Stress and Strain

ME 297
SJSU
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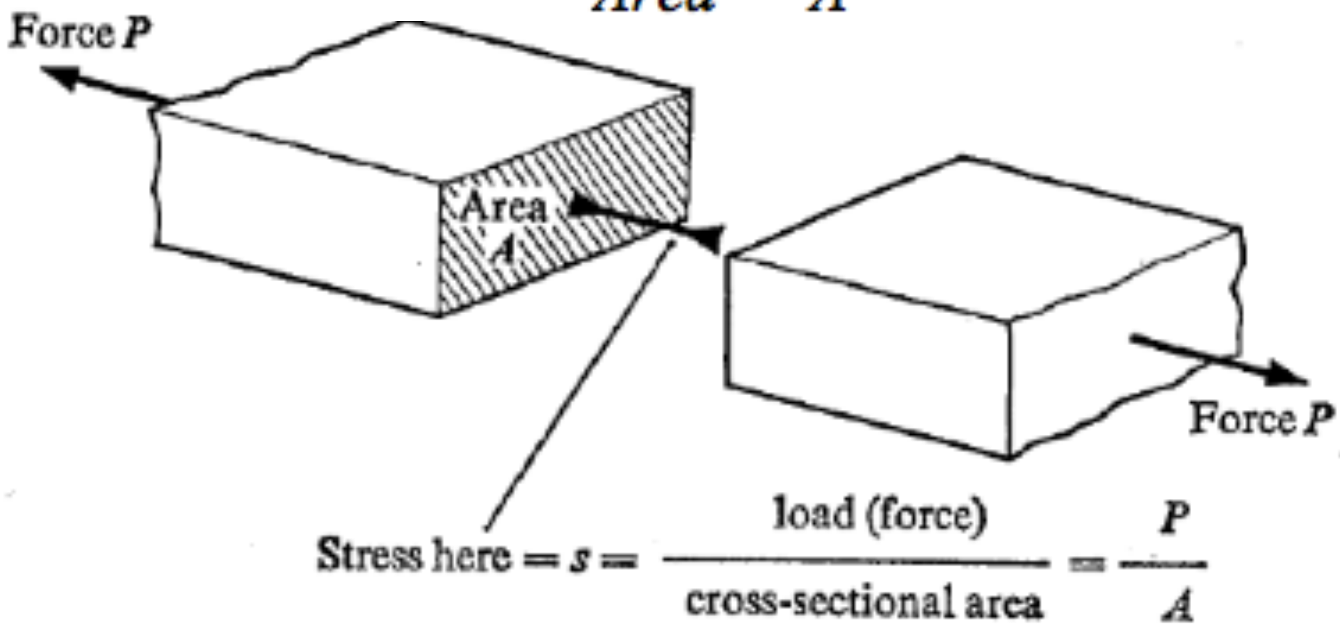
Normal stress and strain

- A normal stress σ, results when a member is subjected to an axial load applied through the centroid of the cross section.
- The average normal stress in the member is obtained by dividing the magnitude of the resultant internal force F by the cross sectional area A. Normal stress is

$$\sigma_{AVG} = \frac{Force}{Area} = \frac{P}{A}$$



Tensile and compressive stress

- **NORMAL stress:** is the stress σ acting in a direction perpendicular to the cut surface.
- **Normal stressed may be**
  - **tensile**
  - **compressive.**
- **Sign convention for normal stresses:**
  - **Tensile stresses are positive (+)**
  - **Compressive stresses are negative (-)**
- **Units of stress:**
  - **psi (or ksi or Msi)**
  - **Pa = N/m^2**
  - **mPa = N/mm^2**
  - **1 psi ~= 7000 Pa**

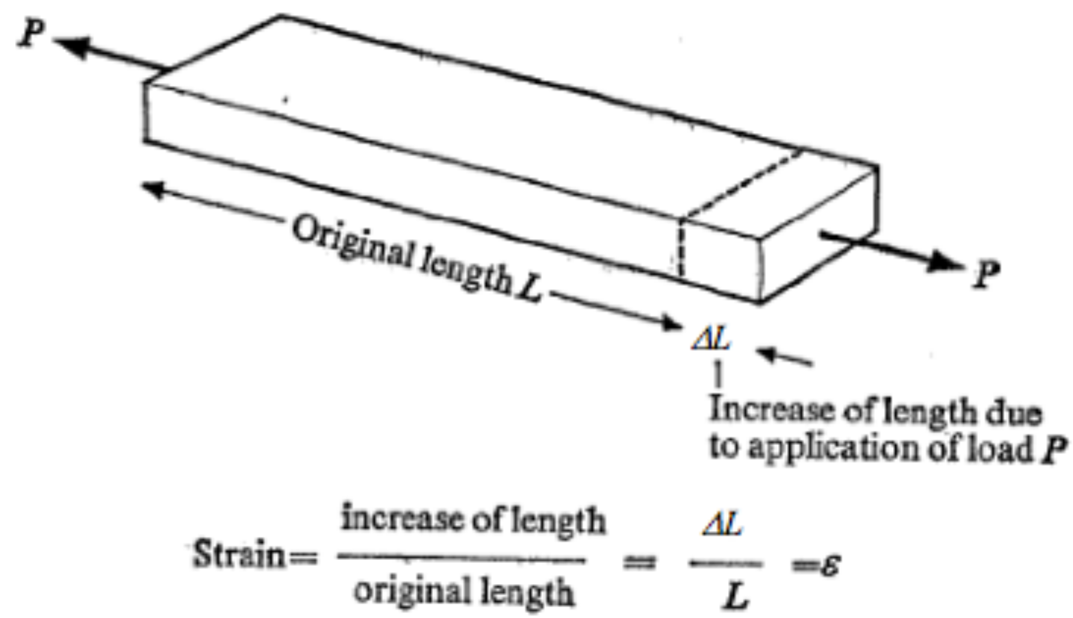
Deformation of Axial Members

- For a prismatic bar of length L in tension by axial force F we define the stress:

$$\sigma = \frac{F}{A}$$

- *Now, define strain ε as normalized elongation:*

$$\varepsilon = \frac{\Delta L}{L}$$



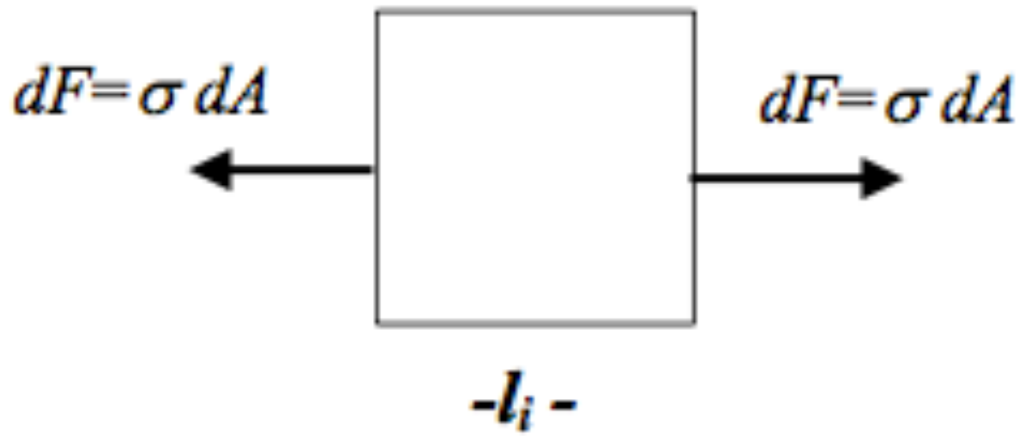
Stress and strain for differential elements

$$\sigma = \frac{F}{A} = \frac{dF}{dA}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{\Delta l_i}{l_i}$$

$$L = \sum_i l_i$$

$$\Delta L = \sum_i \Delta L_i$$



For homogenous material and small deflections, stress is proportional to strain

$$\sigma = \varepsilon \mathrm{E}$$

E = Young's modulus or modulus of elasticity

Then elongation is: $\varepsilon = \frac{\Delta L}{L} = \frac{\sigma}{E} = \frac{1}{E}\frac{F}{A} \rightarrow \Delta L = \frac{FL}{EA}$

*D*eformation is proportional to the load and the length and inversely proportional to the cross sectional area and the elastic modulus of the material.

