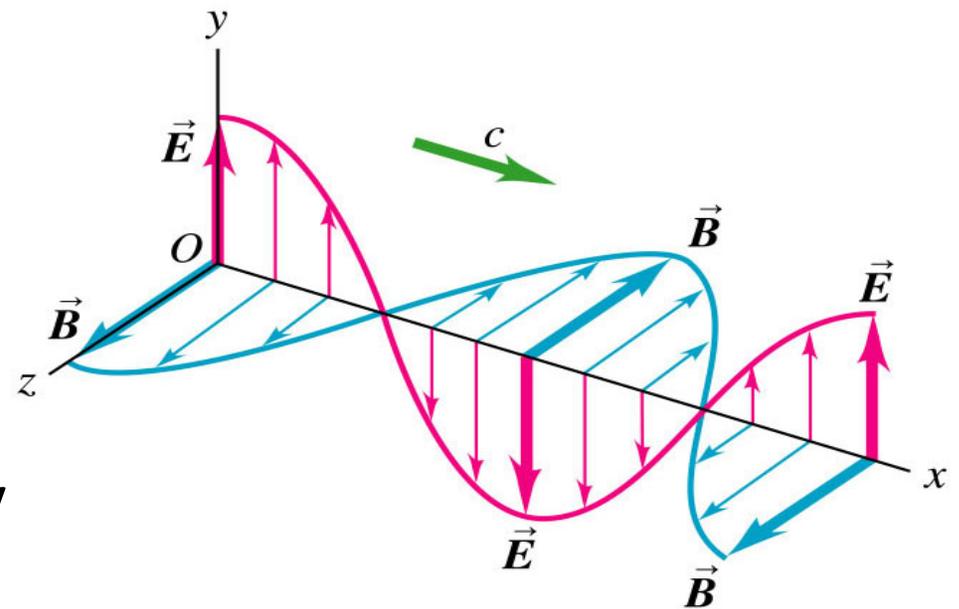
SJSU

Nature of light: a brief history

- 1642-1727 **Stream of particles** called corpuscles (Newtonian)
- 1803 Idea of light being a **wave** grew strong Young's work.
- 1873 Maxwell established (theoretically) that light is an **electromagnetic wave**
- 1887 Hertz showed experimentally that light is indeed composed of electromagnetic waves.
- 1900 Max Planck showed that the electromagnetic radiation is emitted in **quanta (packets) of energy**. He named them photons.
- Quantum optics was born again.
- 1905 Following Planck's work Einstein in 1905 was able to explain the photoelectric effect.
- 1930 By birth of quantum electrodynamics the two pictures are "kind of" reconciled but **the debate is still going on.**

Light as electromagnetic Wave

- Light or electromagnetic wave consists of time-varying electric and magnetic field components which oscillate in phase perpendicular to each other and perpendicular to the direction of energy propagation.
- Light is a wave
- A wave has frequency
- A wave has wavelength
- Color of the light is determined by its frequency



\vec{E} : y-component only
 \vec{B} : z-component only

How to handle dual nature of the light

- Light behaves like an energy carrying particle (photon) when generated or absorbed.
- Light behaves as an electromagnetic wave when it is propagating in a medium.
- None of these scenarios are absolute absorption and re-generation are always present during the propagation process.

What is a wave?

- A self sustaining energy-carrying disturbance of a medium through which it propagates.
 - Longitudinal wave: the medium is displaced in the direction of motion of the wave.
 - Transverse wave: the medium is displaced in a direction perpendicular to that of the motion of the wave.
- Why waves can propagate faster than the medium in which they propagate?
- When a wave propagates, the disturbance advances, not the medium (Leonardo da Vinci).

Harmonic waves

Harmonic waves are smooth patterns that repeat endlessly.

They involve the sine and cosine functions.

Wavefunction : is a mathematical expression that represents the wave motion in space and time.

Wavefunctions of the simple harmonic waves are **sin** and **cos** functions.

$$y = A \cos(kx - \omega t + \phi)$$

$$y = A \sin(kx - \omega t + \phi)$$

Information hidden in a wave function

$$y = \underbrace{A}_{\text{Amplitude}} \cos\left(\underbrace{kx - \omega t + \phi}_{\text{Phase}}\right) \text{ or } y = A \sin\left(kx - \omega t + \underbrace{\phi}_{\text{Initial Phase}}\right)$$

$$k = \frac{2\pi}{\lambda} \quad \text{Wave number: number of wavelengths in unit length}$$

$$\lambda = \frac{2\pi}{k} \quad \text{Wavelength: Length of one complete cycle; } \lambda_0 \text{ is } \lambda \text{ in vacuum.}$$

$$f = \frac{V}{\lambda} = \frac{c}{\lambda_0} \quad \text{Frequency: number of complete cycles per second}$$

$$\omega = 2\pi f \quad \text{Angular frequency: the amount of phase accumulated/second}$$

$$T = \frac{1}{f} = \frac{\lambda}{V} = \frac{\lambda_0}{c} \quad \text{Temporal period: Time required for a complete a cycle.}$$

For a wave f & T are constant, λ & V change with material.

If signs of kx and ωt are opposite, the wave propagates in $+x$ direction

If signs of kx and ωt are the same wave propagates in $-x$ direction

Light: energy carrying particle or photon

$$E = hf = h \frac{c}{\lambda_0} = h \frac{c}{\lambda n}$$

Where h is the Planck's constant

$$h = 6.62606957(29) \times 10^{-34} \text{ J.s} = 4.135667516(91) \times 10^{-15} \text{ eV.s}$$

f is the frequency in Hz,

λ_0 is the wavelength in m (in vacuum)

λ is the wavelength in m (in the medium)

c is speed of light in m/s (in vacuum)

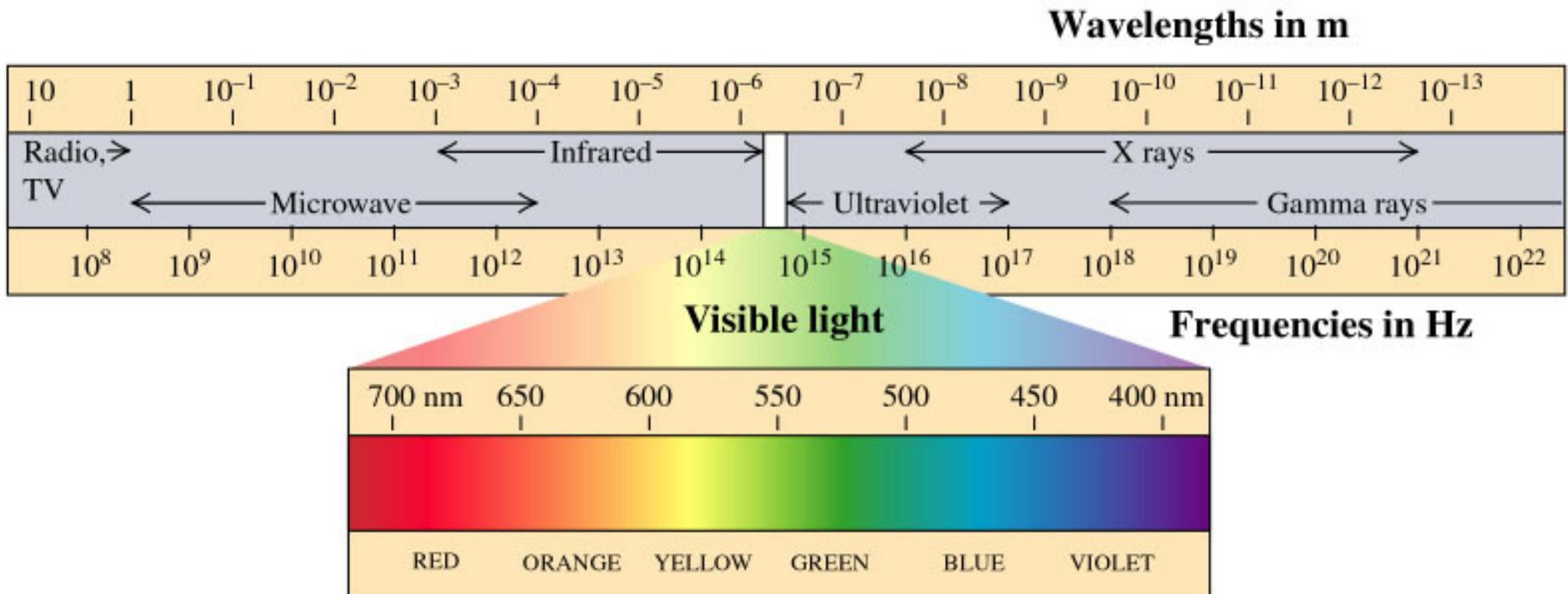
n is the index of refraction

It is very useful to know:
$$E(\text{eV}) = \frac{1240}{\lambda_0(\text{nm})}$$

Electromagnetic spectra

A superposition of many waves with different wavelengths or frequencies with a speed of c for all of them in vacuum:

$$c = 299,792,458 \text{ m/s}$$



Index of refraction and color of the light

$$\underline{\text{Index of refraction}} = \frac{\textit{Speed of light in vacuum}}{\textit{Speed of light in material}}$$

$$\boxed{n = \frac{c}{V} = \frac{\lambda_0}{\lambda}}$$

Since

Speed of light in vacuum > Speed of light in any material

We always have:

$$\boxed{n \geq 1} \text{ and } \lambda_0 > \lambda$$

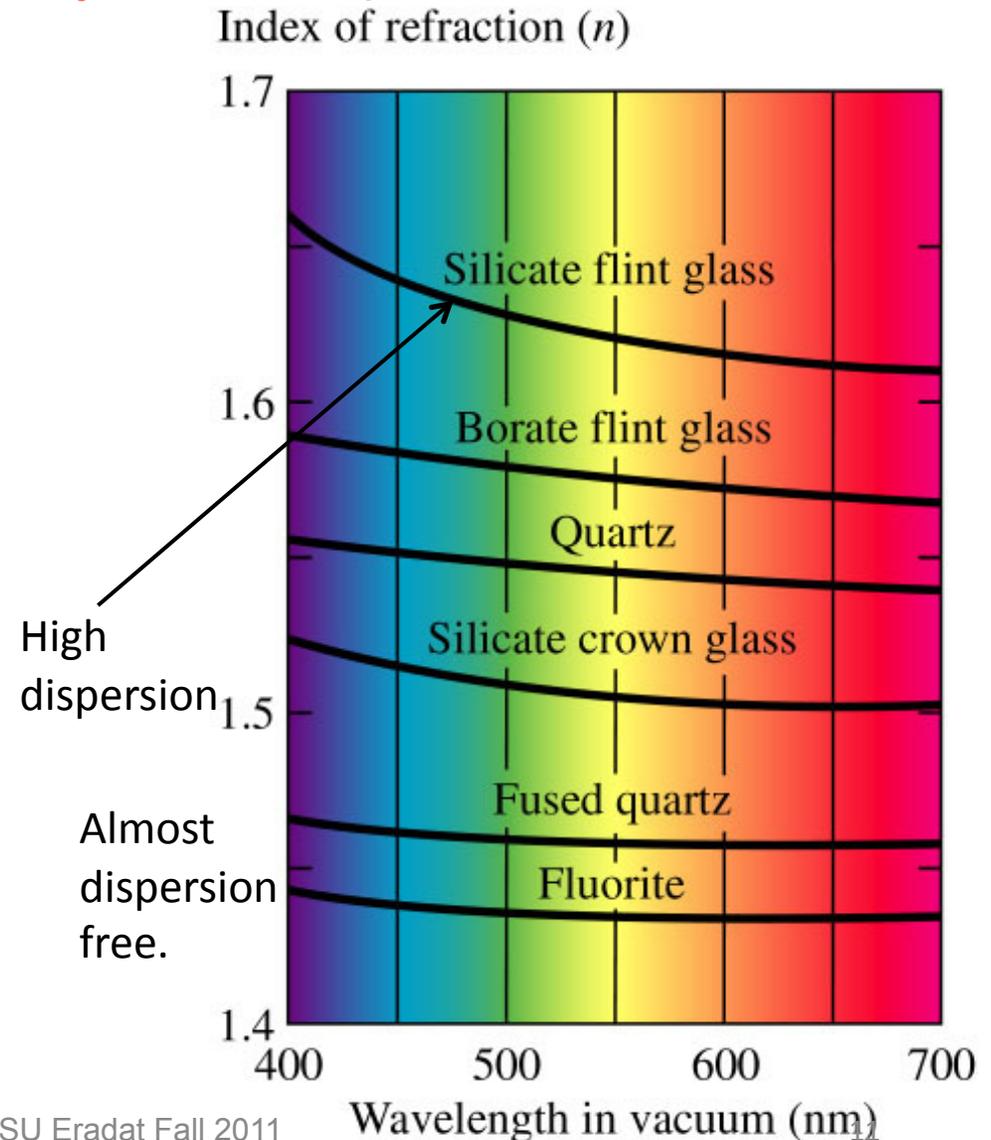
Color of light is determined by the EM wave's frequency.

Wave enters from one medium to another, wavelength changes
but color does not change because the frequency stays the same.

Wavelength dependence of the refractive index (**Dispersion**)

Dispersion is caused by the variation of speed of light (index of refraction) in the medium with frequency. In selection of glasses the number one optical property is the dispersion of the glasses.

Beware of the wavelength in vacuum for the dispersion data.



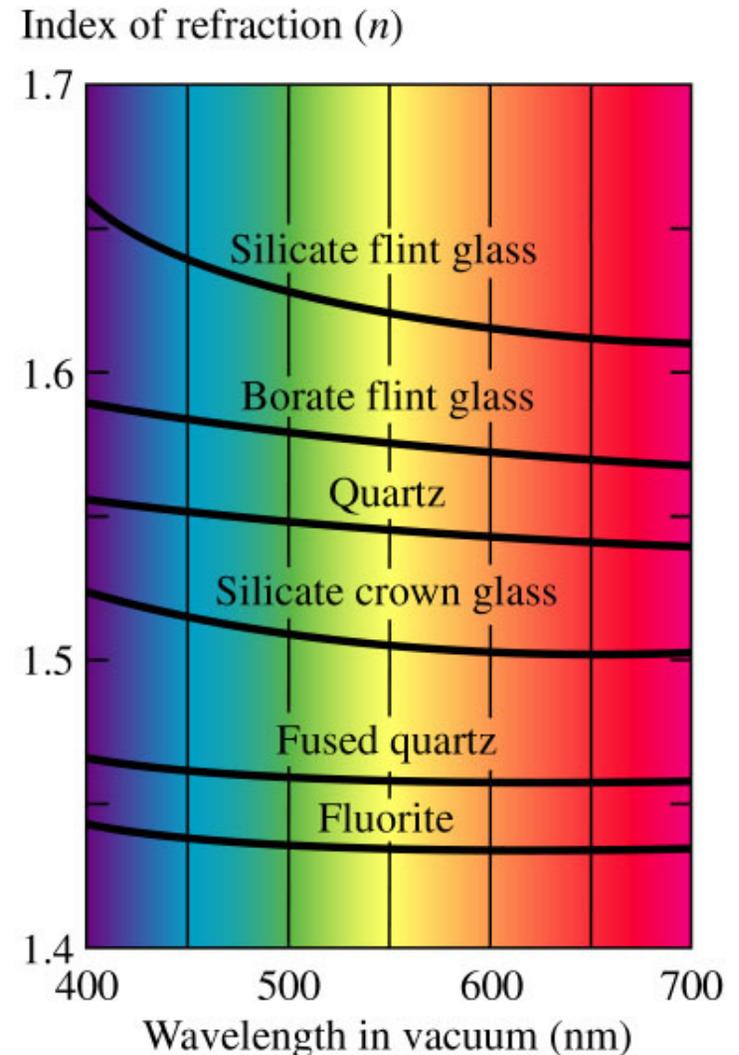
Type of dispersion

- **Normal dispersion** or positive dispersion (common in the visible range for most of the transparent material): index increases with frequency

$$\frac{dn}{d\lambda} < 0 \quad \text{or} \quad \frac{dn}{df} > 0$$

- Anomalous dispersion or Negative dispersion (common for X-rays): index decreases with frequency