Modulus of elasticity

$$\text{Young's Modulus} = \frac{Stress}{Strain} \rightarrow E = \frac{\sigma}{\varepsilon} = \frac{F / A}{\Delta L / L}$$

Modulus of elasticity has unit of pressure Pascal (SI) or psi (engineering)

$1\,psi = 1\ \text{lb/in}^2 = 6\ 894.75729$ pascals

E ~ 10,000,000 psi (10 Msi ~ 70 GPa) for aluminum

E ~ 10,000,000 psi (10 Msi ~ 70 GPa) for glass

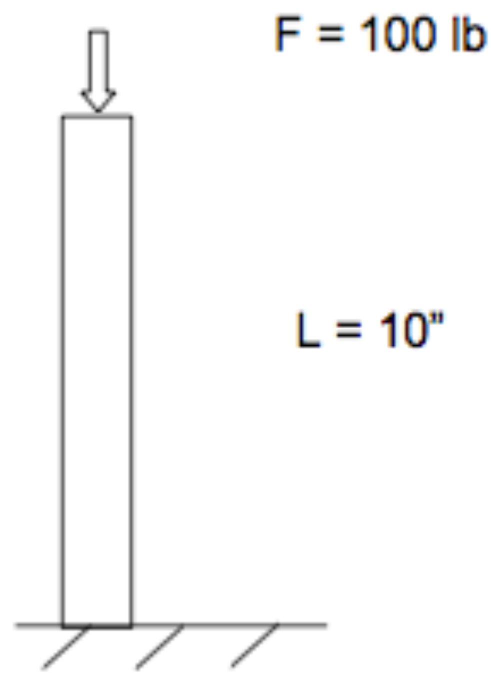
E ~ 30,000,000 psi (30 Msi ~200 GPa) for steel

Example

L = 10"   A = 1 in^2
W = 100 lbs
E = 10,000,000 psi (aluminum)

$$\Delta L = \frac{(100lb)(10")}{(10Msi)(1in^2)} = 100\mu in = 0.0001"$$

$$\Delta L \cong \frac{(450N)(250mm)}{(70GPa)(625mm^2)} \cong 2.5\mu m$$



Axial rigidity, Stiffness, Compliance

Axial rigidity = EA

A bar under tension is analogous to an axially loaded spring: $F = K\delta$,

K is the spring stiffness

δ is the string elongation under the force F.

*T*he above equation can be expressed as follows:

$$F = \frac{AE}{L}\Delta L = K\Delta L \rightarrow K = \frac{F}{\Delta L}$$

K, **stiffness** of an axially loaded bar, is **the force required to produce a unit deflection.**

C **compliance** is the deformation due to a unit load. *C*ompliance of a **axially loaded bar** is: $C = \frac{1}{K} = \frac{L}{AE}$

Combining members: Parallel

$$\Delta L = \frac{F_1}{K_1} = \frac{F_2}{K_2}$$

$$F = F_1 + F_2$$

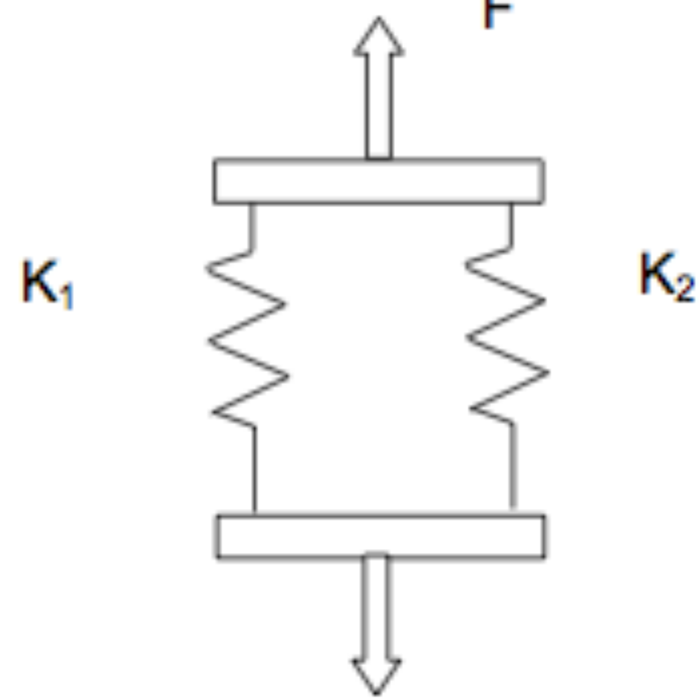
$$F = K_1 \Delta L + K_2 \Delta L$$

$$F = (K_1 + K_2)\Delta L$$

$$F = K_e \Delta L$$

$$K_e = K_1 + K_2$$

<u>Adding in parallel adds stiffness</u>



Combining members: series

$$\Delta L = \Delta L_1 + \Delta L_2$$

$$F = F_1 = F_2$$

$$\Delta L_1 = C_1 F$$

$$\Delta L_2 = C_2 F$$

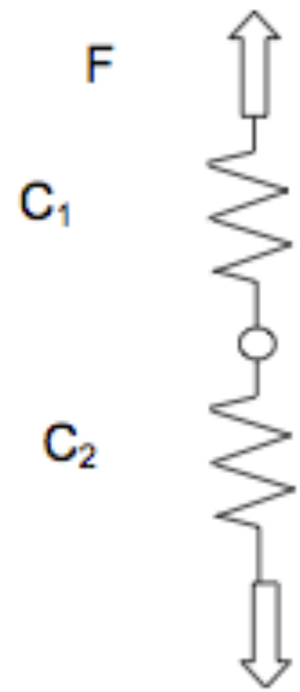
$$\Delta L = C_1 F + C_2 F$$

$$= (C_1 + C_2) F$$

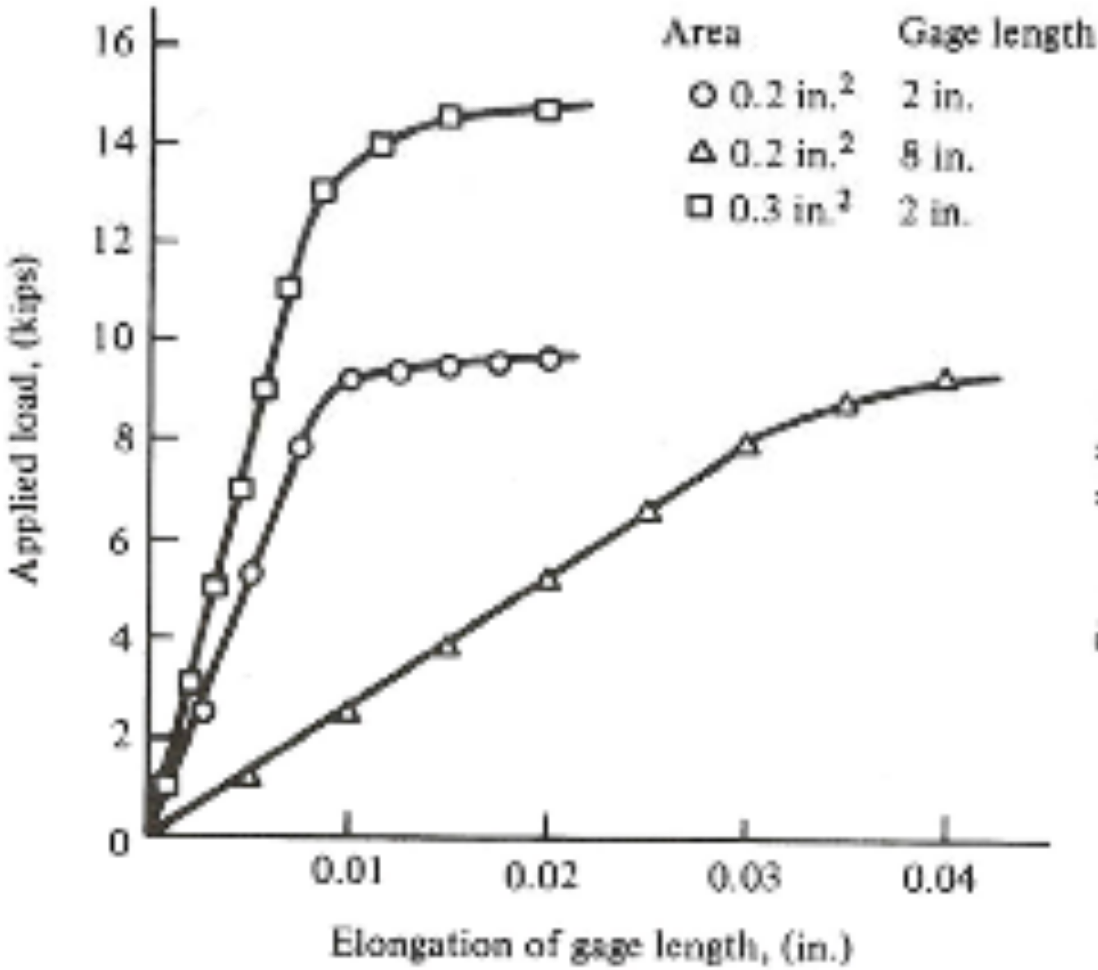
$$= C_e F$$

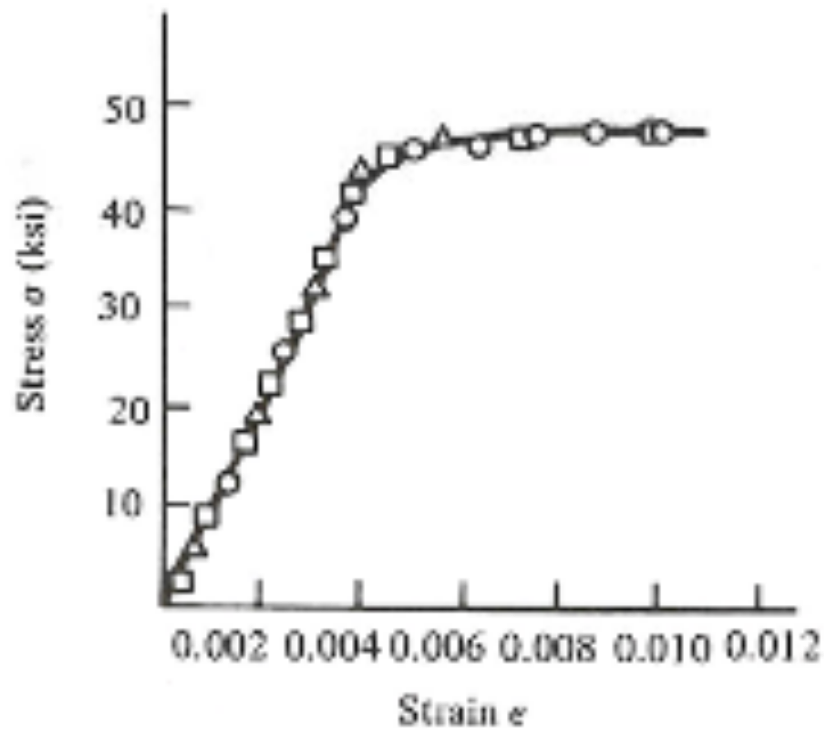
$$C_e = C_1 + C_2$$

<u>Adding in serial adds compliance</u>



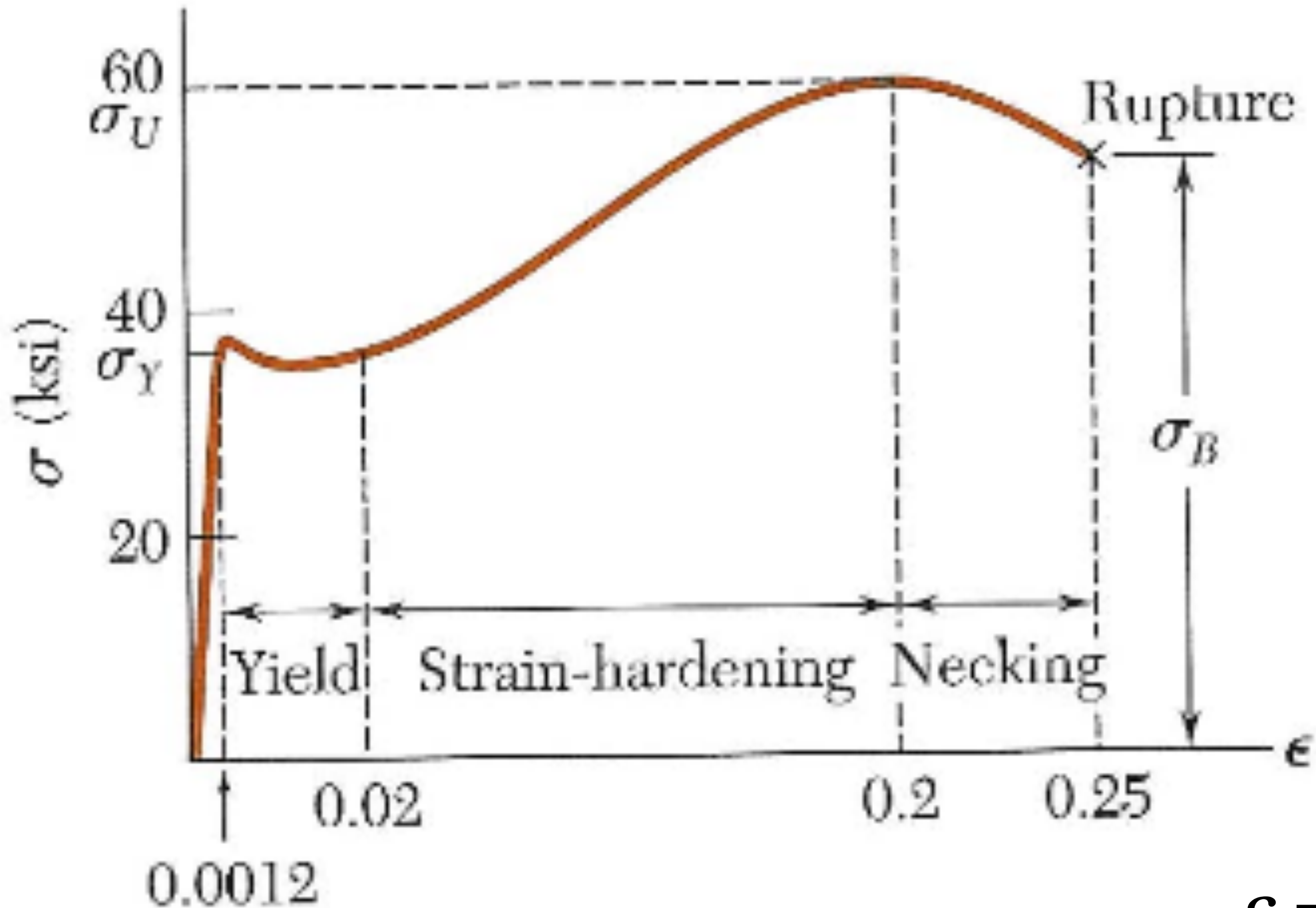
Materials


Area
Gage length
0.2 in.²
2 in.
0.2 in.²
8 in.
0.3 in.²
2 in.
Applied load, (kips)
Elongation of gage length, (in.)