$$\frac{dn}{d\lambda} > 0 \quad \text{or} \quad \frac{dn}{df} < 0$$

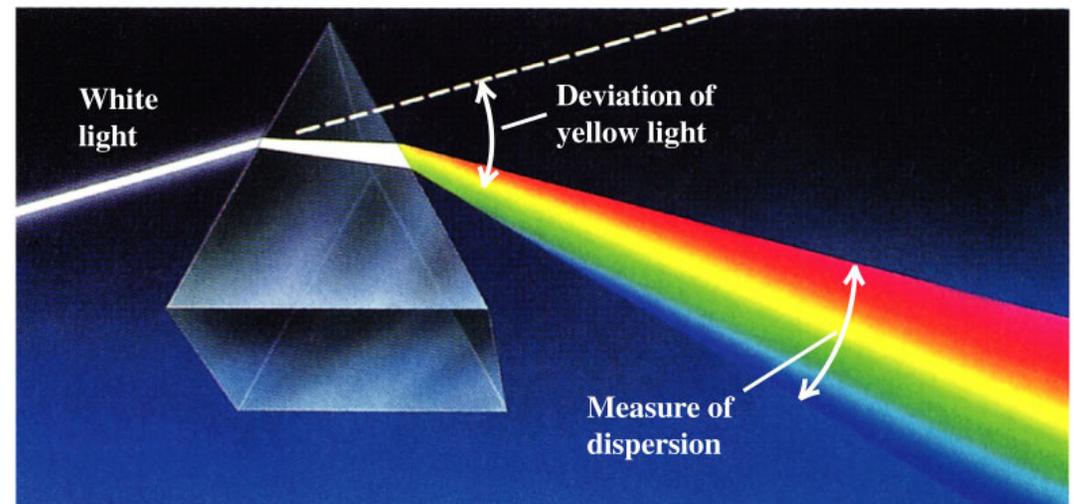
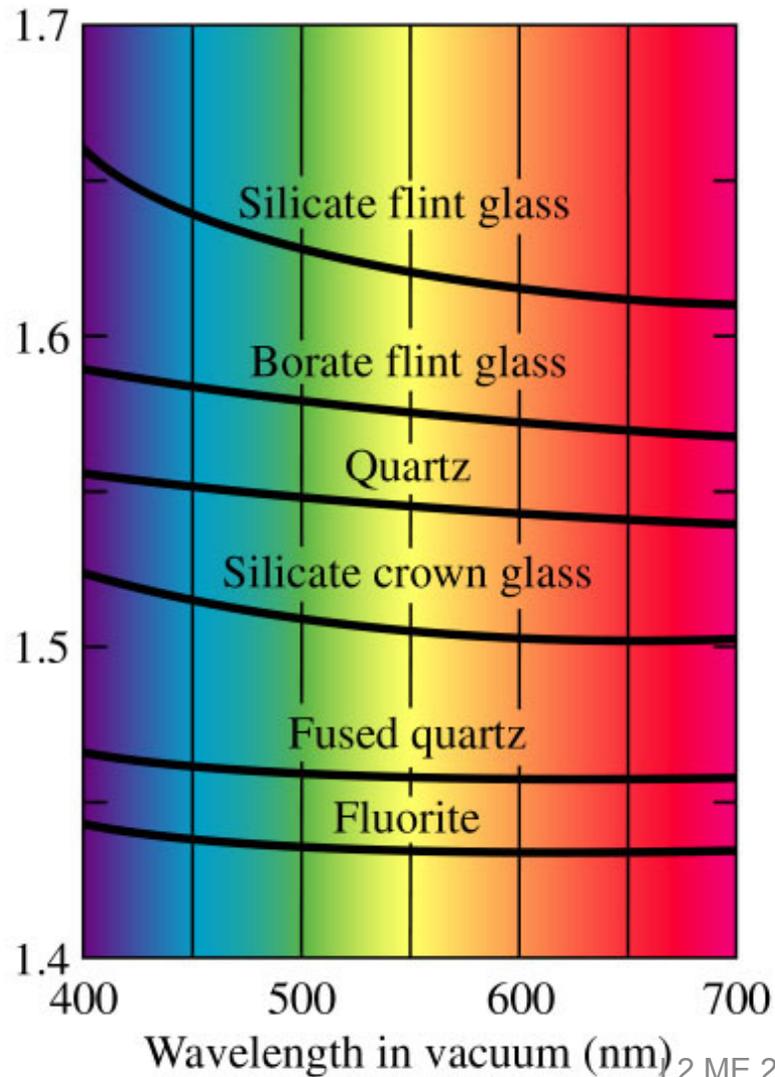


publishing as Addison Wesley.

Application of dispersion:

Which material is the best choice for a spectrometer?

Index of refraction (n)



Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley.

Abbe V number and dispersion

Abbe V number is defined using index of a glass for a set of spectral lines.

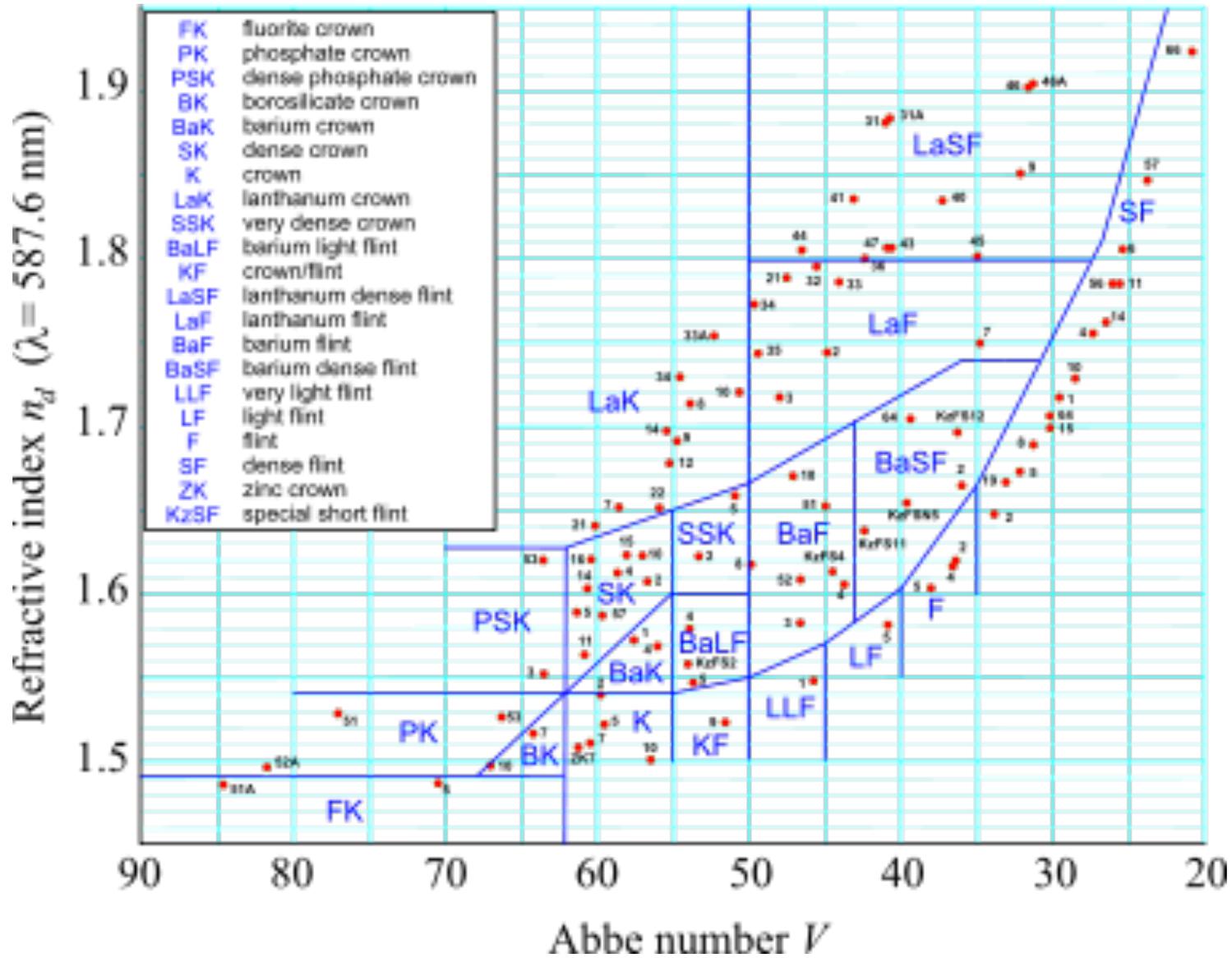
$$\left\{ \begin{array}{l} V_d = \frac{n_d - 1}{n_F - n_C} \\ \lambda_{\text{He D}_3\text{-line}} = 587.6\text{nm}; \lambda_{\text{H F-line}} = 486.1\text{nm}; \lambda_{\text{H C-line}} = 656.3\text{nm} \end{array} \right.$$

$$\left\{ \begin{array}{l} V_e = \frac{n_e - 1}{n_{F'} - n_{C'}} \\ \lambda_{\text{Hg E-line}} = 546.073\text{ nm}; \lambda_{\text{Cd F'-line}} = 480.0\text{ nm}; \lambda_{\text{Cd C'-line}} = 643.8\text{ nm} \end{array} \right.$$

Low dispersion (low chromatic aberration) materials have high values of V.

Typical values of V range from around 20 for very dense flint glass, around 30 for polycarbonate plastics, and up to 65 for very light crown glass, and up to 85 for fluor-crown glass. Abbe numbers are only a useful measure of dispersion for visible light, and for other wavelengths, or for higher precision work, the group velocity dispersion is used.

Abbe Diagram



Geometrical optics vs. physical optics

- Rays: geometrical optics
- Wavefronts: physical optics

Geometrical optics: $\lambda \ll$ feature sizes

Or the accuracy required can be achieved by geometrical treatment.

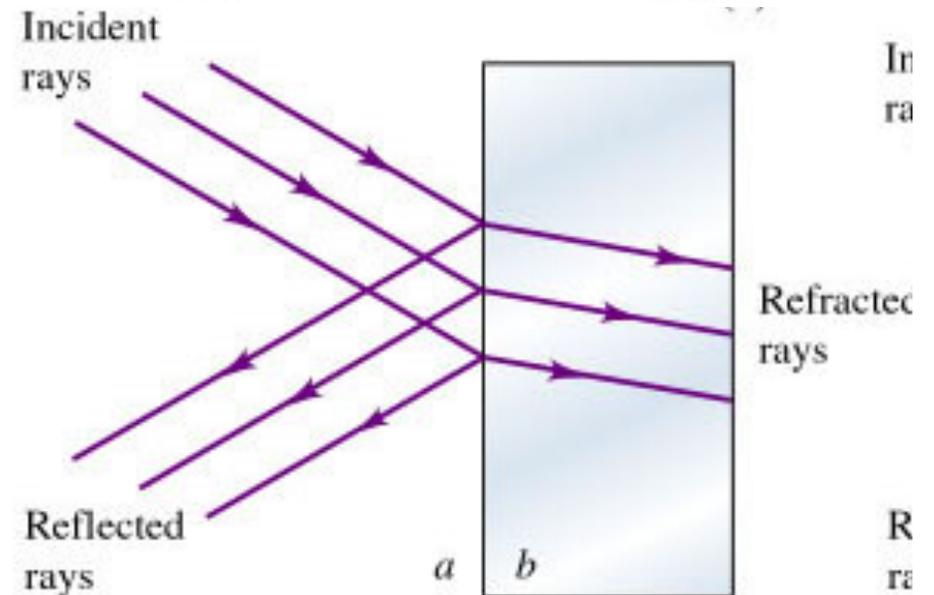
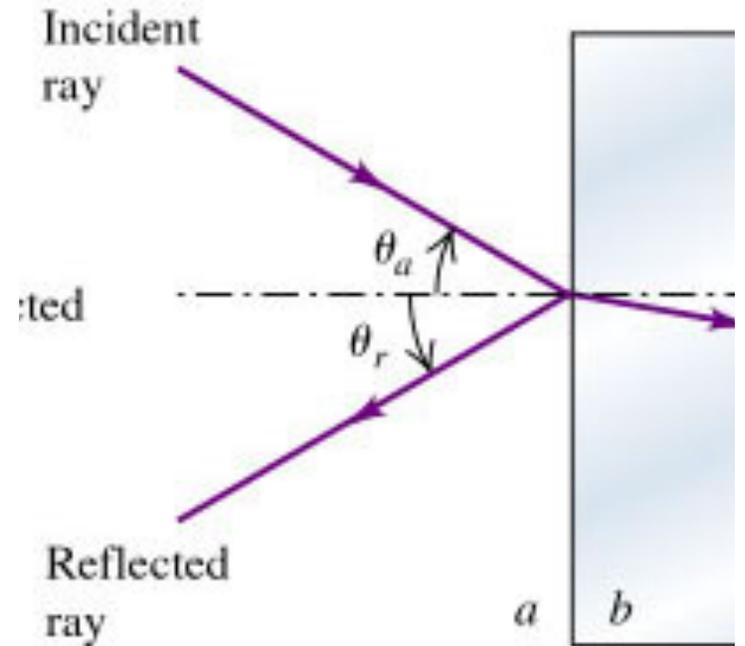
Physical optics: $\lambda \simeq$ feature sizes

about λ - 100λ

Or whenever a very high accuracy is demanded

Reflection & Refraction

- When light hits interface of two dielectric (transparent) media it may be
 - reflected
 - Specular reflection
 - Diffused reflection
 - Refracted



(b)

Laws of Reflection & Refraction

AKA Snell's Laws

- 1) The incident (ray), reflected, and normal to the surface are all in one plane that is perpendicular to the interface.
- 2) Law of reflection: Angle of incidence = Angle of reflection

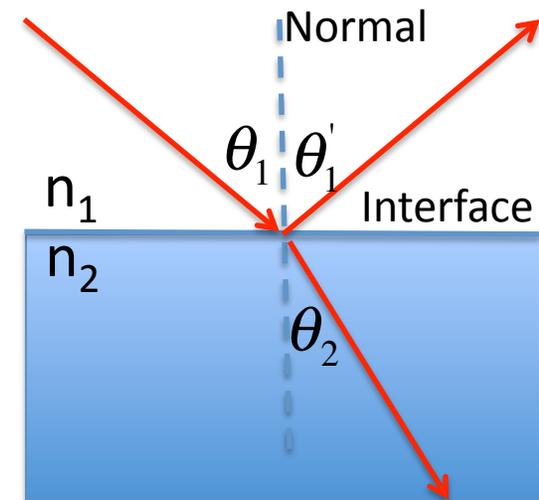
$$\theta_1 = \theta_1'$$

- 3) Law of refraction: the reflected and refracted beams at an interface of two dielectrics follow this relationship:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

For small angles ($\leq 6^\circ$ or ≤ 100 mR): $n_1 \theta_1 = n_2 \theta_2$

Reciprocity principle: left-to-right works the same way as right-to-left



The purpose of an optical system

- Image forming systems: resolving image of a specified minimum-sized object in the image space over a desired (spatial /angular) field of view in the object space.
- Non-imaging systems: optimal transfer of light radiation between a source and a target.

Some definitions regarding imaging systems I

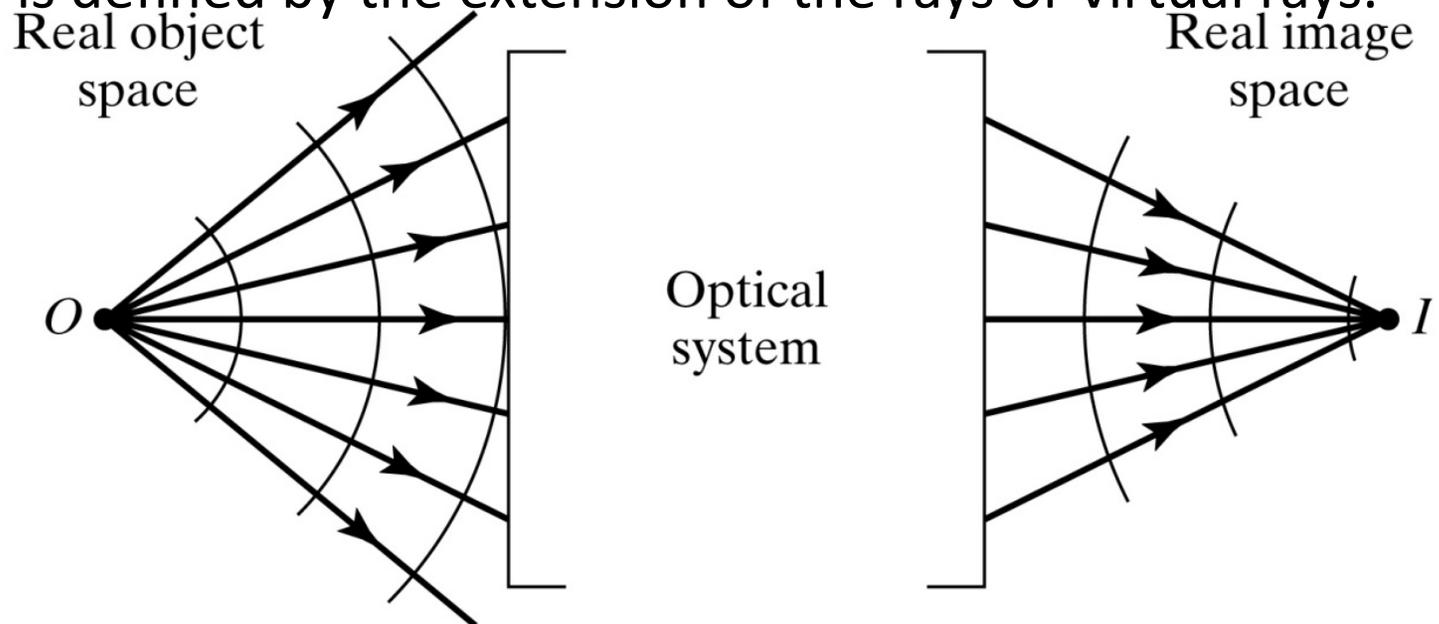
Optical system: any number of reflecting and/or refracting surfaces that may alter direction of the rays leaving an object point.

Object point: location of object

Object space: the space between the object and front surface of the optical system.

Real object: is defined by real rays leaving it.

Virtual object: is defined by the extension of the rays or virtual rays.



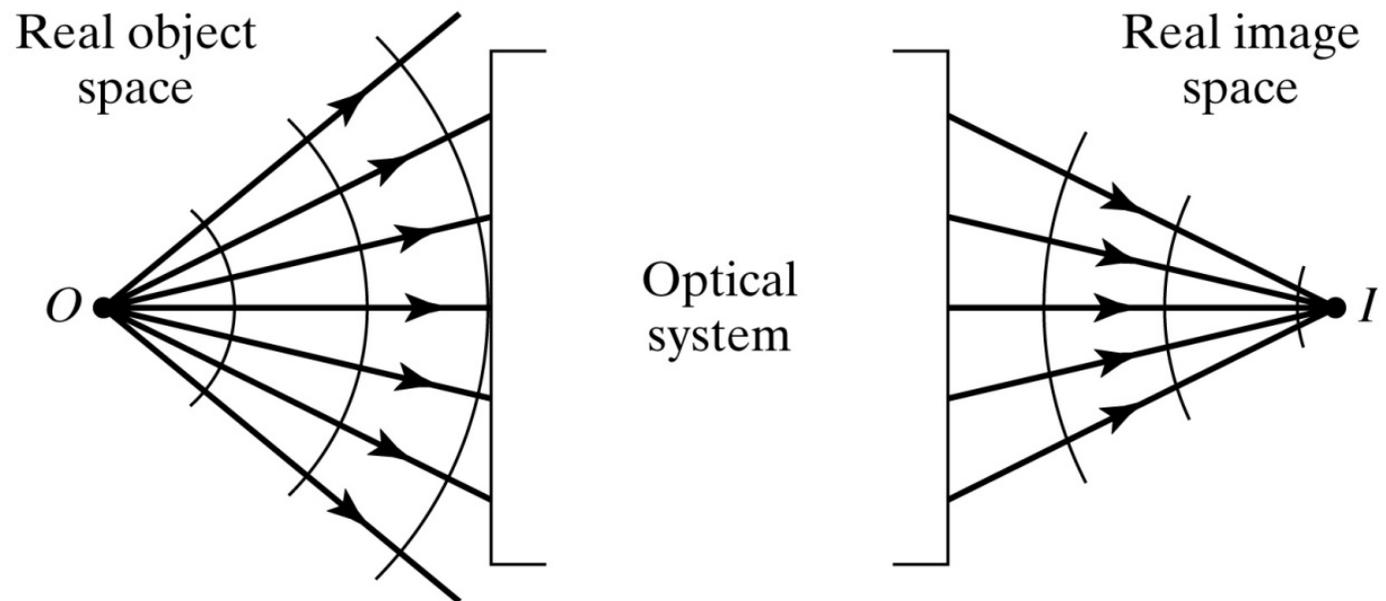
Some definitions regarding imaging systems II

Image point: location of the image.

Image space: the space between the back surface of the optical system and the image.

Real image: is defined by real rays intersecting

Virtual image: is defined by the extension of the rays or virtual rays intersecting..



© 2007 Pearson Prentice Hall, Inc.

Some assumptions (idealizations) for start and more definitions

Each individual medium in the optical system is **homogeneous** and **isotropic (not in reality)**.

Homogeneous: single phase material, no inclusions or bubbles. No severe scattering or diffusion of light inside the material.

Isotropic: same optical properties across the media means **constant index of refraction**.

Concave surfaces: center of the curvature is on the reflecting side or source side (incoming light hits the cave).

Convex surfaces: center of curvature is on the opaque side or propagation side (light hits the cone).

Optical Path Length: OPL and phase gain

OPL = distance traveled \times index of refraction of the environment

$$OPL = \sum_i n_i x_i$$

OPL is constant for all **rays that converge to an image point.**

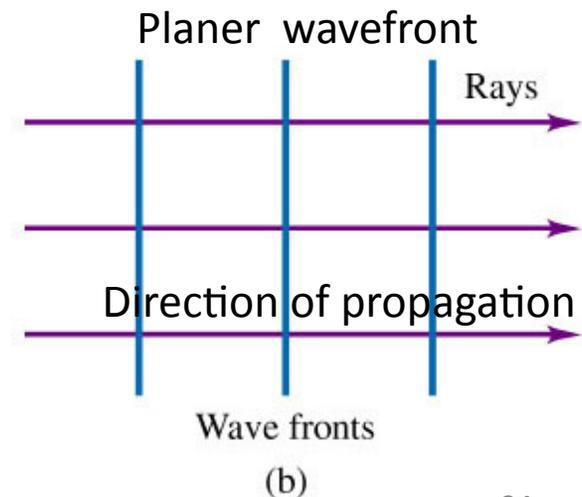
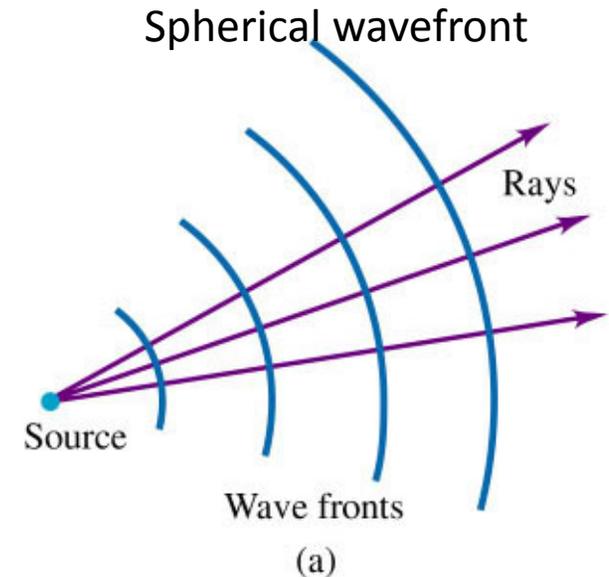
OPL is constant for all the **points on a wavefront.**

The phase a wave gains by traveling a distance x in an environment of index of refraction n is:

$$\phi = \frac{2\pi}{\lambda} x = \frac{2\pi}{\lambda_0} nx = \frac{2\pi}{\lambda_0} OPL$$

Wavefronts (phase-based definition)

- Wavefront: loci of the adjacent points on the wave with the **same phase**
- In **homogeneous** (same nature) and **isotropic** (uniform in all directions) media wavefronts do not change as wave propagates.
- Most common wavefronts are planar, spherical and cylindrical



Wavefront (ray-based definition)

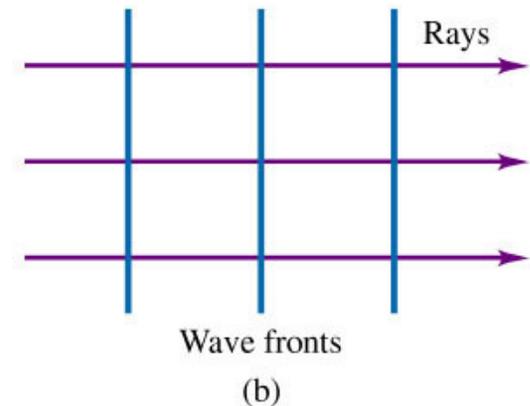
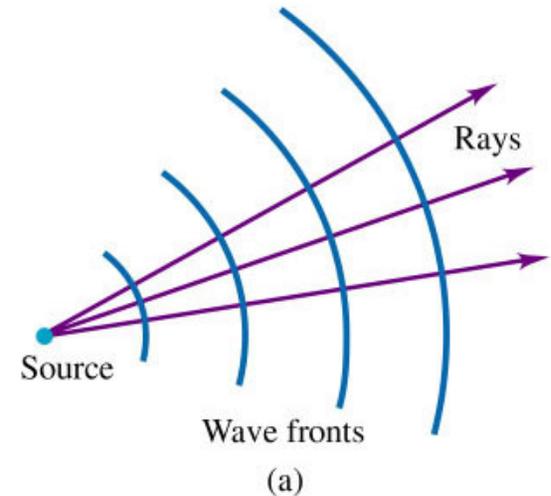
Wavefronts: the family of surfaces normal to the rays that are leaving the object point.

Wavefronts are locus of points such that each ray contacting them represents same transit time of light from the source

(Isochronous rays). For points on a wavefront **transit time from the source is constant:**

$$t = \frac{x}{v} = \frac{x}{c/n} = \frac{nx}{c} = \text{constant}$$

The Optical Path Length (OPL) from the source is constant for the points on the wavefront.



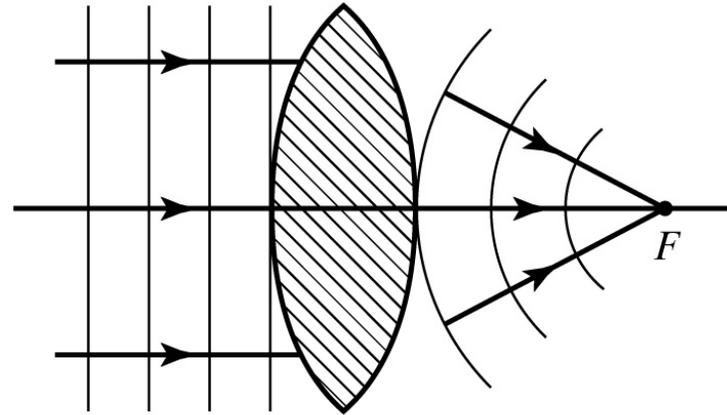
shing as Addison Wesley.

Imaging

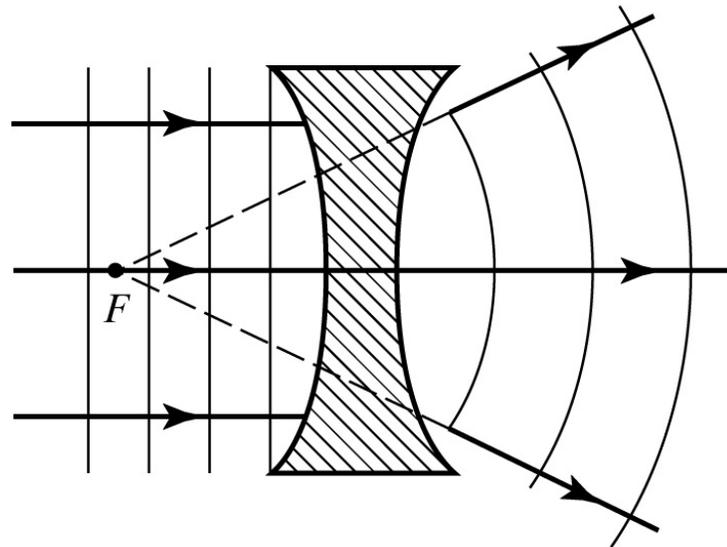
- **Fermat's principle:** All the rays leaving an object point and arriving at the image point of the same object point via an optical system have to be **isochronous**. Otherwise we can't process the image and see it.
- **Principle of reversibility:** if object and image points switch places light rays will go through the same path only in opposite direction.
- **Conjugate points for an optical system:** points that are images of one another.

Wavefront analysis of thin lenses; Application of Fermat's principle

Plane wavefronts arriving
at a thin lens are curved
to stay **isochronous** or to
stay in-phase or to
accommodate the
Fermat's principle.



(a)



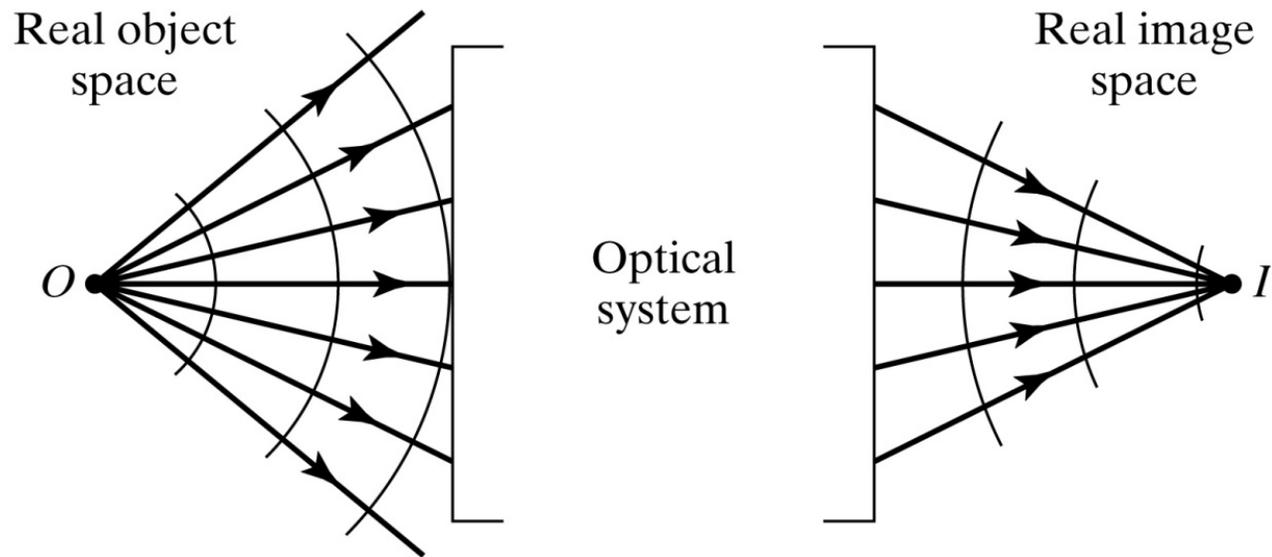
(b)

© 2007 Pearson Prentice Hall, Inc.

Ideal vs. actual imaging systems I

Ideal optical system: all the rays –and only those rays - intersecting an optical system participate in image formation.

To reconstruct an **actual image** of an object, the optical system must generate an ideal image for each point on the object.



© 2007 Pearson Prentice Hall, Inc.

Ideal vs. actual imaging systems II

Actual images are far from ideal and are **blurred** because of:

1. Reflection losses at refracting surfaces

$$E_r = \frac{n_a - n_b}{n_a + n_b} E_i \rightarrow I = \left(\frac{n_a - n_b}{n_a + n_b} \right)^2 I_i \rightarrow n_a = 1, n_b = 1.5 \rightarrow I = 0.04 I_i$$

For 4 surfaces or 2 lenses: $(0.96)^4 = 0.85$ only 85% is transmitted

2. Diffuse reflection from reflecting surfaces
3. Scattering by inhomogenities of the material
4. **Aberrations** when the **system fails to produce one-to one image** of the all points on the object.
5. **Diffraction when** an otherwise perfect image, is blurred due to **using limited portion of the wavefront** to construct the image. Diffraction poses limitation on the perfect focusing.

Ray tracing

- Tracking path of light using the Snell's laws for an optical system to find:
 - Image location for imaging optics
 - Light path and distribution of it at a desired location for non-imaging optics
- **Paraxial** when only rays with small angles (less than 100 mRa or 6°) relative to the optical axis are considered and small angle approximation is used.

$$\sin \theta \simeq \tan \theta \simeq \theta \text{ (in radians)}$$

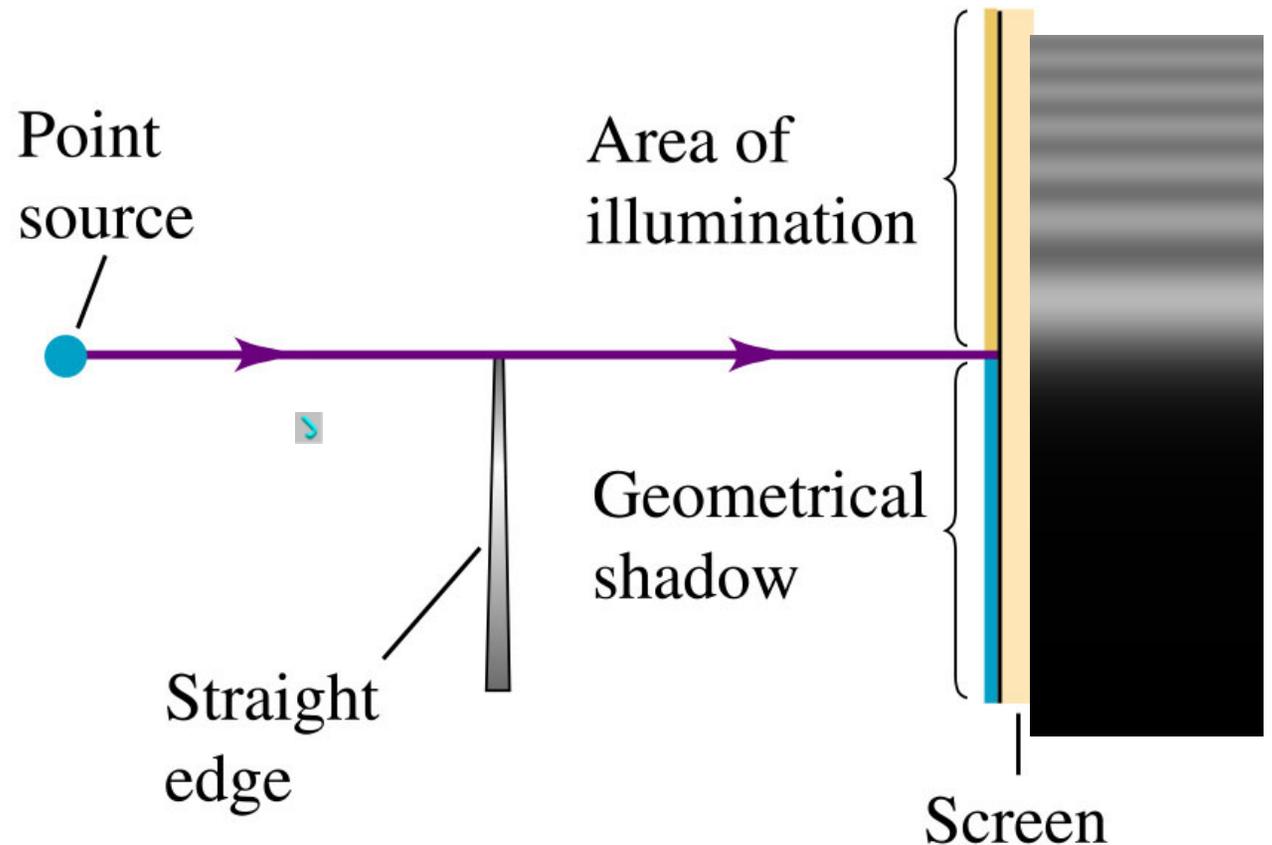
- **“Exact”**: no approximation

Straight edge diffraction

A razor blade for example

How the shadow should look like based on geometrical optics?