0
2
4
6
8
10
12
14
16
0.01
0.02
0.03
0.04


Stress σ (ksi)
Strain e
10
20
30
40
50
0.002
0.004
0.006
0.008
0.010
0.012

Material: Low carbon steel

$$\sigma = \frac{F}{A}$$



(a) Low-carbon steel

$$\varepsilon = \frac{\Delta L}{L}$$

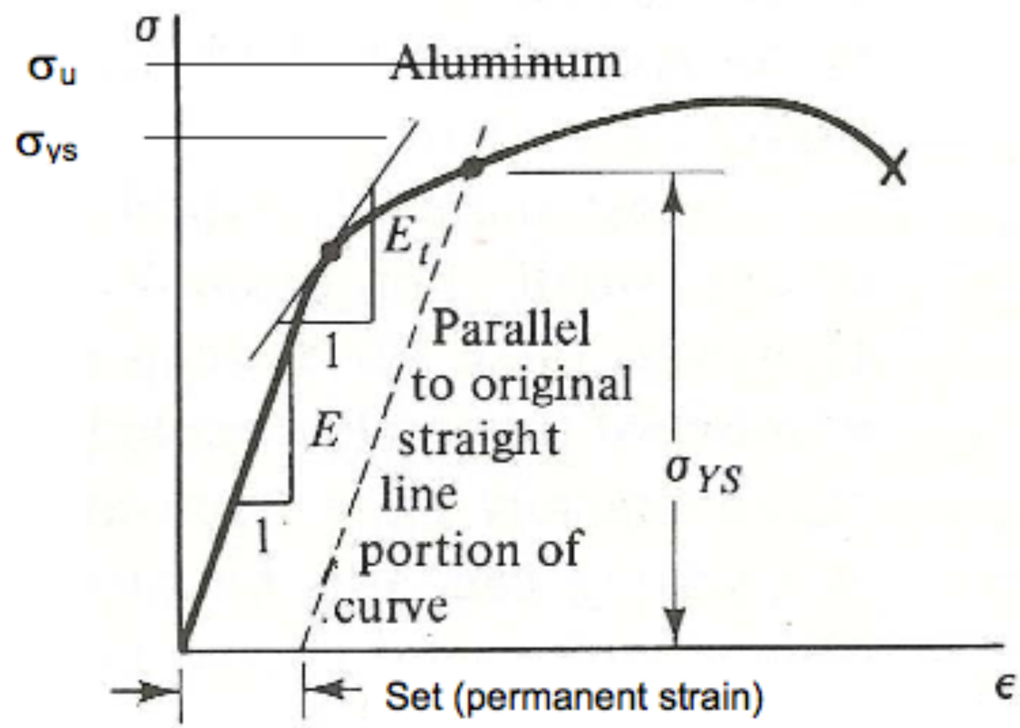
Performance of the material under stress

$E = \frac{Stress}{Strain} = \frac{d\sigma}{d\varepsilon}$ for small loads and small deflections

σ_{YS}, **Yield Strength** : The maximum stress that can be applied without exceeding a specified value of permanent strain (typically 0.2% = .002 in/in).

σ_{PEL}, **Precision elastic limit** or **micro - yield strength** : The maximum stress that can be applied without exceeding a permanent strain of 1 ppm or 0.0001%

σ_{U}, **Ultimate Strength** :
The maximum stress the material can withstand (based on the original area).