What is the actual form of the shadow?

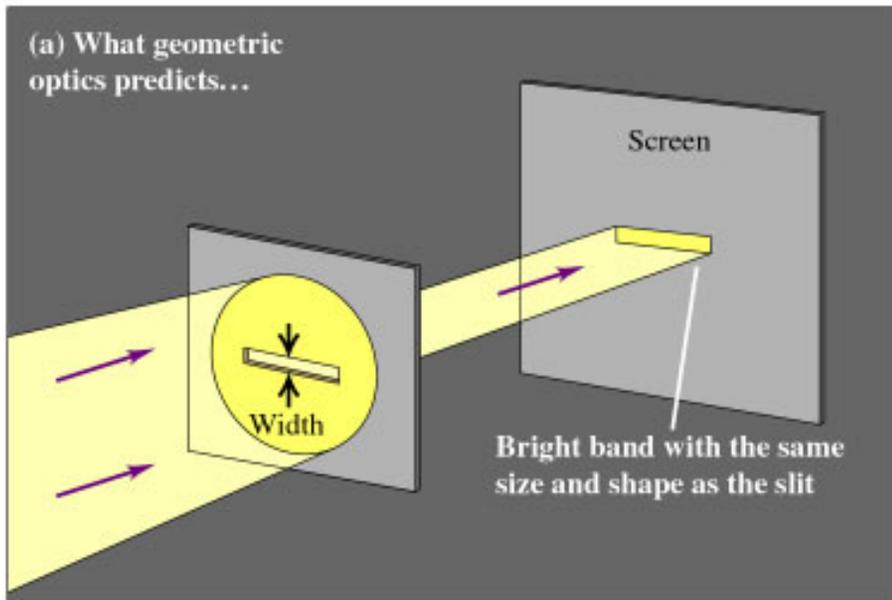


Why we do not see a sharp pattern on the shadow edges in our daily experiences?

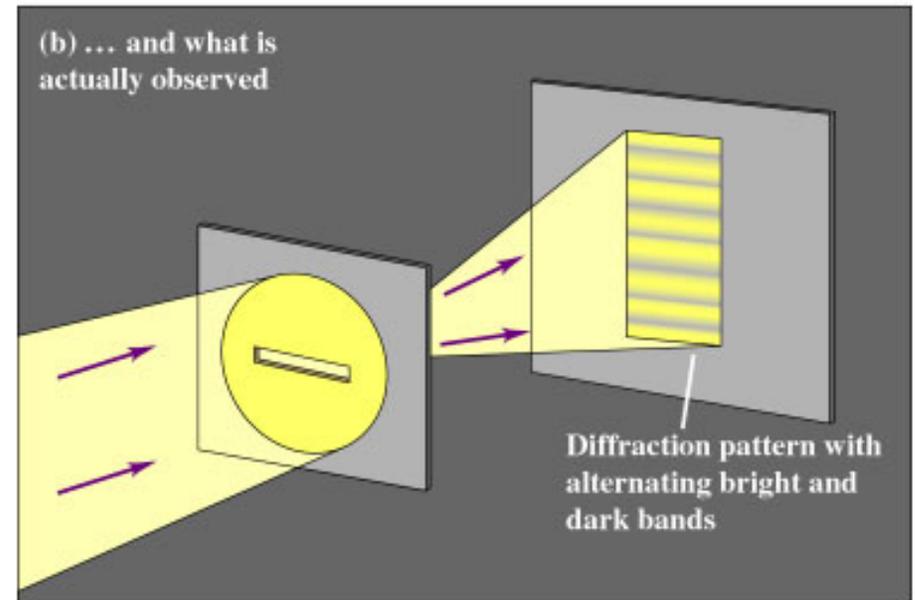
Fresnel and Fraunhofer Diffraction

- Diffraction occurs when light passes through an aperture or around an edge.
- Diffraction is due to obstruction of the wavefront.
 - **Fraunhofer Diffraction:** when source and observer are so far away from the obstructing surface that the outgoing rays can be considered parallel or the wavefront is planar. (We will restrict our studies to the Fraunhofer Diff.).
 - **Fresnel Diffraction:** when the source or the observer is relatively close to the obstructing surface. The wavefront is no longer seen planar.

Diffraction from a single slit



INCORRECT



CORRECT

For small slits
How about large slits?

Intensity in the single slit diffraction as a function of system parameters

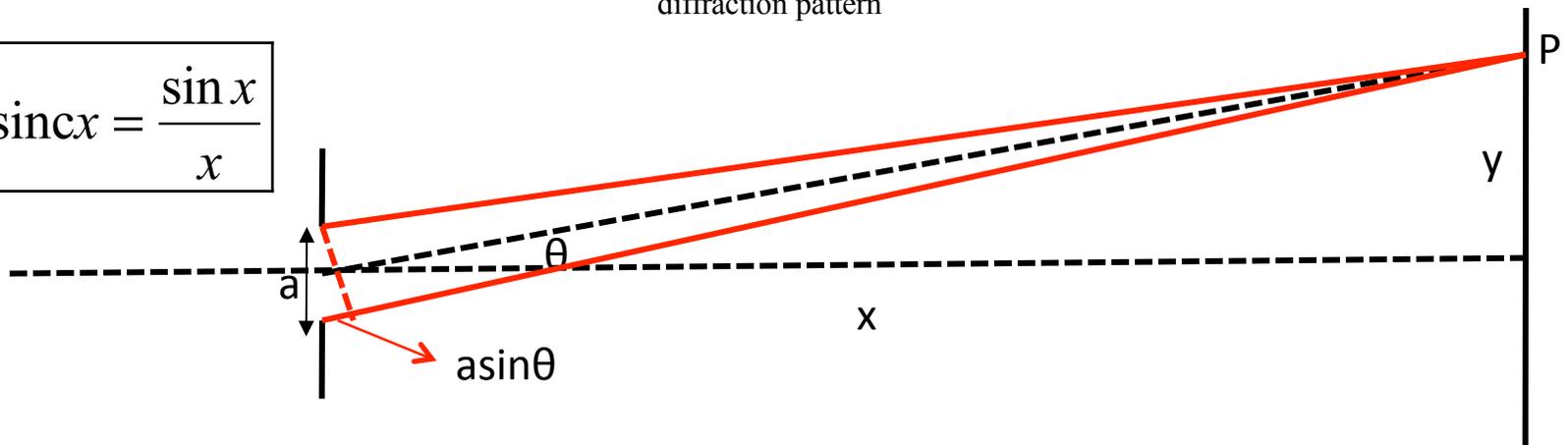
β is the phase difference

$$\text{phase difference} = \frac{2\pi}{\lambda} \text{path difference} \rightarrow \boxed{\beta = \frac{2\pi}{\lambda} a \sin \theta}$$

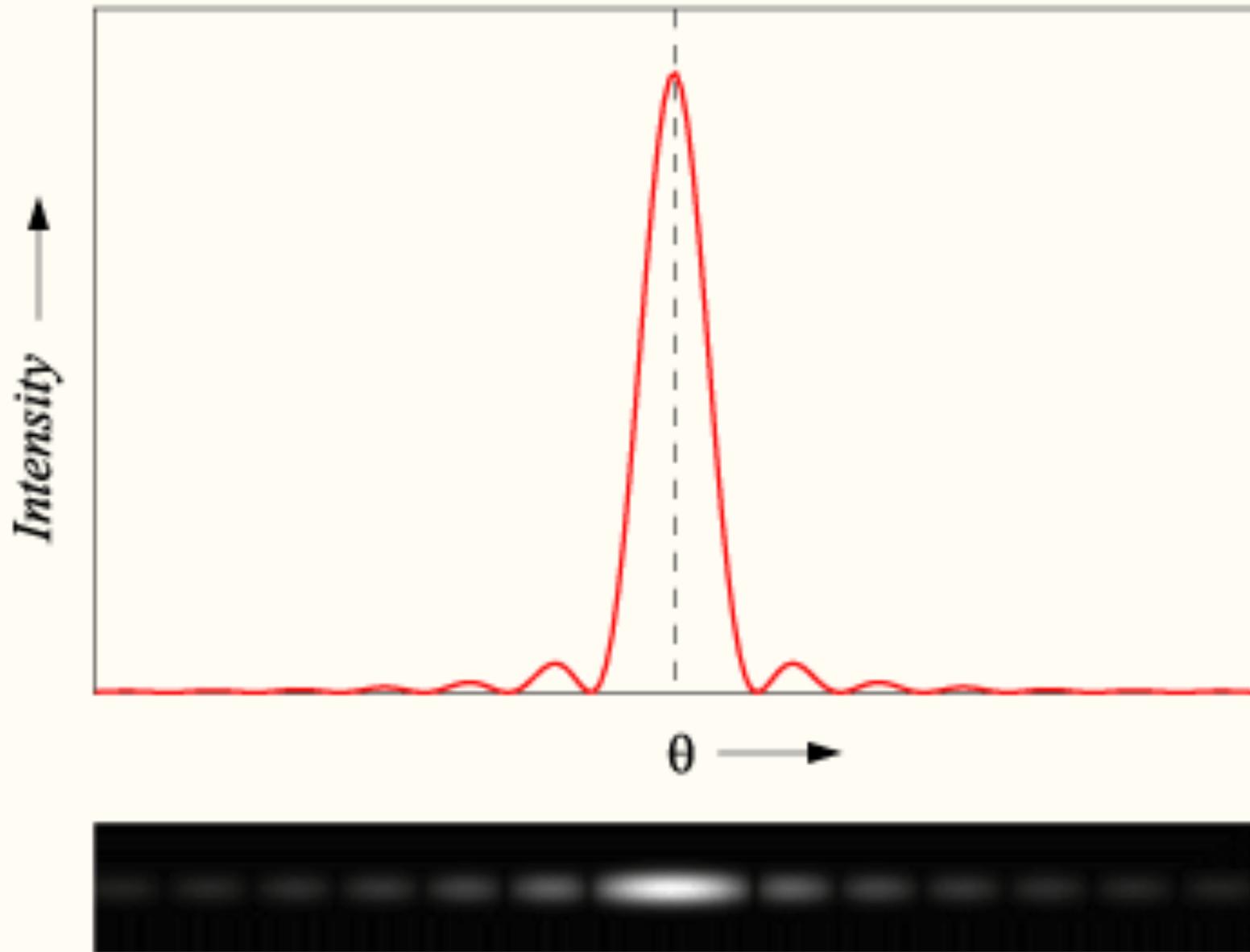
$$I = I_o \left[\frac{\sin \frac{\beta}{2}}{\frac{\beta}{2}} \right]^2 = I_o \left[\frac{\sin \frac{\pi a \sin \theta}{\lambda}}{\frac{\pi a \sin \theta}{\lambda}} \right]^2 \rightarrow \boxed{I = I_o \text{sinc}^2 \frac{\beta}{2}}$$

Intensity of the single slit diffraction pattern

where $\boxed{\text{sinc} x = \frac{\sin x}{x}}$



Single-slit diffraction pattern



Location of the dark fringes on the single slit diffraction pattern

$$I = I_0 \left[\frac{\sin \frac{\pi a \sin \theta}{\lambda}}{\frac{\pi a \sin \theta}{\lambda}} \right]^2$$

Location of a dark fringe:

$$I = 0 \rightarrow \sin \frac{\pi a \sin \theta}{\lambda} = 0$$

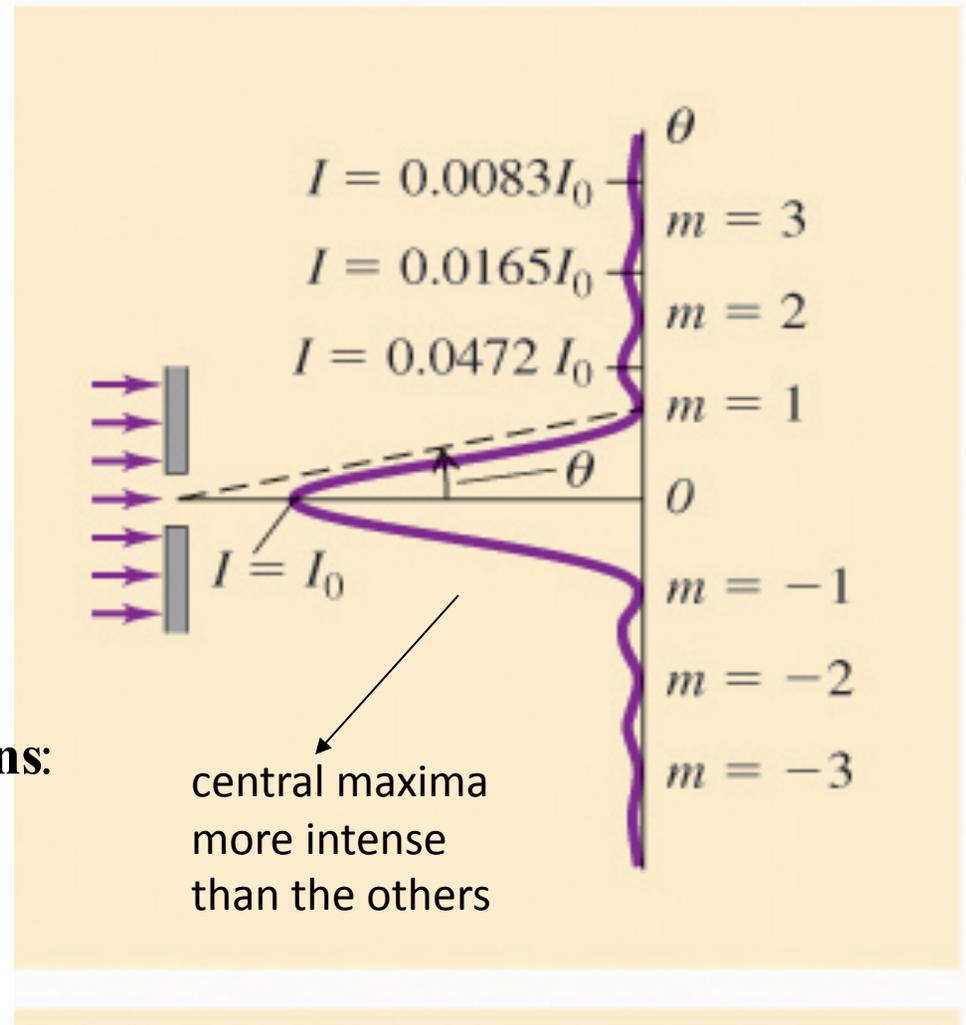
$$\frac{\pi a \sin \theta}{\lambda} = m\pi \text{ where}$$

b) For the case of $m = \pm 1, \pm 2, \pm 3, \dots$

Dark fringes exist at angular locations:

$$\sin \theta = \frac{m\lambda}{a} \text{ with } m = \pm 1, \pm 2, \pm 3, \dots$$

Central maximum occurs at $\theta = 0$



Other features of the single-slit diffraction

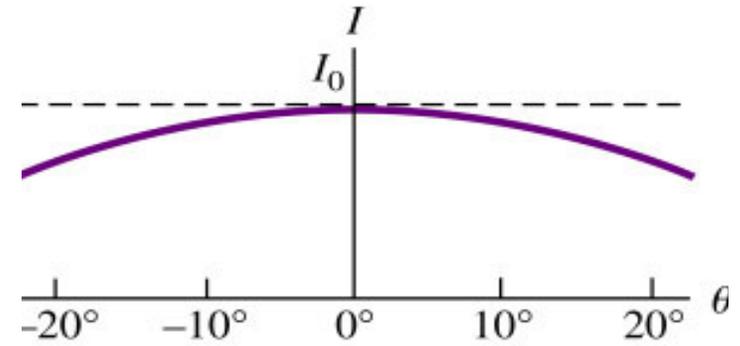
1) Angular width of the central maxima

is inversely proportional to the $\frac{a}{\lambda}$

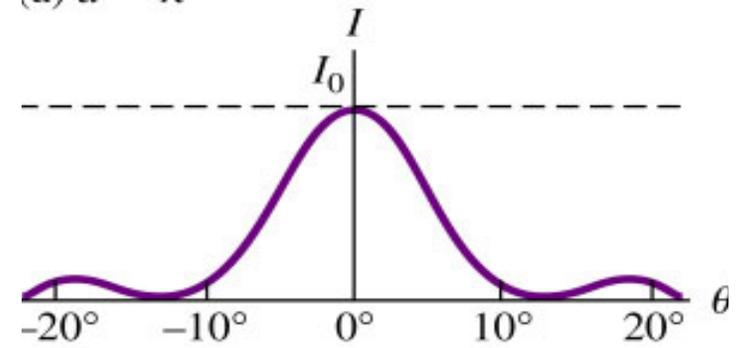
2) When $a < \lambda$ central maximum spreads over 180°

Approximate intensities at the side maxima

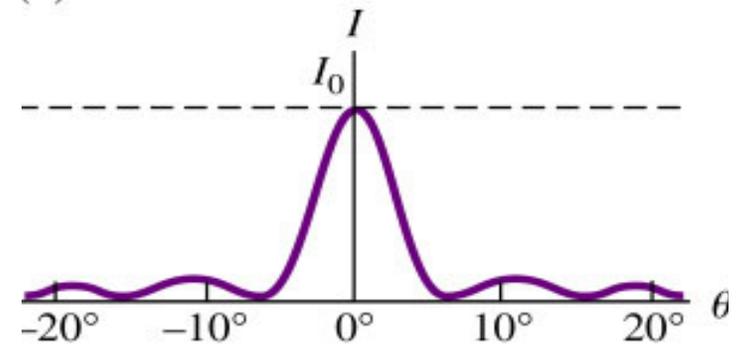
$$I \approx \frac{I_0}{\left(m + \frac{1}{2}\right)^2 \pi^2}; \quad m = 0, 1, 2, 3, \dots$$



(a) $a = \lambda$



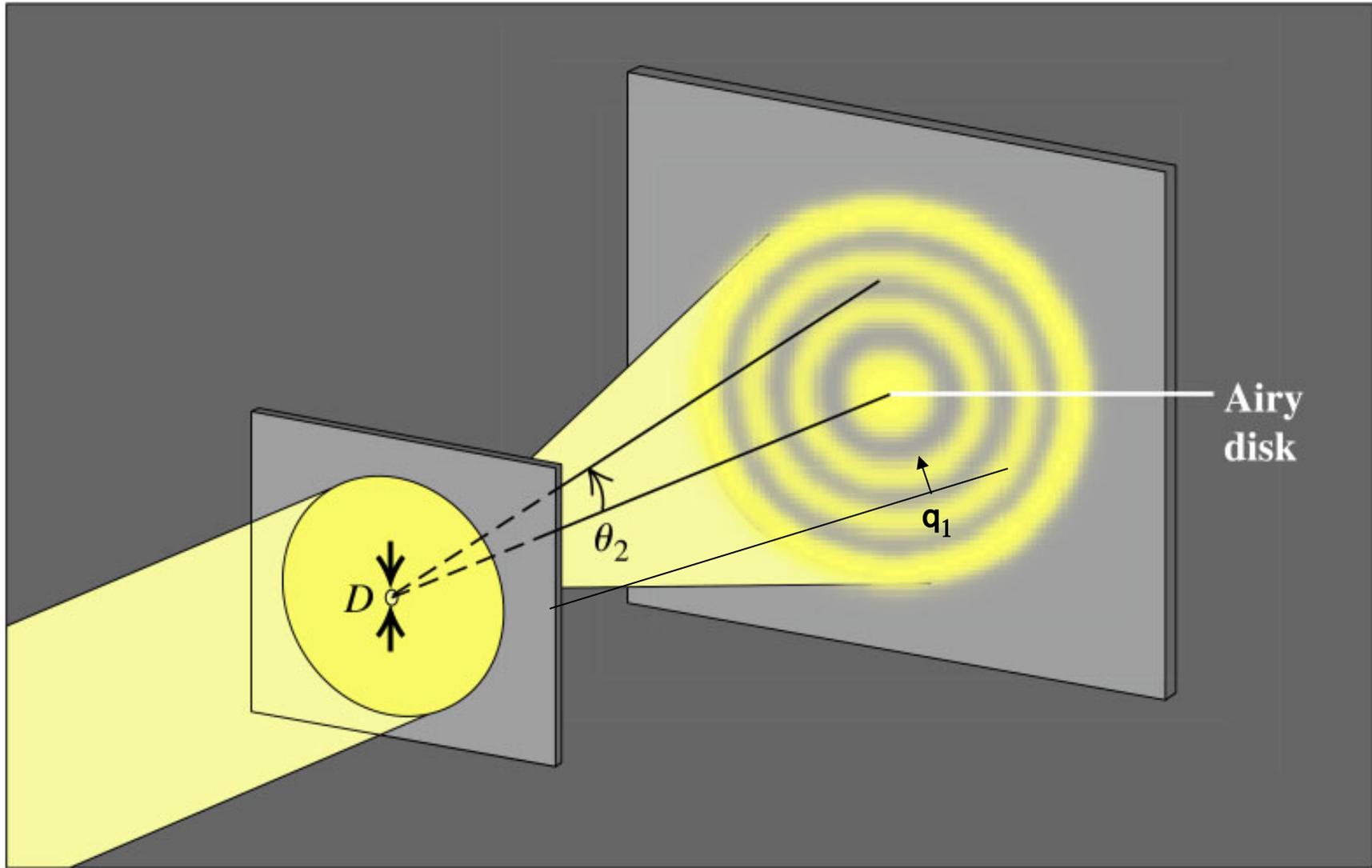
(b) $a = 5\lambda$



(c) $a = 8\lambda$

© Pearson Wesley.

Circular aperture diffraction pattern



Diffraction pattern formed by a circular aperture

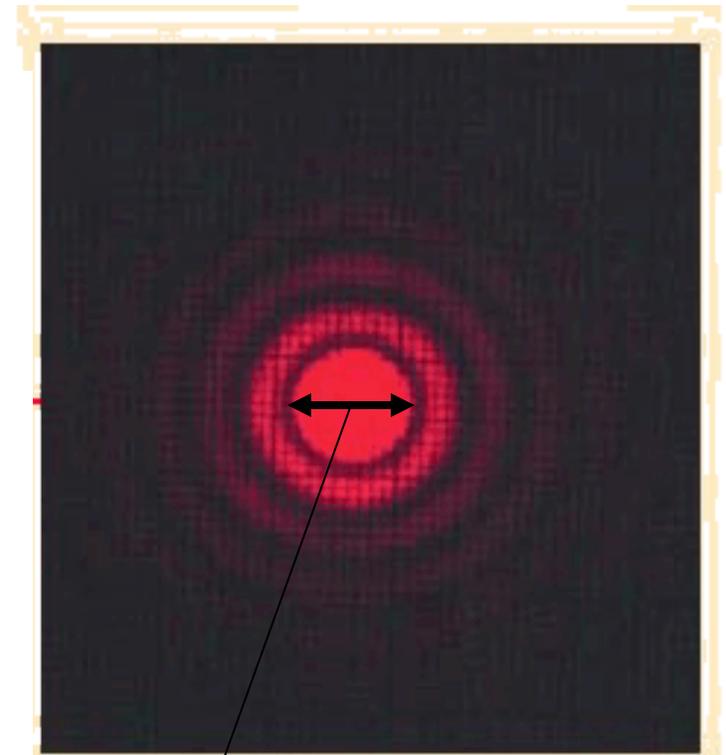
Angle of Airy disk: $\sin \theta_1 = 1.22 \frac{\lambda}{D}$

Second dark ring: $\sin \theta_2 = 2.23 \frac{\lambda}{D}$

Third dark ring: $\sin \theta_3 = 3.24 \frac{\lambda}{D}$

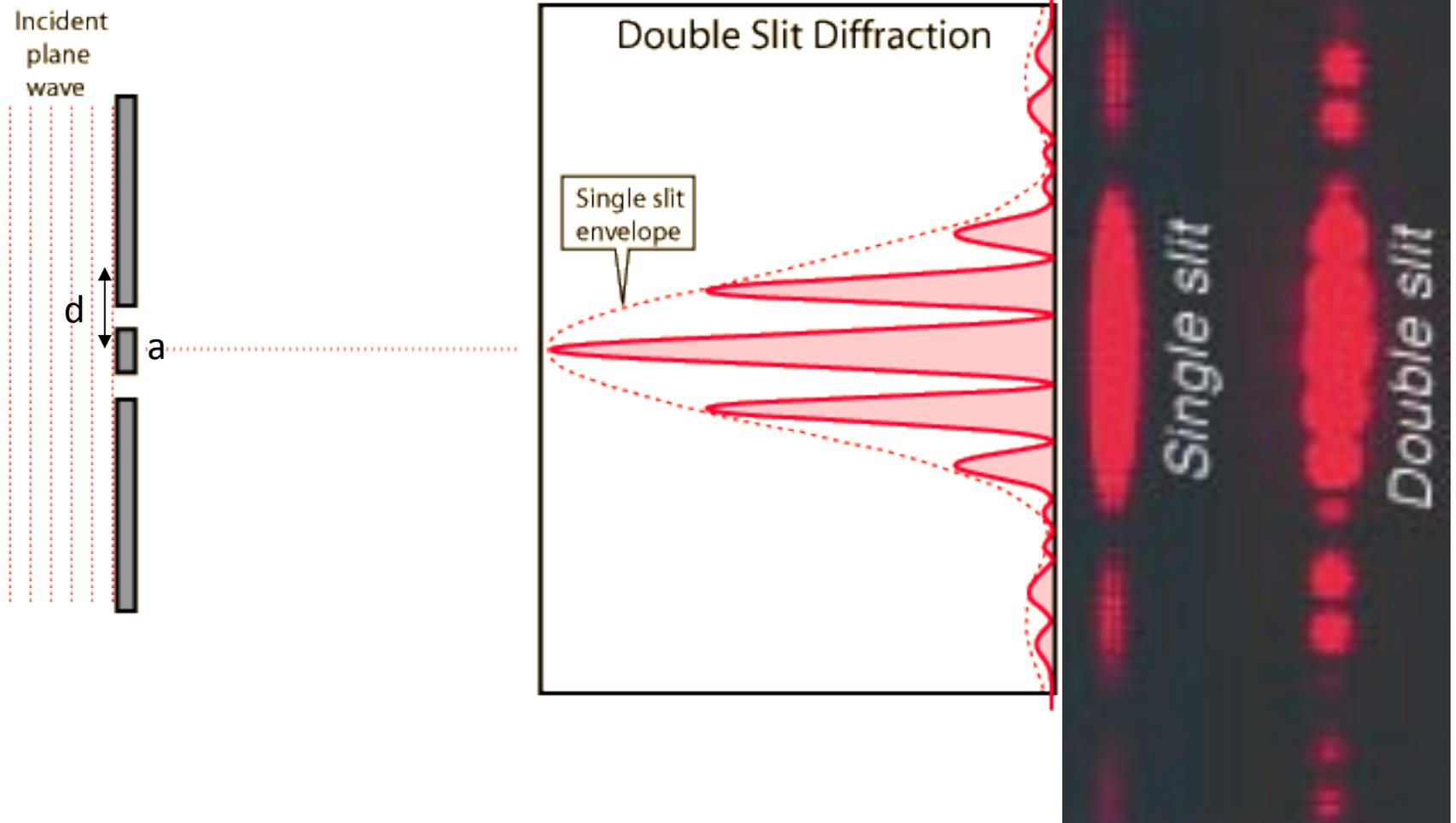
Three bright rings at

$\sin \theta = 1.63 \frac{\lambda}{D} \quad 2.68 \frac{\lambda}{D} \quad 3.70 \frac{\lambda}{D}$

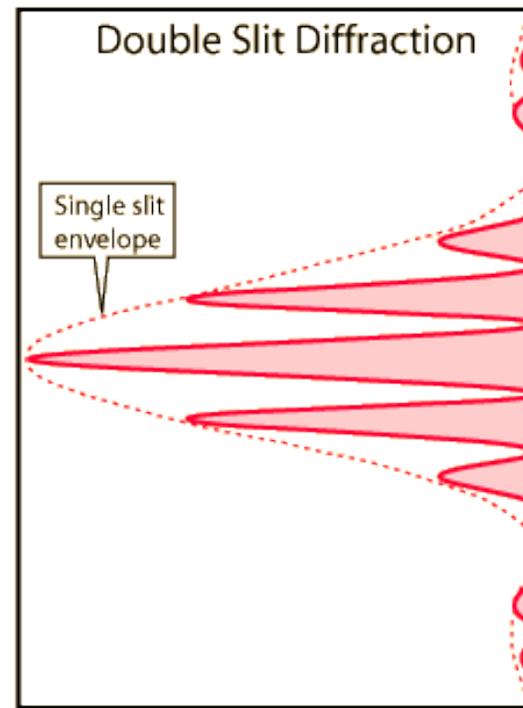
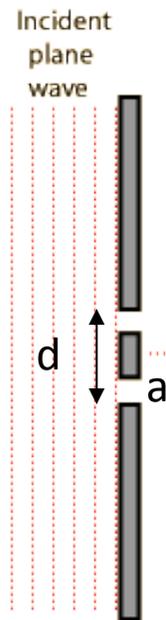


Airy disk 85% of the light falls within this disk

Double Slit Diffraction/Interference



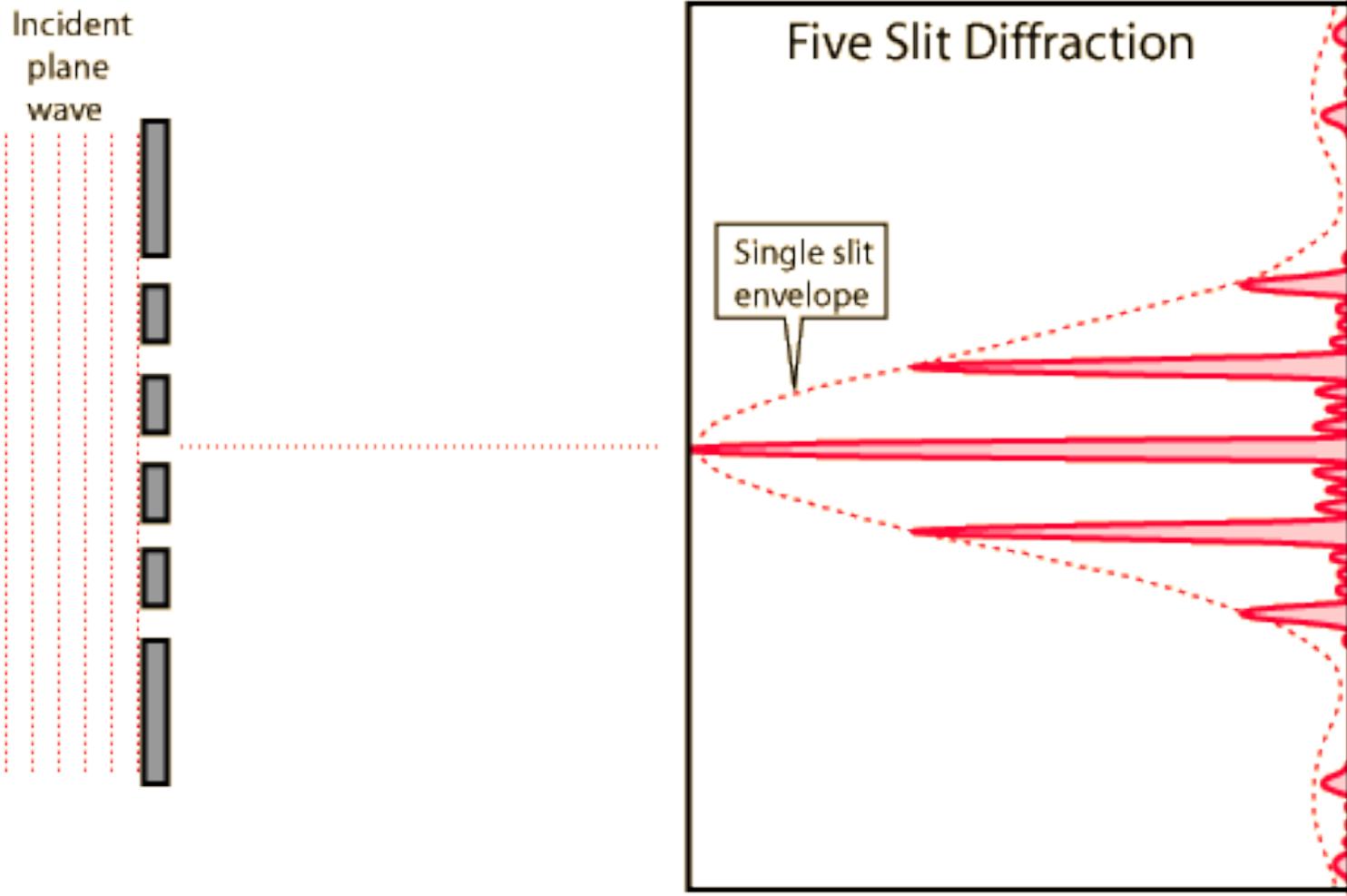
Double Slit Diffraction/Interference



Superposition of two patterns: $I = I_0 \underbrace{\cos^2 \frac{\phi}{2}}_{\text{Interference contribution}} \underbrace{\left[\frac{\sin(\beta/2)}{(\beta/2)} \right]^2}_{\text{Diffraction contribution}}$

where $\phi = \frac{2\pi d}{\lambda} \sin \theta$; $\beta = \frac{2\pi a}{\lambda} \sin \theta$