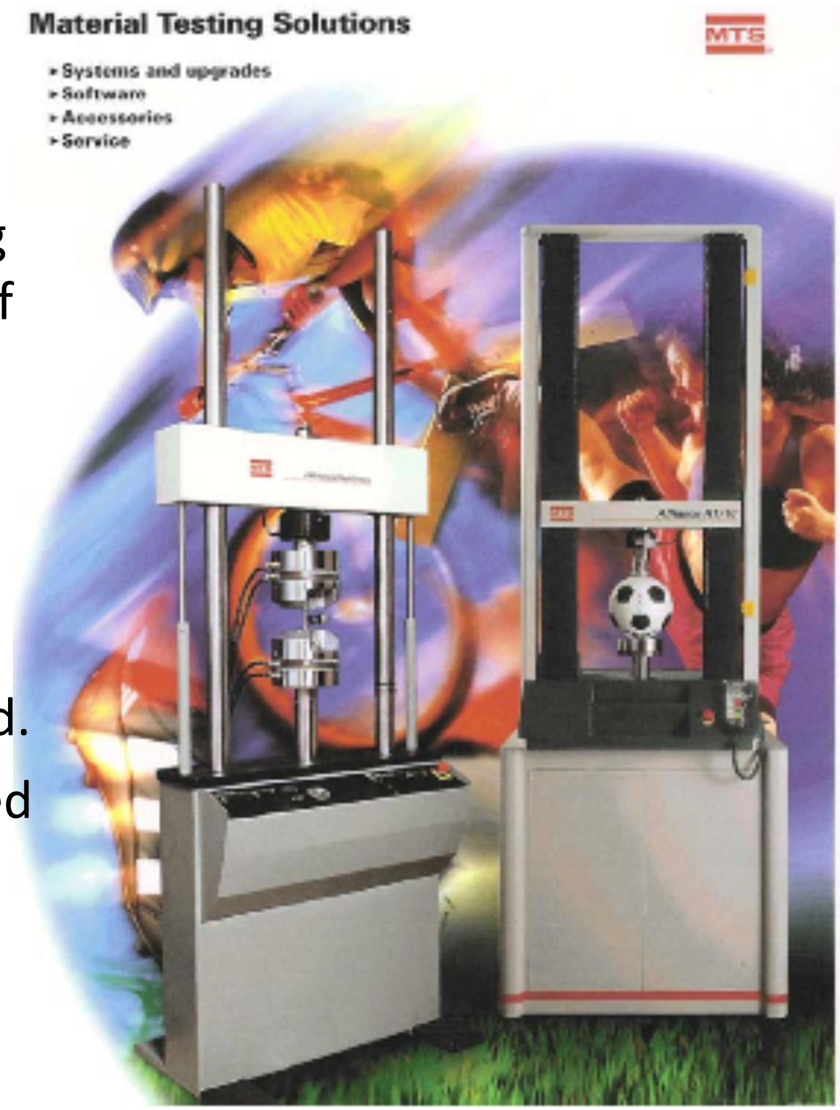


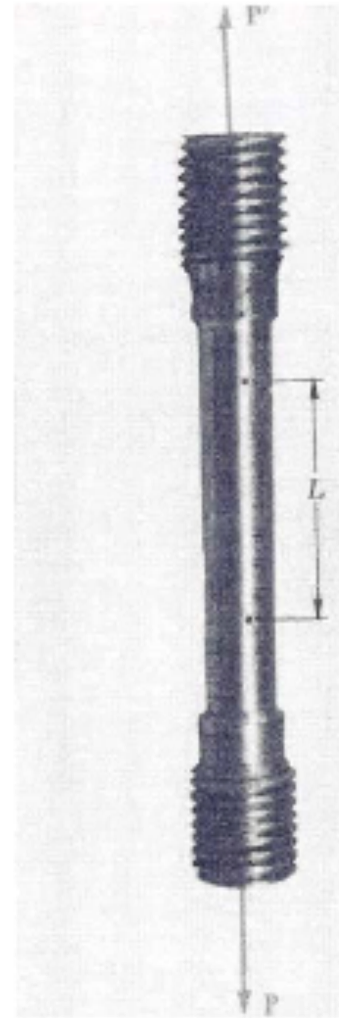
Resistive properties of the material

- Resistive properties of materials relate the stresses to the strain.
- They can only be determined by experiment.
- Tensile test:
  - A load is applied along the longitudinal axis of a circular test specimen.
  - The applied load and the resulting elongation of the member are measured.
  - The process is repeated with increased load until the desired load levels are reached or the specimen breaks.
- Load-deformation data obtained from tensile and/or compressive tests do not give a direct indication of the material behavior, because **they depend on the specimen geometry**.
- Using the relationships we previously discussed, loads and deformations may be converted to stresses and strains.

Tensile testing

1. A load is applied along the longitudinal axis of a circular test specimen.
2. The applied load and the resulting elongation of the member are measured.
3. The process is repeated with increased load until the desired load levels are reached or the specimen breaks.



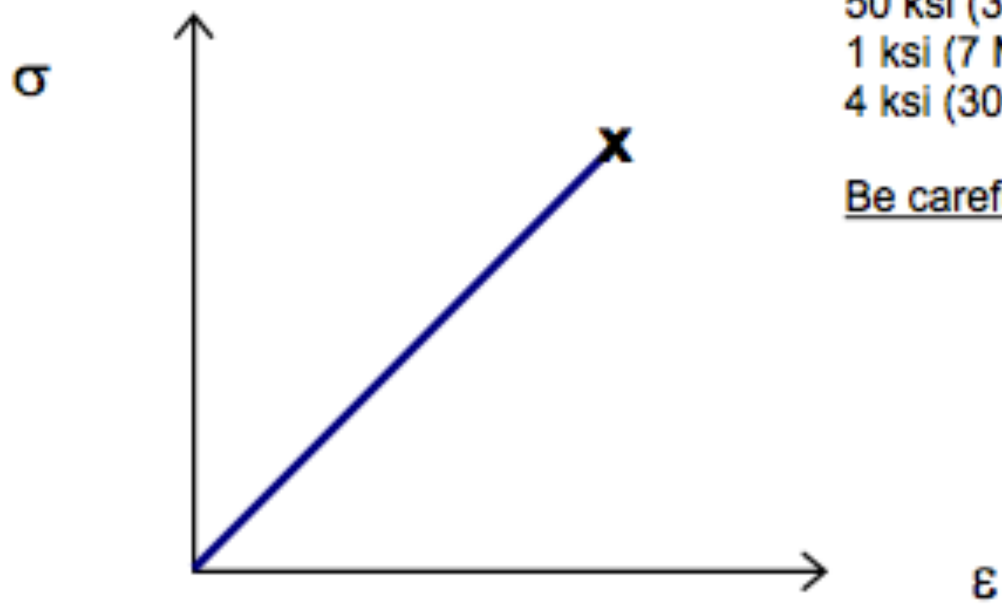


Strength of the material: stress which will cause the material to break

Material	Tensile strength p.s.i.	Tensile strength MN/m²
Metals		
STEELS		
Steel piano wire (very brittle)	450,000	3,100
High tensile engineering steel	225,000	1,550
Commercial mild steel	60,000	400
WROUGHT IRON		
Traditional	15,000–40,000	100–300
CAST IRON		
Traditional (very brittle)	10,000–20,000	70–140
Modern	20,000–40,000	140–300
OTHER METALS		
Aluminium: cast	10,000	70
wrought alloys	20,000–80,000	140–600
Copper	20,000	140
Brasses	18,000–60,000	120–400
Bronzes	15,000–80,000	100–600
Magnesium alloys	30,000–40,000	200–300
Titanium alloys	100,000–200,000	700–1,400

Strength of glass

Depends on critical flaw size



<u>Rule of thumb for glass strength</u>
50 ksi (350 MPa) compression
1 ksi (7 MPa) in tension
4 ksi (30 MPa) tensile (short duration)

<u>Be careful with this!</u>

Poisson's ratio

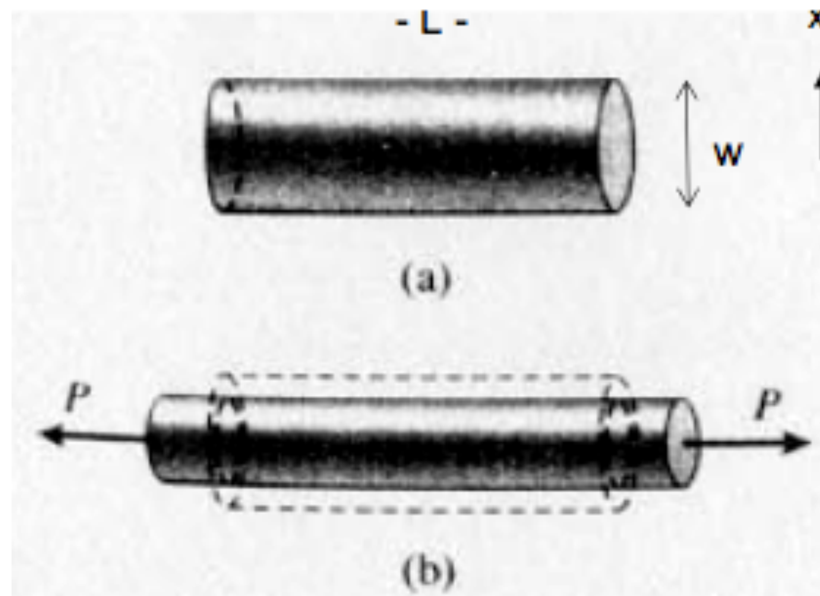
The ratio of lateral or transverse strain to the longitudinal strain.

Initial, unloaded state: $\begin{cases} L: \text{ lengt} \\ w: \text{ Width} \end{cases}$

Deformed state after loading: $\begin{cases} L' = L + \Delta L \\ w' = w + \Delta w \end{cases}$

Longitudinal strain: $\varepsilon_z = \dfrac{\Delta L}{L}$

Transverse strain: $\varepsilon_x = \dfrac{\Delta w}{w}$



Poisson's ratio: $v = -\dfrac{\varepsilon_x}{\varepsilon_z}$

Poisson's ratio for some material

Poisson's ratio for most materials ranges from 0.25 to 0.35.