Five slit diffraction

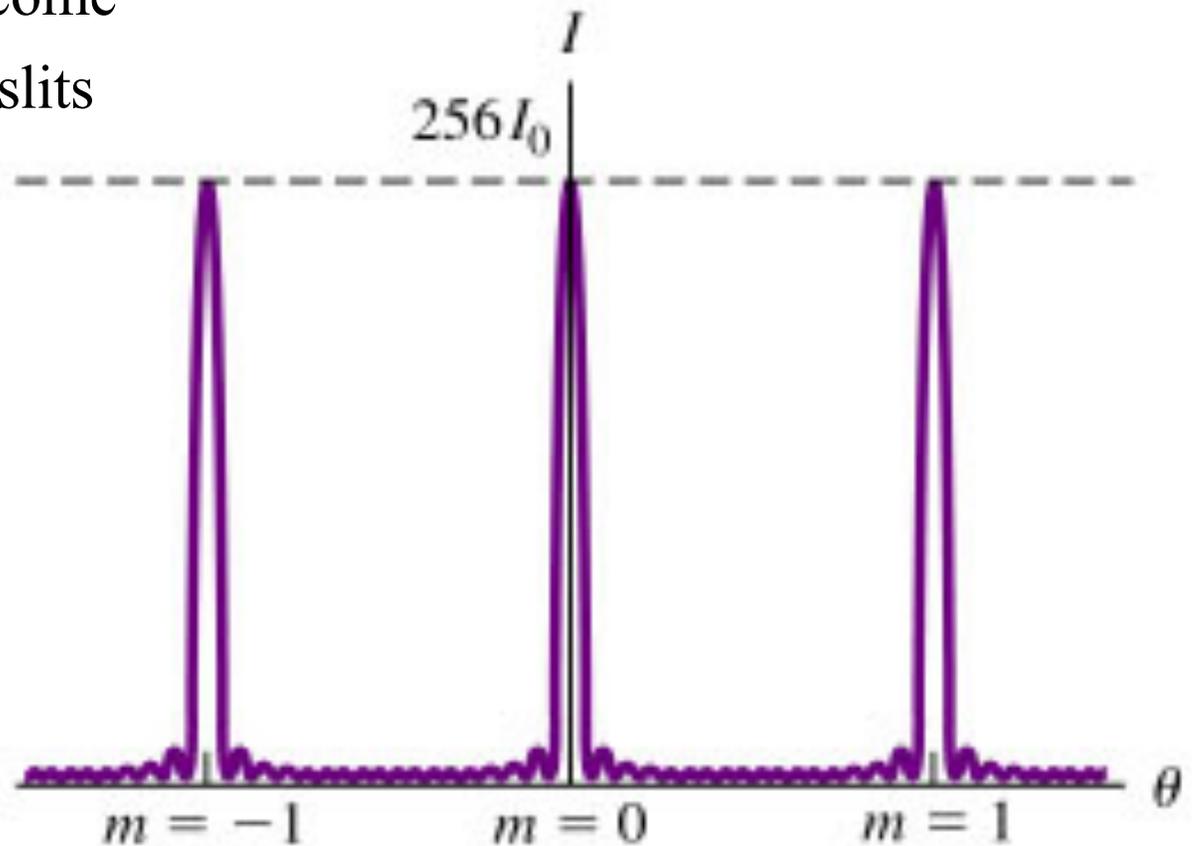


Even more slits: $N = 16$

Large N corresponds to higher intensity principal maxima $\rightarrow I_N \propto I_0 N^2$

The principal peaks become narrower as number of slits increase

$$\rightarrow FWHM_{\text{maxima}} \propto \frac{1}{N}$$



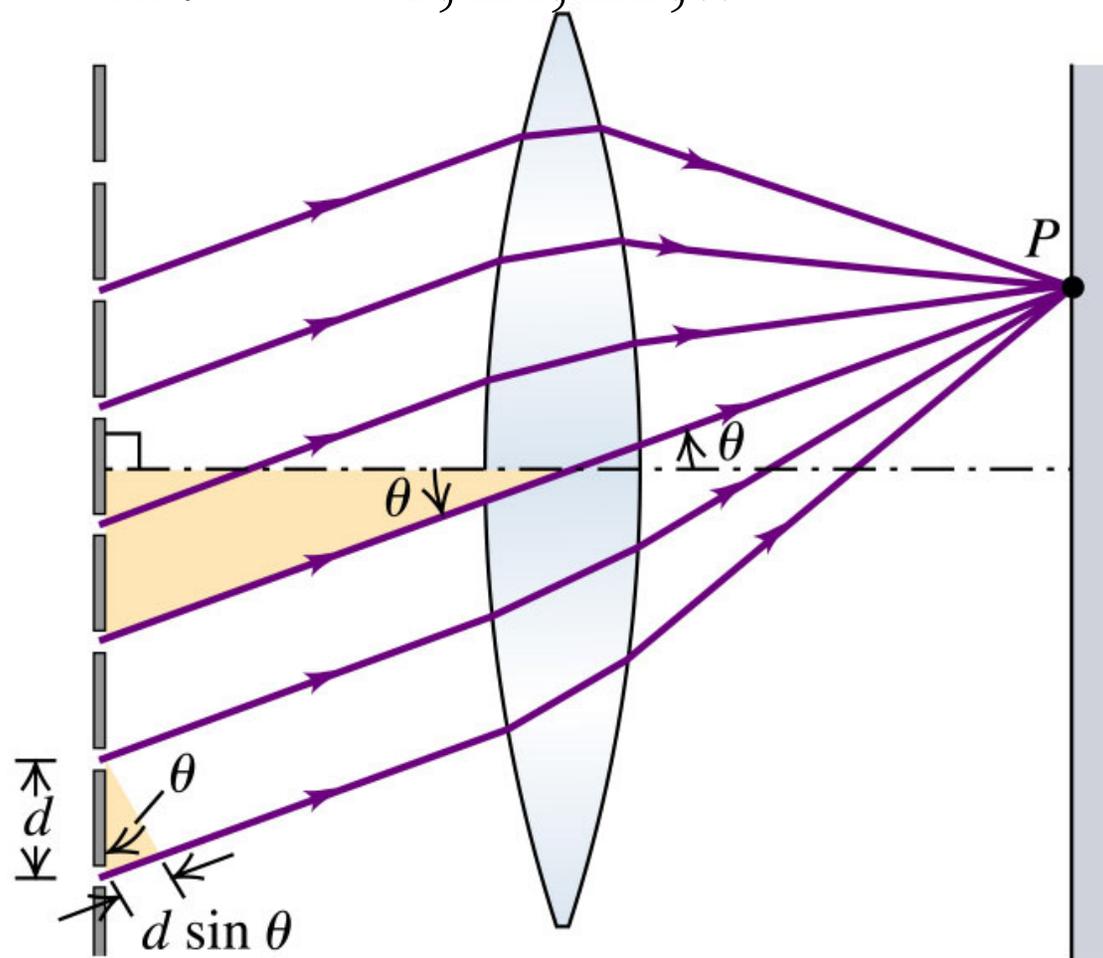
(c) $N = 16$

Multiple-slit Fraunhofer diffraction

Constructive interference happens when

$$d \sin \theta = m\lambda \quad m = 0, \pm 1, \pm 2, \dots$$

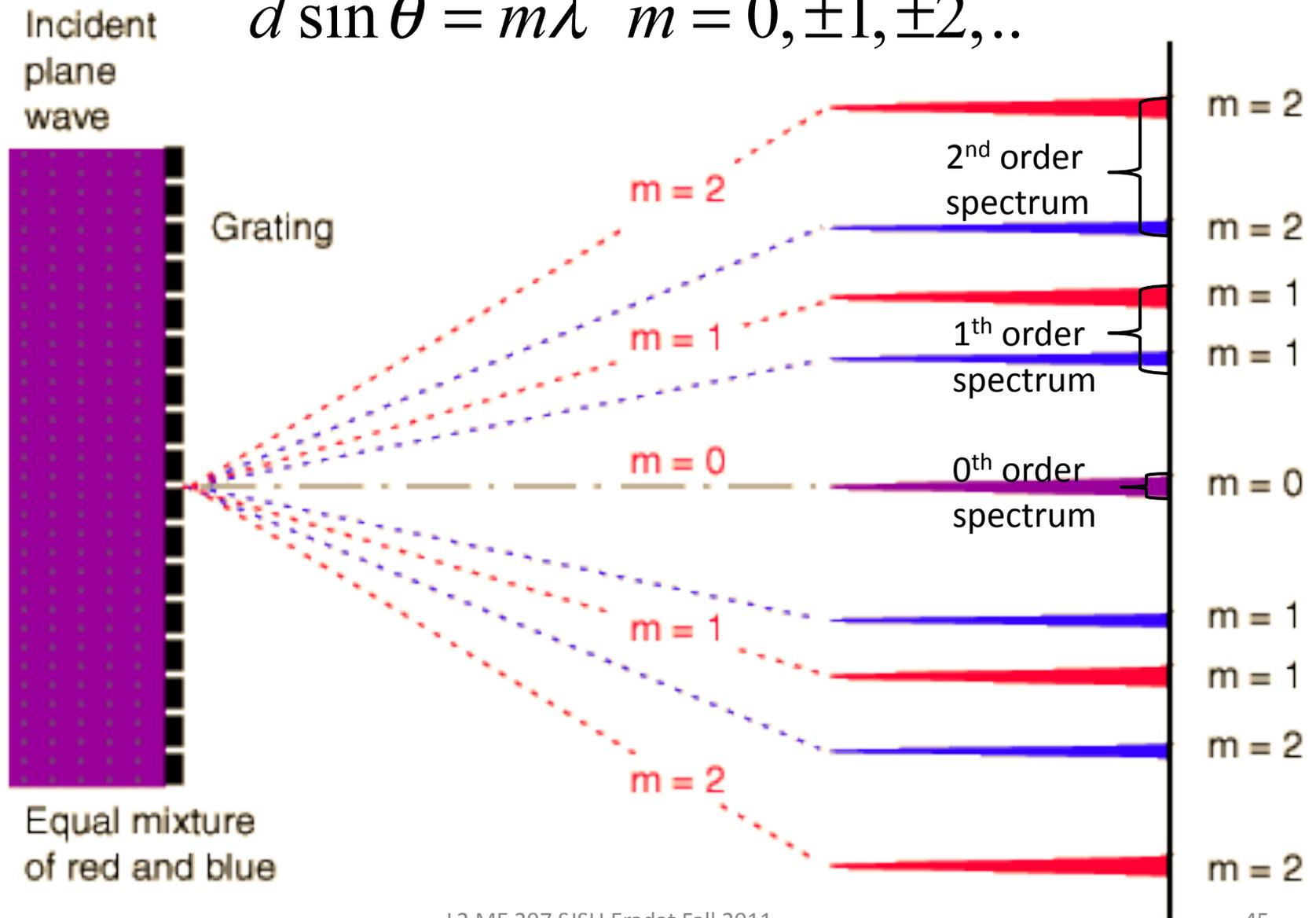
Parallel rays arrive
Or
Planar wave fronts
arrive



Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley.

Separation of colors with diffraction grating

$$d \sin \theta = m\lambda \quad m = 0, \pm 1, \pm 2, \dots$$



Matrix methods in paraxial optics

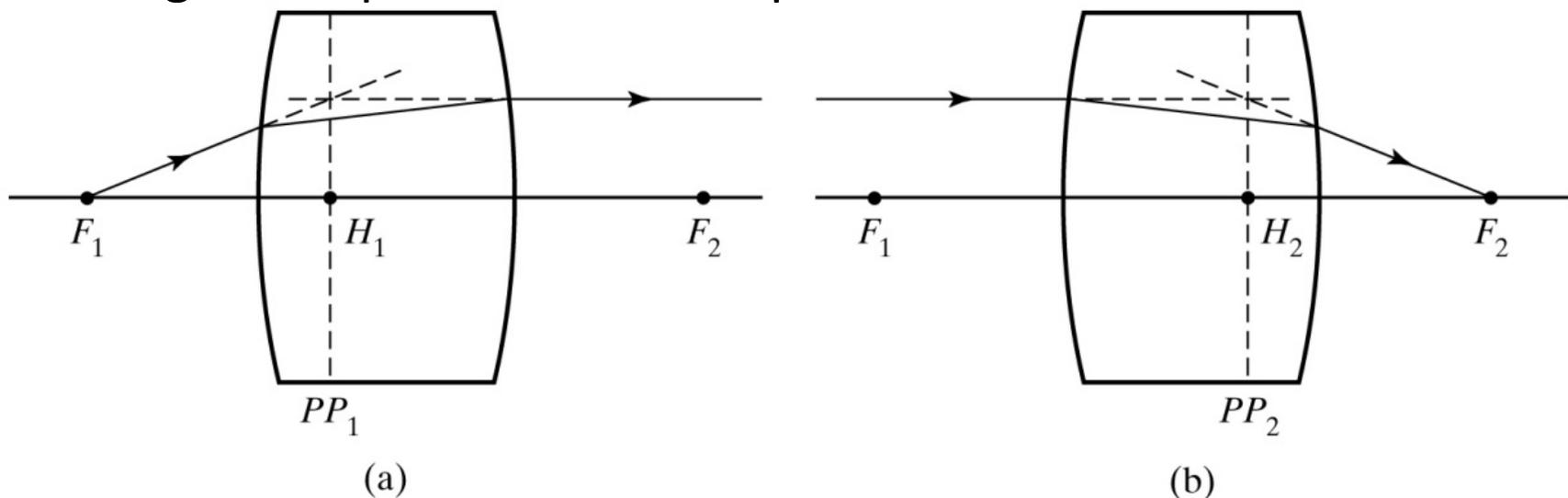
- Describing a single thick lens in terms of its cardinal points.
- Describing a single optical element with a 2x2 matrix.
- Analysis of train of optical elements by multiplication of 2x2 matrices describing each element.
- Computer ray-tracing methods, a more systematic approach

Cardinal points and cardinal planes

- Imaging properties of a thick lens can be deduced from the six cardinal points on its axis. Planes normal to the axis at the cardinal points are called cardinal planes. They are:
- First and second set of focal points and focal planes.
- First and second set of principal points and principal planes.
- First and second set of nodal points and nodal planes.