Cork $\Rightarrow \nu \approx 0.0$

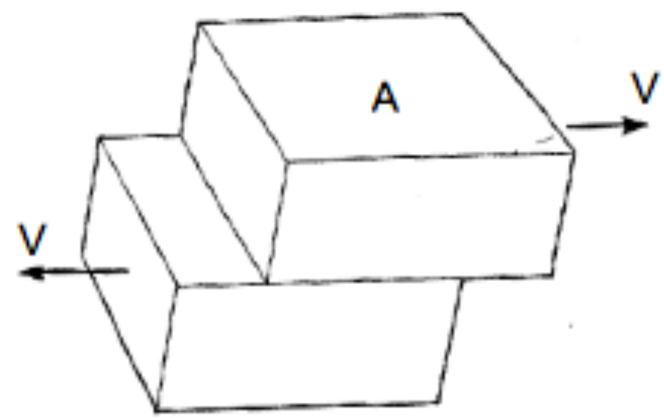
Steel $\Rightarrow \nu = 0.27 - 0.30$

Aluminum $\Rightarrow \nu = 0.33$

Rubber $\Rightarrow \nu \approx 0.5$ (limiting value for Poisson's ratio, volume is conserved)

Shear stress and strain

Shear force V is the force spread over area A



Shear stress: $\tau = \dfrac{V}{A}$

For small elements: $\begin{cases} \text{Stress: } \sigma = \dfrac{dF}{dA} \\ \text{Shear stress: } \tau = \dfrac{dV}{dA} \end{cases}$

Units same as stress: psi or Pa

$1ksi \simeq 7MPa = 7N/mm^2$

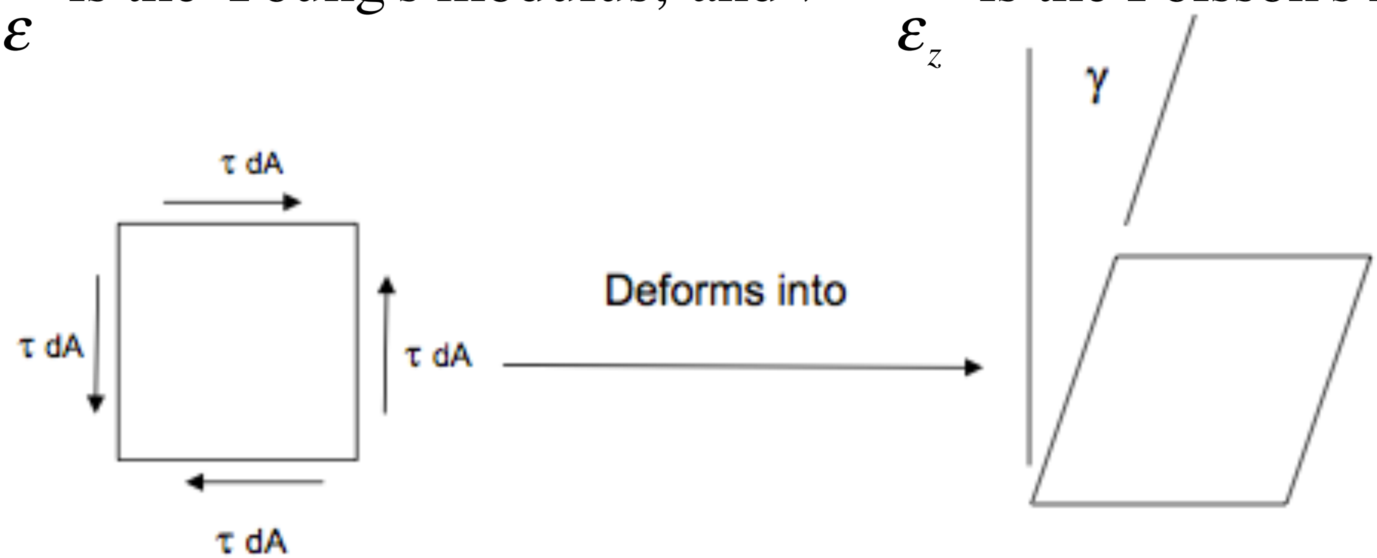
Shear strain

Shear strain: $\gamma = \frac{\tau}{G}$ where $\tau = \frac{V}{A}$ is the shear stress

G is the **shear modulus** or **modulus of rigidity**

For **linear, isotropic material**: $G = \frac{E}{2(1+v)} \simeq \frac{F/A}{2(\varepsilon_z + \varepsilon_x)}$

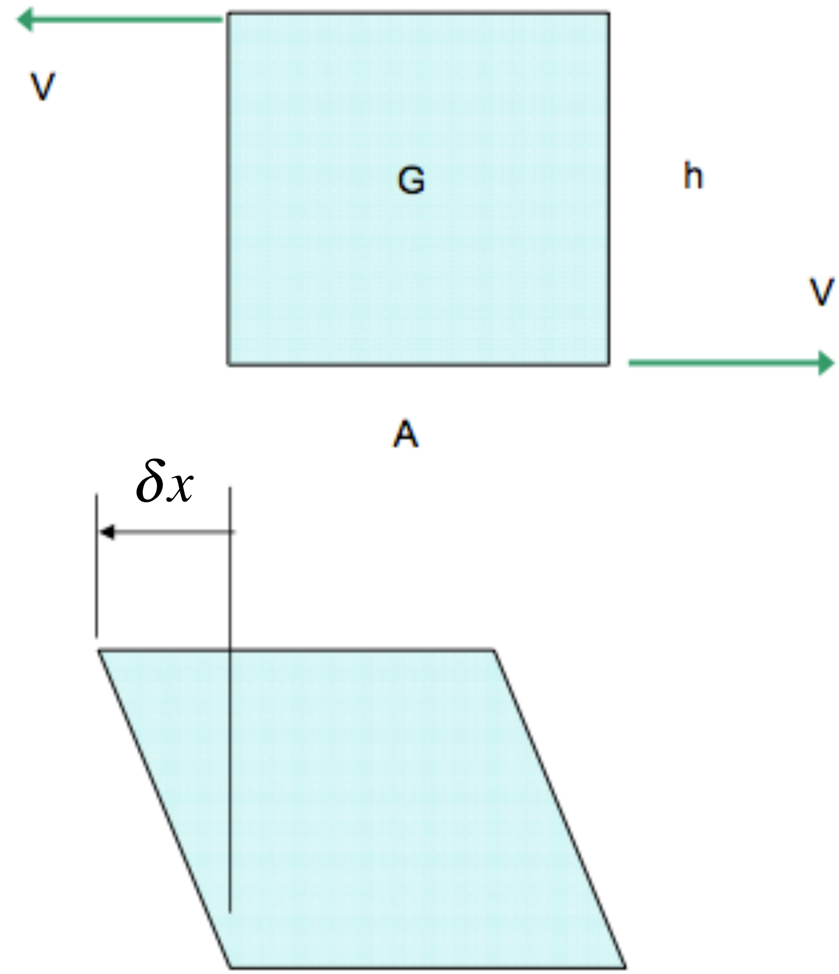
Where $E = \frac{\sigma}{\varepsilon}$ is the Young's modulus, and $v = \frac{\varepsilon_x}{\varepsilon_z}$ is the Poisson's ratio.



Shear stiffness

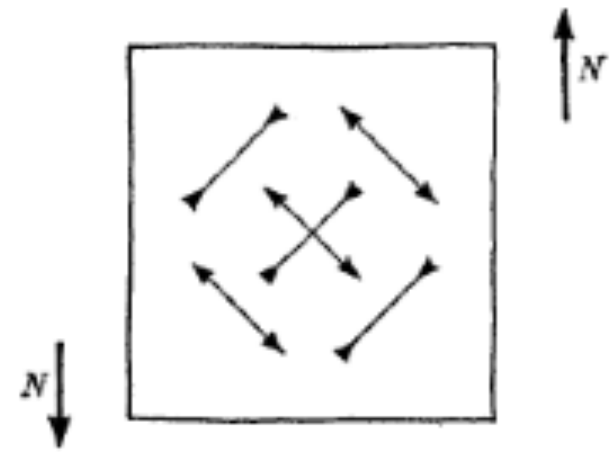
V : the shear force

$$\delta x = \gamma h = \frac{\tau}{G} h = \frac{V / A}{G} h = \frac{Vh}{AG}$$



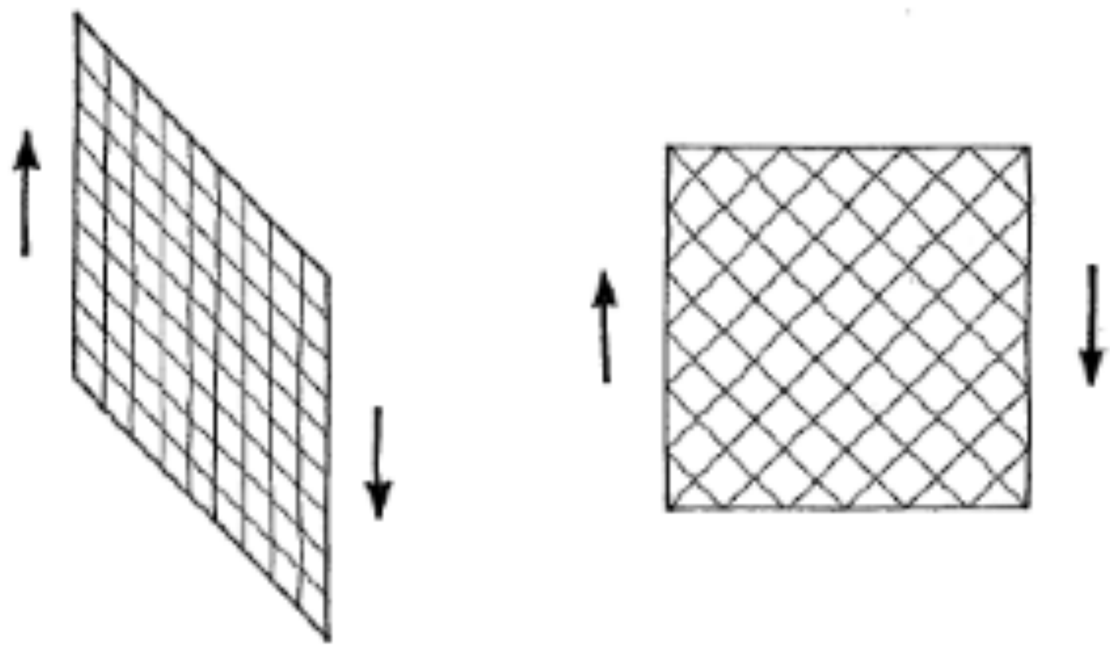
Effect of shear in the structure

- Shear will cause tension and compression stress in directions at 45^0 to the direction of shear force.

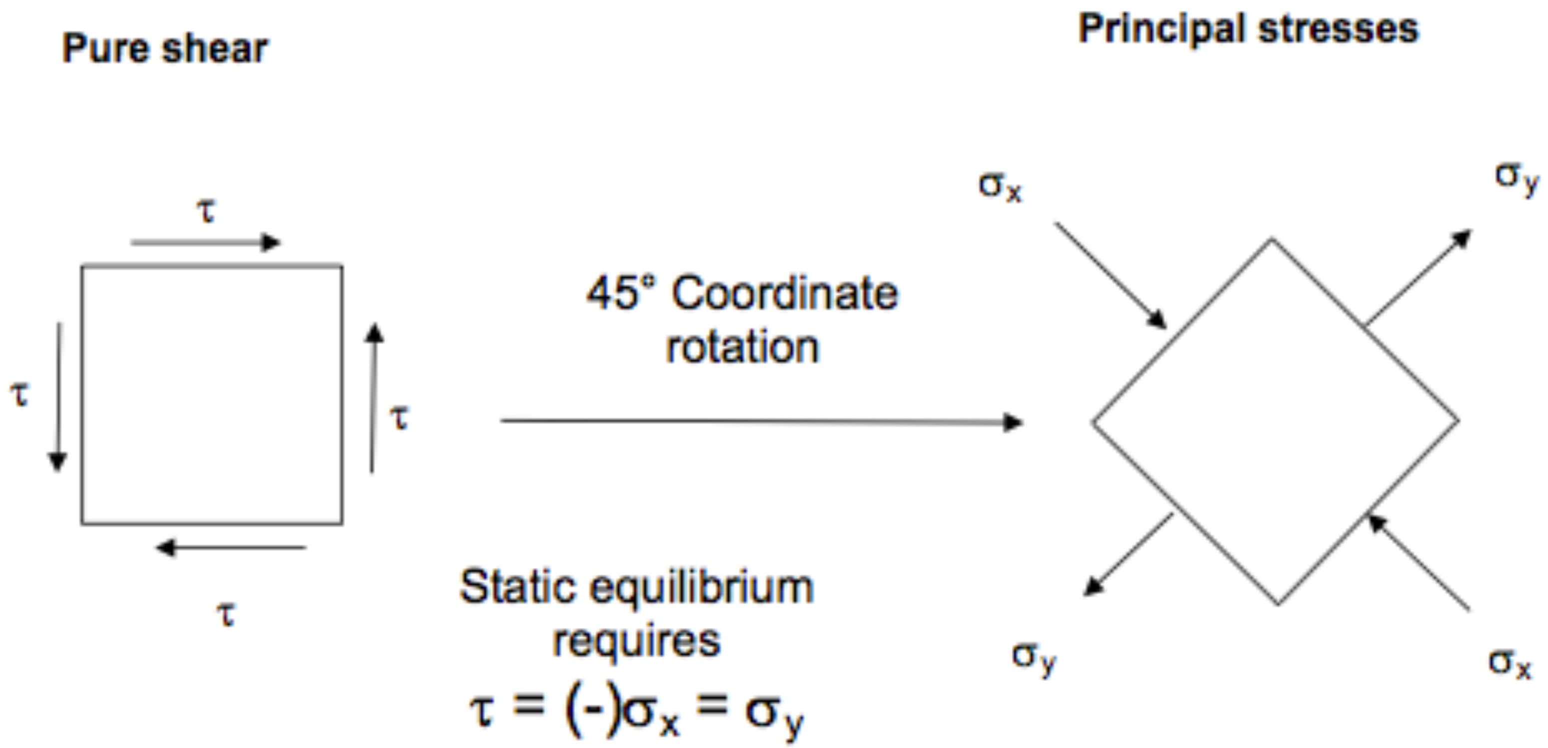


Structures under shear

- Which structure is more rigid under the shear shown in the picture?