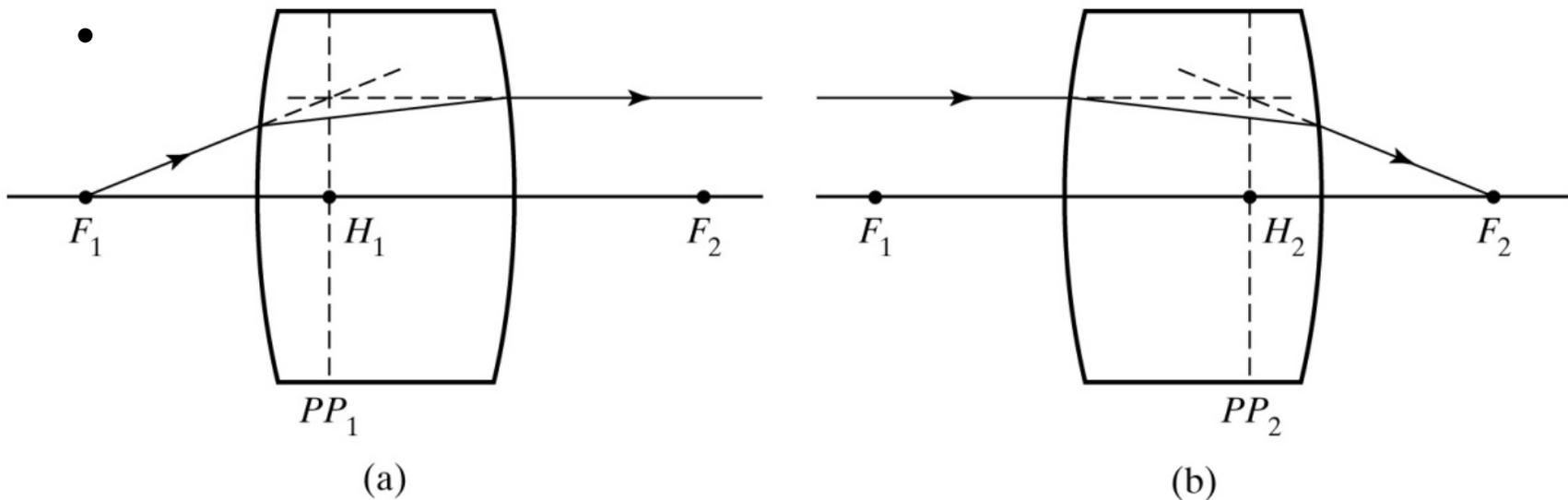
Focal points and focal planes

- Object focal plane: loci of the object points when image is at infinity
- Image focal plane: loci of the image points when object is at infinity
- Image and object focal points: intersection of the object and image focal planes with the optical axis.



Principal points and principal planes

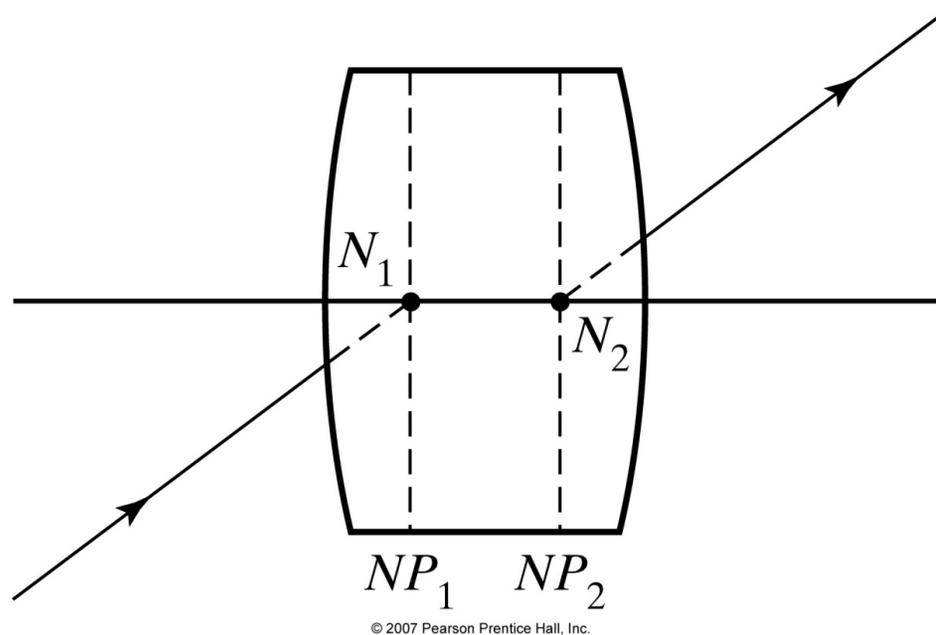
- The rays determining the focal points change direction at their intersection with the principal planes.
- Principal points are at the intersection of the principal planes the optical axis.



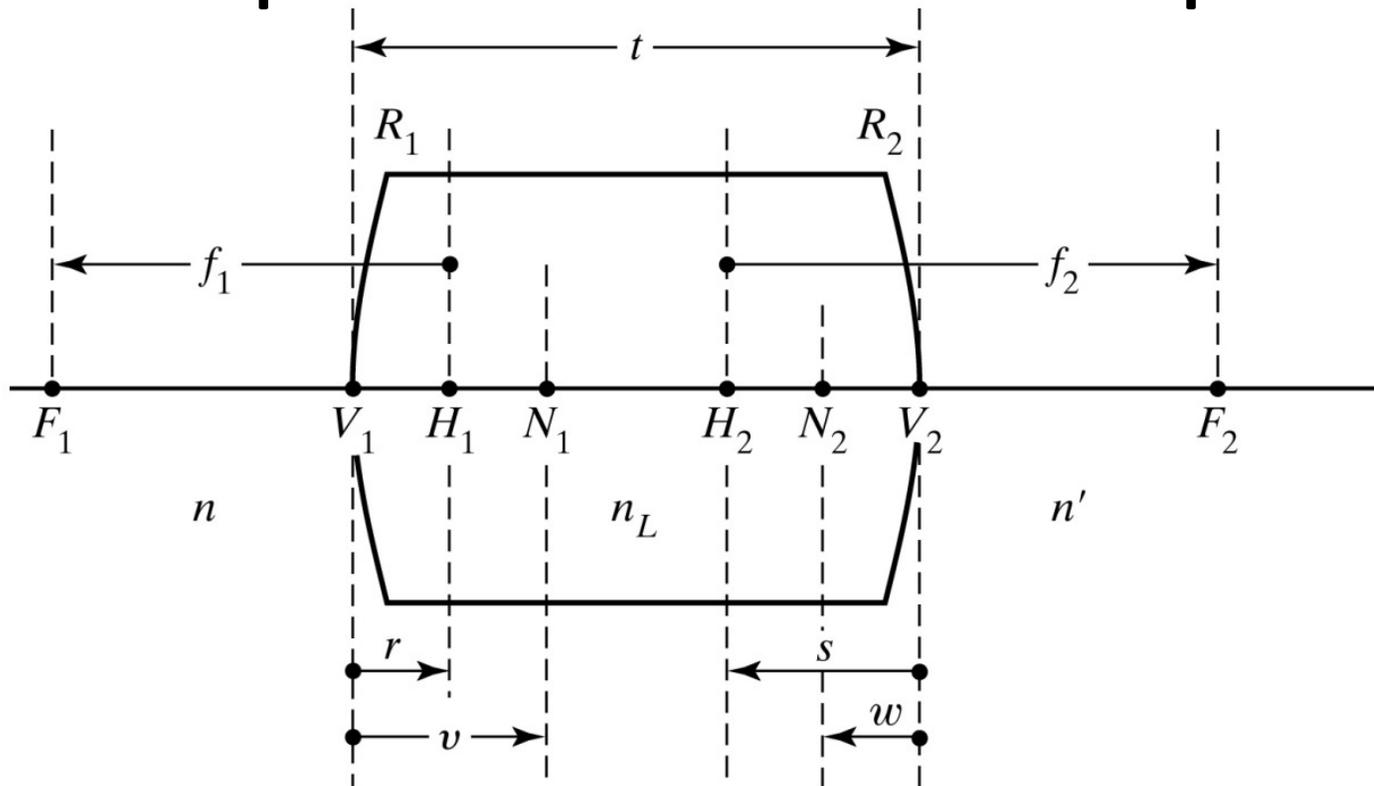
© 2007 Pearson Prentice Hall, Inc.

Nodal points and nodal planes

- Nodal points of a thick lens or any optical system permit correction to the ray that aims the center of the lens.
- Any ray that aims the first nodal point emerges from the second nodal point undeviated but slightly displaced.



Cardinal points and cardinal planes



© 2007 Pearson Prentice Hall, Inc.

- All the distances that are directed **to the left** are **negative (-)**
- All the distances that are directed **to the right** are **positive (+)**
- Notice that focal distances are not measured from the vertices

Basic equations for the thick lens II

$$\frac{1}{f_1} = \frac{n_L - n'}{nR_2} - \frac{n_L - n}{nR_1} - \frac{(n_L - n)(n_L - n')}{nn_L} \frac{t}{R_1R_2} \quad \text{and} \quad f_2 = -\frac{n'}{n} f_1$$

If $n = n'$ then $f_2 = -f_1$

Location of the principal planes:

$$r = \frac{n_L - n'}{n_L R_2} f_1 t;$$

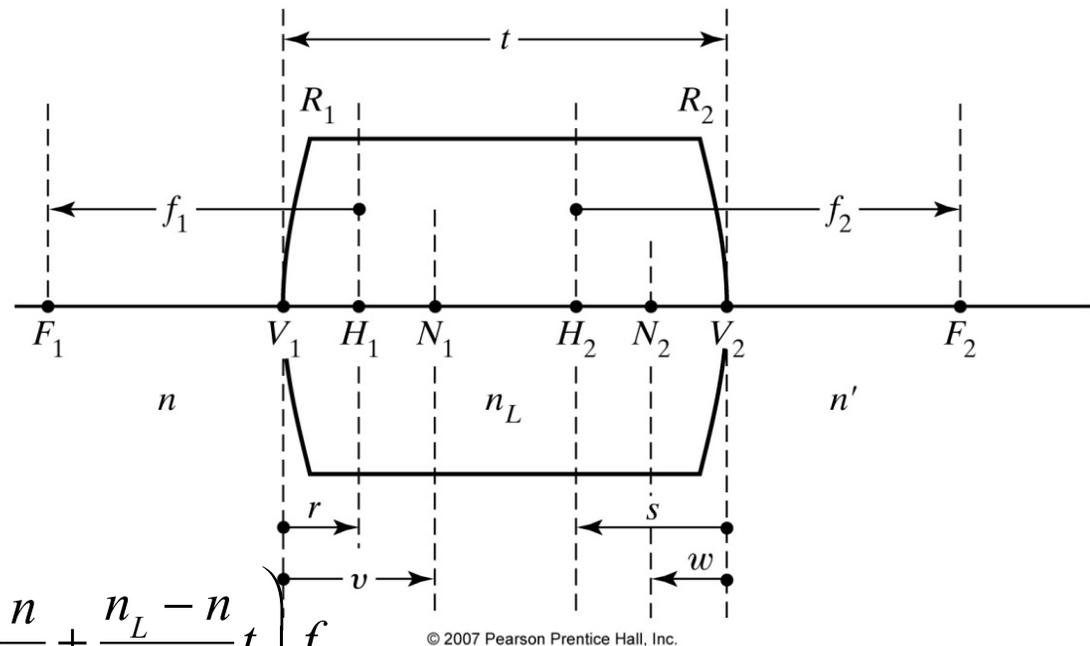
$$s = \frac{n_L - n}{n_L R_1} f_2 t$$

The positions of the nodal points:

$$v = \left(1 - \frac{n'}{n} + \frac{n_L - n'}{n_L R_2} t \right) f_1; \quad w = \left(1 - \frac{n}{n'} + \frac{n_L - n}{n_L R_1} t \right) f_2$$

Image and object distances and lateral magnification:

$$-\frac{f_1}{s_o} + \frac{f_2}{s_i} = 1 \quad \text{and} \quad m = -\frac{ns_i}{n's_o}$$



Basic equations for the thick lens II

For an ordinary thin lens in air: $n = n' = 1$ and $r = v$, $s = w$

we arrive at the usual thin lens equations:

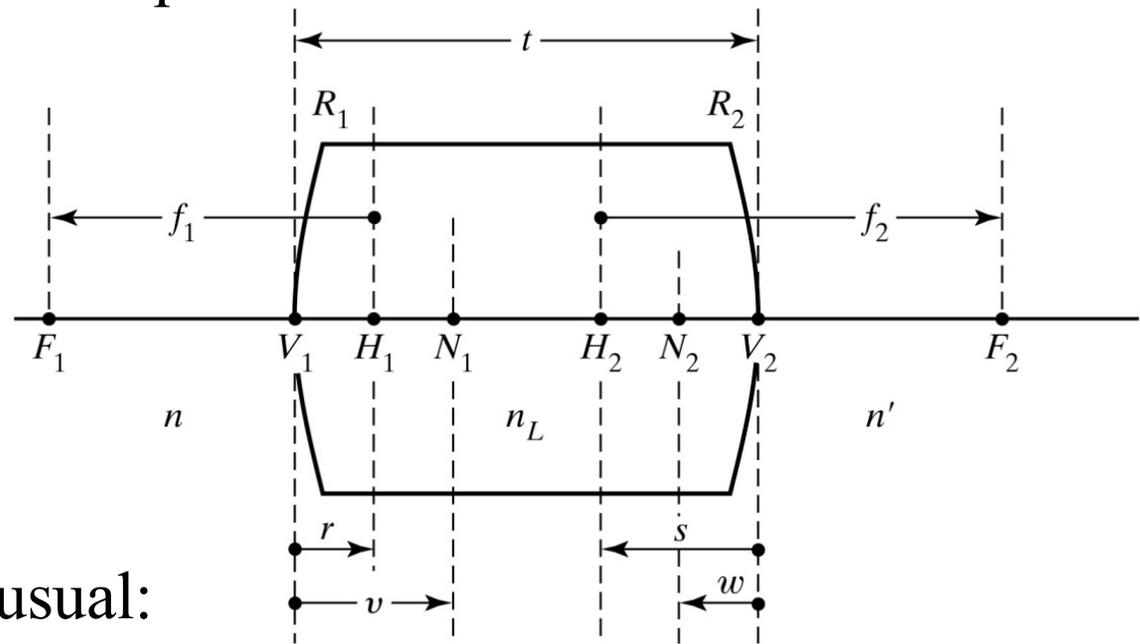
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$m = -\frac{s_i}{s_o}$$

$$f = f_2 = -f_1$$

The sign convention is as usual:

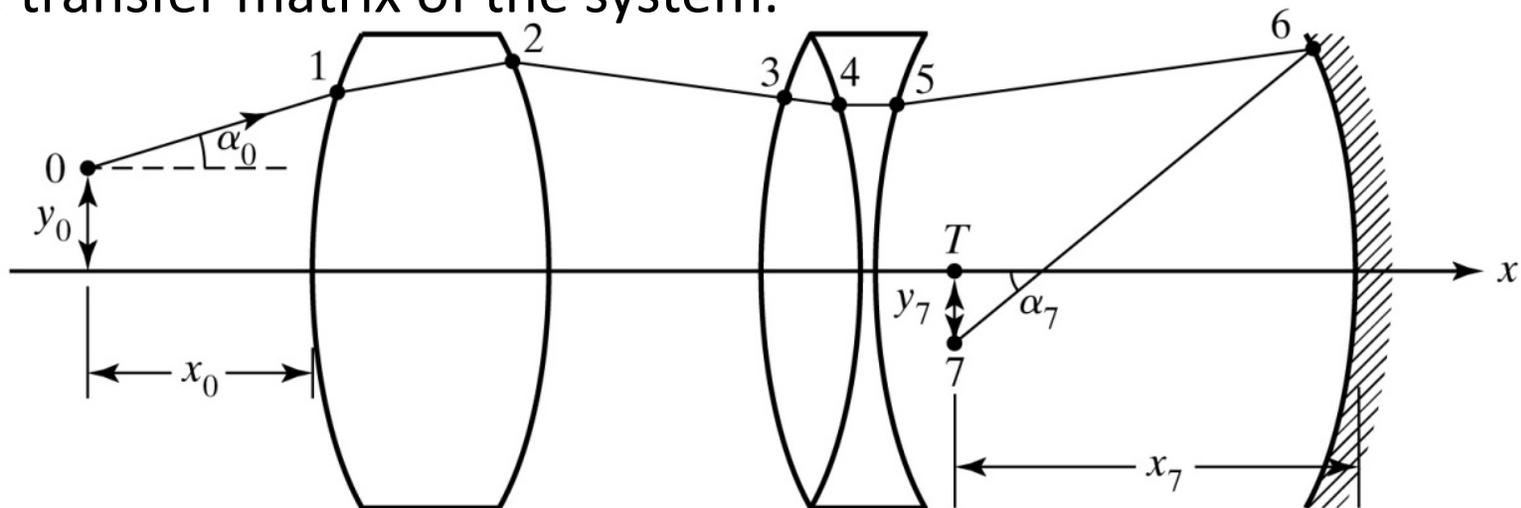
real + and virtual - as long as the distances are measured relative to their corresponding principal planes.



© 2007 Pearson Prentice Hall, Inc.

The matrix methods in paraxial optics

- For optical systems with many elements we use a systematic (programmable) approach called matrix method.
- For each ray as it progresses through the optical system we follow two parameters .
- A ray is defined by its height y and its angle α with the optical axis.
- We can express y_{final} and α_{final} in terms of y_1 and α_1 multiplied by the transfer matrix of the system.



© 2007 Pearson Prentice Hall, Inc.

L2 ME 297 SJSU Eradat Fall 2011

Example: The translational matrix

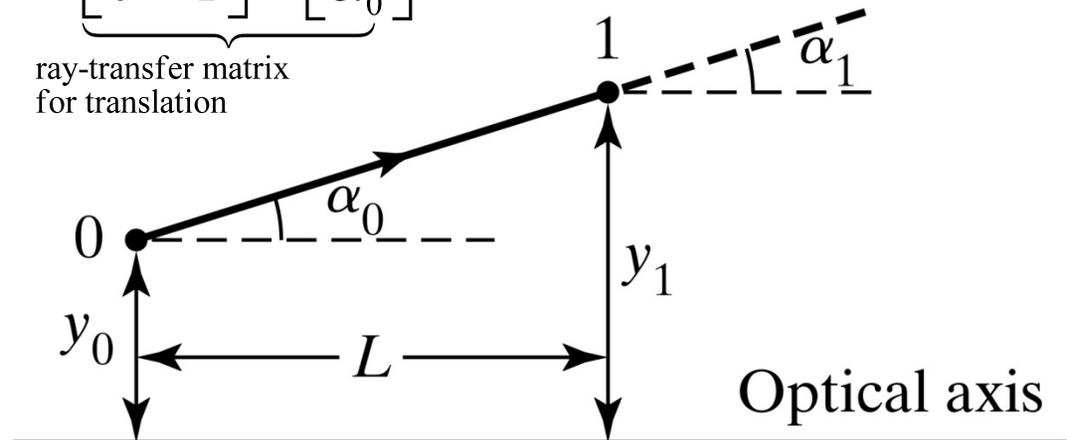
Consider simple translation of a ray in a homogeneous medium.

Translation from point 0 to 1 with paraxial approximation:

$$\alpha_1 = \alpha_0 \text{ and } y_1 = y_0 + L \tan \alpha_0 = y_0 + L\alpha_0$$

We rewrite the equations:

$$\left. \begin{array}{l} y_1 = (1)y_0 + (L)\alpha_0 \\ \alpha_1 = (0)y_0 + (1)\alpha_0 \end{array} \right\} \rightarrow \begin{bmatrix} y_1 \\ \alpha_1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}}_{\text{ray-transfer matrix for translation}} \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix}$$

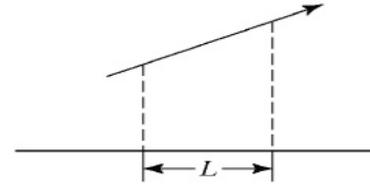


© 2007 Pearson Prentice Hall, Inc.

TABLE 18-1 SUMMARY OF SOME SIMPLE RAY-TRANSFER MATRICES

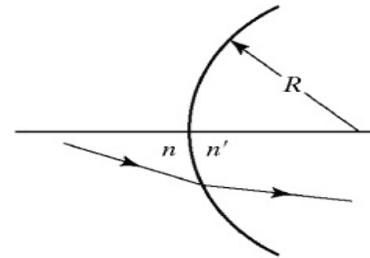
Translation matrix:

$$M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} = \mathfrak{R}$$



Refraction matrix,
spherical interface:

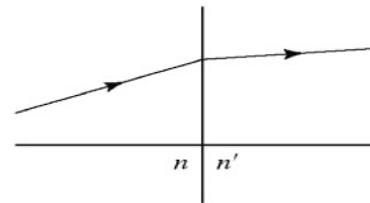
$$M = \begin{bmatrix} 1 & 0 \\ \frac{n - n'}{Rn'} & \frac{n}{n'} \end{bmatrix} = \mathfrak{R}$$



(+R) : convex
(-R) : concave

Refraction matrix,
plane interface:

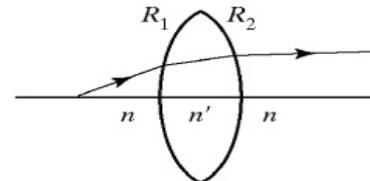
$$M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n}{n'} \end{bmatrix}$$



Thin-lens matrix:

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

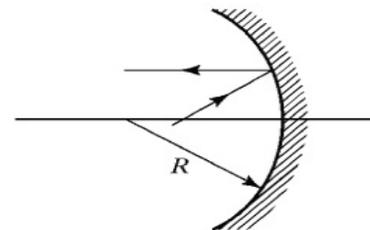
$$\frac{1}{f} = \frac{n' - n}{n} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$



(+f) : convex
(-f) : concave

Spherical mirror
matrix:

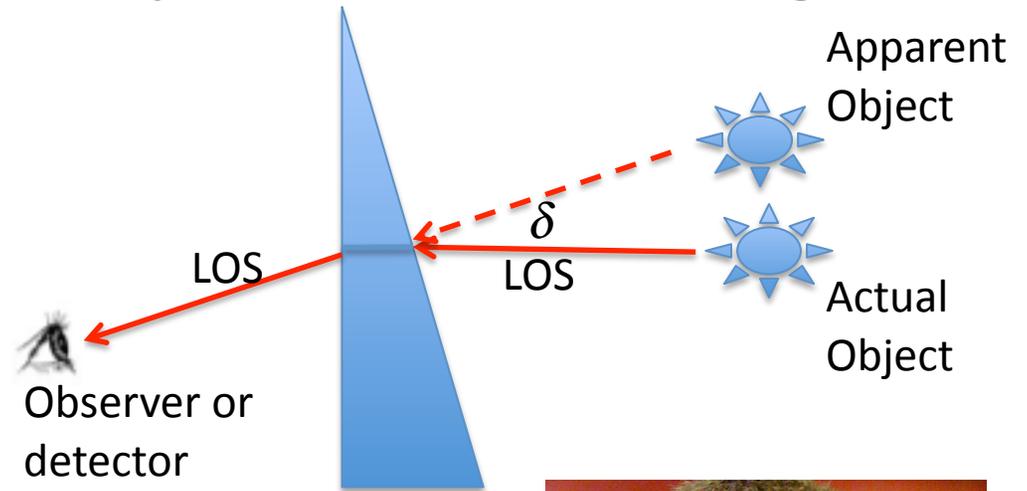
$$M = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$



(+R) : convex
(-R) : concave

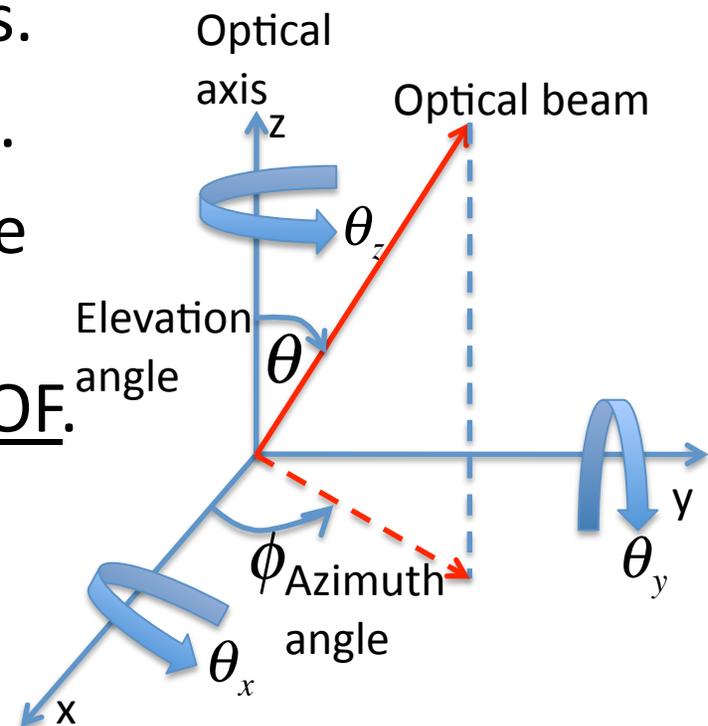
System Line of Sight (LOS) or where the optical system is looking

- To determine the line of sight of the optical system follow the actual path of the light from object to observation point or detector.
- To experience look through somebody's glasses (thick ones 😊)



6 Degrees of freedom (6DOF) of a rigid body and LOS

- Translational DOFs in xyz directions.
- Rotational DOFs around xyz axis.
- Convention: z is the optical axis.
- For every optical component we shall determine how the LOS changes with motion in each DOF.



Prisms and deviation angle

Angle of deviation of a prism when the light is passing through symmetrically is:

$$\sin \frac{A + \delta}{2} = n_{\lambda} \sin \frac{A}{2}$$

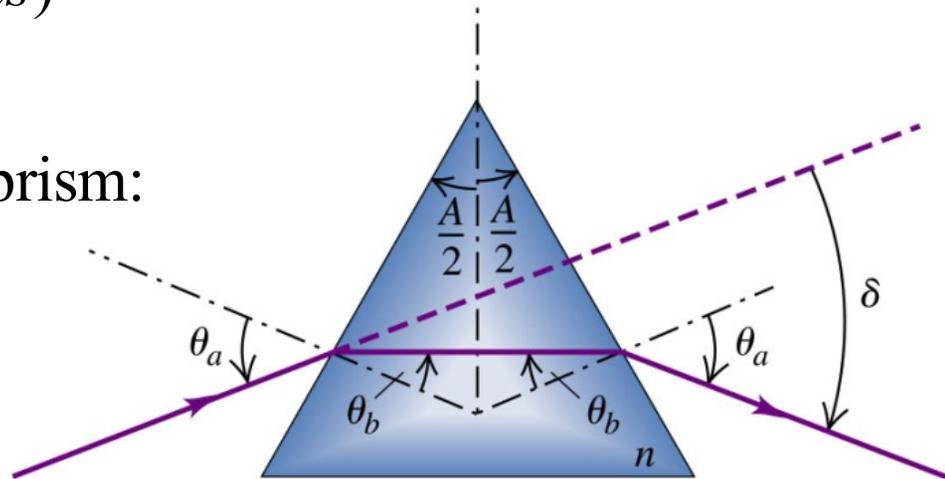
For thin prisms A is small and small-angle approximation is valid

$$\sin A \approx \tan A \approx A(\text{in radians})$$

and we get:

Deviation angle for a thin prism:

$$\underline{\delta = (n_{\lambda} - 1)A}$$



Chromatic dispersion of a thin prism

When polychromatic light hits the prism, each wavelength is deviated slightly at a different angle.

Deviation angle for a thin prism:

$$\underline{\delta_{\lambda} = (n_{\lambda} - 1)\alpha}$$

Here α is apex for a thin prism.

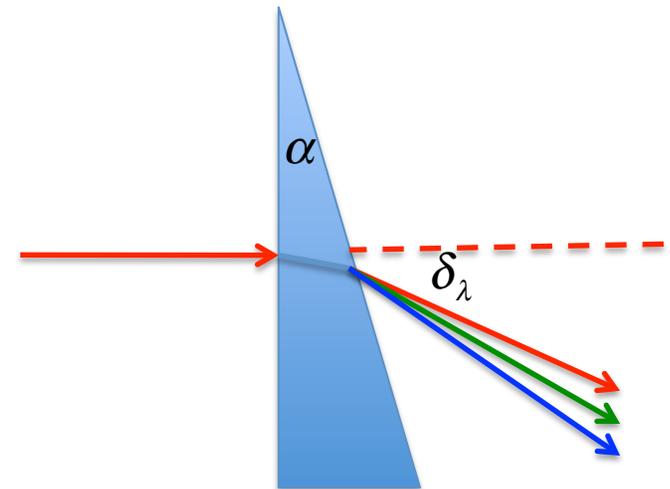
Differentiating we get $\underline{d\delta_{\lambda} = \alpha dn_{\lambda}}$

Substituting α we get the chromatic dispersion:

$$d\delta_{\lambda} = \frac{\delta_{\lambda}}{n_{\lambda} - 1} dn_{\lambda} = \frac{\delta_{\lambda}}{V}$$

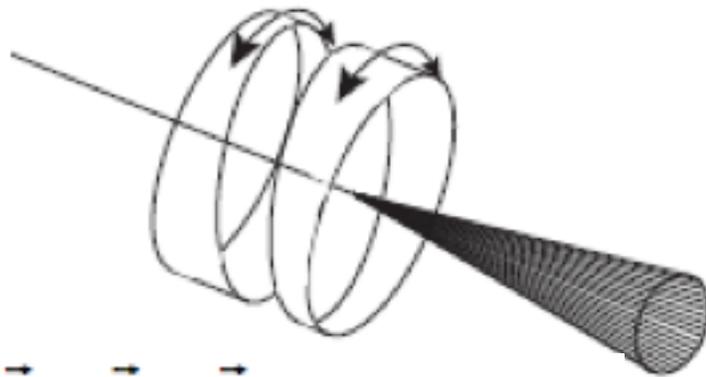
where V is the Abbe-V number for the glass.

(Ref: Smith P92-96, Yoder P360)



Risely Prisms

- Steer the line of sight by using rotation of prisms.
- Rotation of one prism moves LOS in a circle.
- Separate rotation of a second prism allows two-axis control of LOS

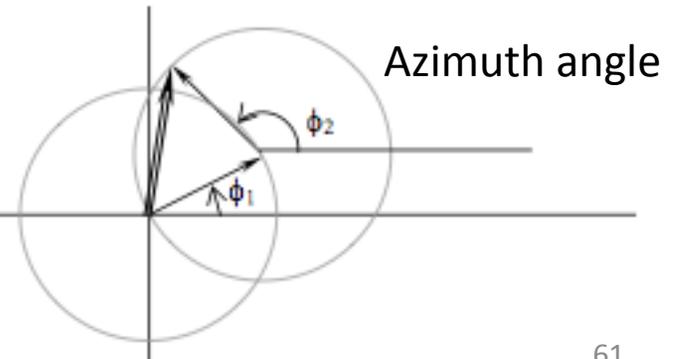
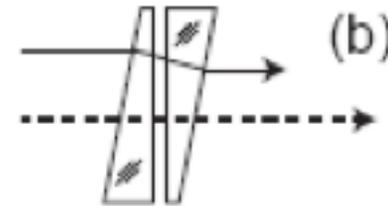


$$\vec{\delta} = \vec{\delta}_1 + \vec{\delta}_2$$

$$\delta x = \delta_1 \cos \phi_1 + \delta_2 \cos \phi_2$$

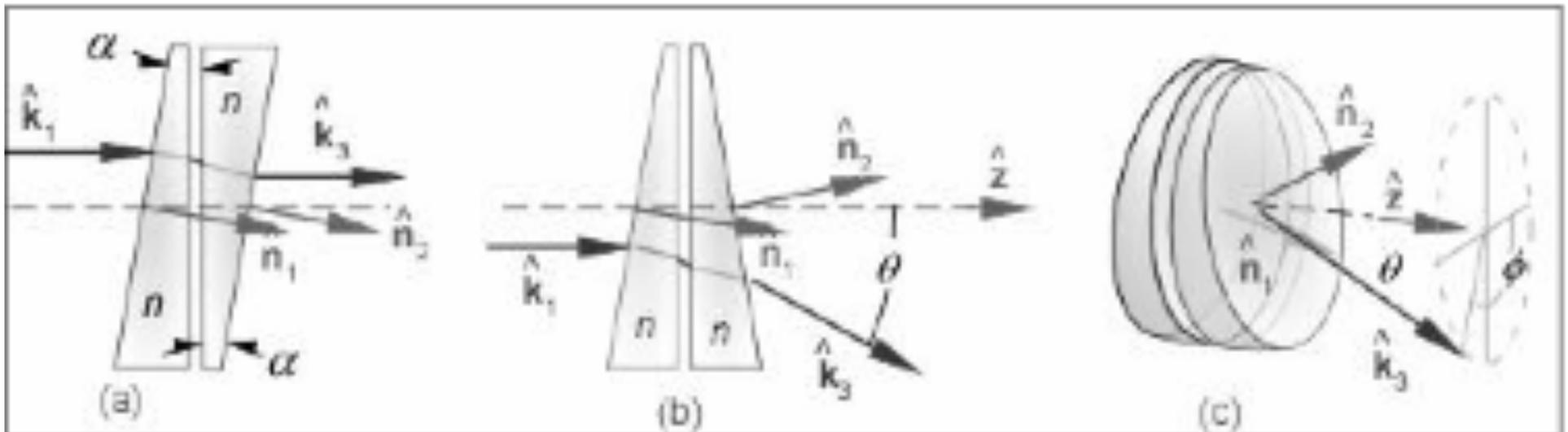
$$\delta y = \delta_1 \sin \phi_1 + \delta_2 \sin \phi_2$$

Ref: Burge, OptSci website



Risely Prisms

- An ideal Risley prism pair with identical wedge angles and indices of refraction shown. Only one DOF, rotation about the optical axis, is allowed that affects the line of sight.
 - a) with the wedge normals n aligned ($\phi' = 0^\circ$). The direction cosine k of the beam is offset, but undeflected at this orientation.
 - b) With wedge normals pointing opposite to each other ($\phi' = 180^\circ$), the beam has maximum elevation deviation.
 - c) A perspective view. Note that the azimuth angle ϕ' is swept out by co-rotation of the prism pair (dashed line).



Ref. Ostaszewski et. al

Applications of optical wedges

- A thin prism is known as an optical wedge can be used to change slightly the direction of light travel, and therefore it can be used in pairs as an alignment device.
- Optical wedges are also used in stereoscopic instruments to allow the viewer to observe the three-dimensional effect without forcing the eyes to point in different directions.
- A variable wedge can be integrated into a commercial pair of binoculars to stabilize the line of sight (LOS) in the presence of the user's slight hand movements.