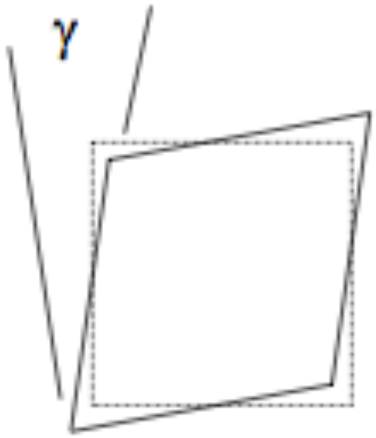


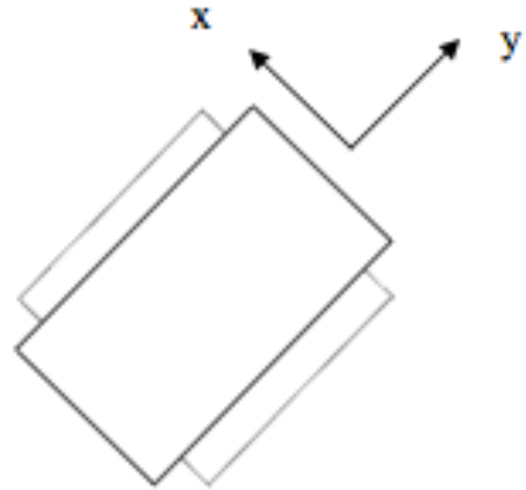
Shear and normal stress



- At every point in a stressed body there are at least three planes, called *principal planes, with normal vectors , called principal directions,* ***where the corresponding stress vector is perpendicular to the plane****, i.e., parallel or in the same direction as the* ***normal vector*** *, and where there are no normal shear stresses .*
- *The three stresses normal to these principal planes are called principal stresses.*
- *Shear stress along the principal axis is zero*

Transformation of normal and shear stress





Shear strain $\gamma = \frac{\tau}{G}$ Shear stress

Normal strains

$$\varepsilon_y = \frac{dy}{dl} = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E}$$

$$\varepsilon_x = \frac{dx}{dl} = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$$

$$(\sigma_x, \varepsilon_x < 0)$$

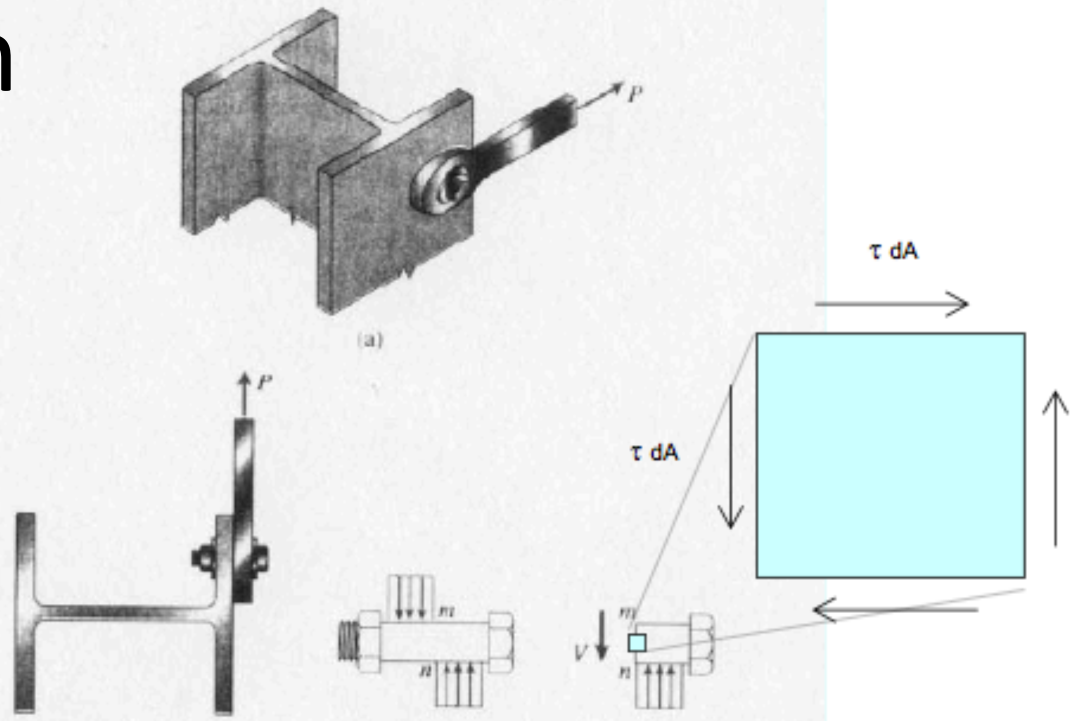
Normal stresses

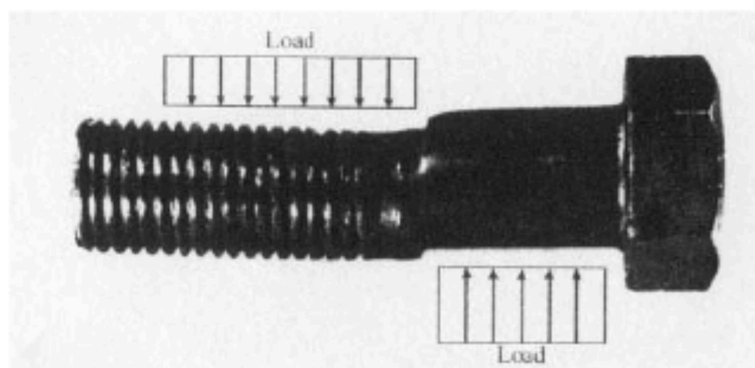
Equivalent, using $G = \frac{E}{2(1+\nu)}$

For more general cases, use **Mohr's circle to transform coordinates and see transformation of normal and shear stress**

Shear strength

- **Shear strength** is a term used to describe the strength of a material or component against the type of yield or structural failure where the material or component fails in shear.
- A shear load is a force that tends to produce a sliding failure on a material along a plane that is parallel to the direction of the force.
- Here the screw may fail in shear.





Bulk modulus

Bulk modulus defines **compressibility** of the material

For an element under uniform hydrostatic pressure P (in all directions)

Change in volume per unit volume $= \dfrac{\Delta V}{V}$

We can show for small changes

$$\Delta \mathrm{V} = \frac{\Delta \mathrm{x}}{x} + \frac{\Delta \mathrm{y}}{y} + \frac{\Delta \mathrm{z}}{z} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Bulk modulus $\mathrm{E_B}$ is a material property, defined as

$$K = E_B \equiv \frac{-P}{\left(\dfrac{\Delta V}{V}\right)}$$ Where P is the pressure

For isotropic materials: $E_B \equiv \dfrac{E}{3(1-2v)}$

Case study: soft rubber between two stiff plates

Constrained layer:

$\Delta x \cong 0, \Delta y \cong 0, \Delta A \cong 0$

$$\Delta V \cong A\Delta t = At\frac{\Delta t}{t} = V\frac{\Delta t}{t} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta t}{t}$$

by definition of E_B

$$P = -E_B \frac{\Delta V}{V} = \frac{F}{A} \Rightarrow -E_B\frac{\Delta t}{t} = \frac{F}{A}$$

$$\Delta t \cong \frac{F\,t}{E_B A}$$

$$E_B = \frac{E}{3(1-2\nu)}$$

for soft rubber, $\nu \sim 0.5$, E_B blows up

For RTV rubber, E ~ 300 psi and EB~100,000 psi

The constraint makes the rubber seem 300 times stiffer!