

Lens design & Fourier optics analysis (under construction)

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PHYS 258 Fourier Optics

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SJSU

Some of the important topics needed to be addressed in a successful lens design project (R.R. Shannon: The Art and Science of Optical Design)

- Focal length (f)
- Field angle or field size
- F/number
- Numerical aperture (NA)
- Wavelength and spectral range
- Magnification and focusing range
- Zoom ranges
- Type of lenses
- Back focus
- Front focus
- Pupil locations
- Illumination
- Irradiance uniformity
 - Vignetting
 - Transmission
- Ghost images
- Distortion
- Variation with conjugates
- Variation with spectral region
- Interference with optical path
- Image quality
 - Aberrations
 - Resolution
 - Optical Transfer Function (OTF)
 - Modulation transfer function (MTF)
 - Energy concentration (Intensity pattern)*
 - Effect of aperture stop
 - at various apertures
- Scattered light
- Polarization
- Veiling glare
- Light baffling
- Off-axis rejection
- Field stop definition
- Diffraction effects
- Tolerances
- Depth of focus
- Interface with variable aperture
- Interface with autofocus system
- Size and configuration
- Zoom Mechanization
- Focus mechanization
- Folding components
- Schedule and delivery time
- Optical interfacing with instruments
- Cost of
 - Design
 - prototype
 - Production
- Materials
 - Availability
 - Cost
 - Continued supply
 - Suitability for processing
 - Compatibility with operation conditions
 - Environmental considerations
 - Hazardous material
- Environment
 - Temperature range
 - Storage conditions
 - Atmospheric pressure
 - Humidity
 - Vibration and shock
- Availability of subcontractors
- Level of technology
- Weight
- Moment about mounting
- Coatings
 - Transmission
 - Reflectivity
 - Absorption
 - Availability
 - Risk
 - Environmental effects
 - Mechanical and optical quality
- Manufacturability
- Producibility
- Manufacturing processes
- Manufacturability
- Producibility
- Manufacturing processes
- Mounting processes
- Mounting interfaces
- Mechanical interfaces with instrument
- Detector
 - Photographic
 - Sampling array
 - Signal to noise
- Surface finish, cosmetics
- Beam parameters
- Radiation damage
- Irradiance damage
- Prior experience
- Track record
- Prior art
- Patentability
- Patent conflict situation
- Competitive situation
- Marketability
- Interface to other producers
- Lifetime of product
- Rate of production
- Liability issues
- Delay to market
- Timing of disclosure
- Integration with other products
- Customer view of product
- Styling
- Investment requirements and risks
- Funding and financial viability

Lens design parameters

- Field (or object) size
- Axial aperture
 - Controls brightness of the image
 - Vignetting: deliberate reduction of irradiance by off axis by proper selection of element diameters. It is a tool to control aberrations
- Image size
- Design concerns
 - Image quality requirements
 - Mechanical layout
 - Material selection
 - Tolerances
 - **Definition of starting point**
- **Passage of rays through the system is studied with geometrical ray tracing**
- **Image formation through combination (interference) of rays and bundles is studied with physical optics or diffraction optics.**

Basic design steps

Data supplied by customer

Evaluation of parameters by designer for selection of realistic and economic requirements

Initial selection of parameters by designer

Select first order optical specifications to establish paraxial base set of coordinates in which the image is evaluated

Mechanical and fabrication requirements

Select tolerances

- 1) Requirements on construction parameters
- 2) The need to use the lens in a defined environment
- 3) Acceptable irregularities on the lens surface to control absorption and scattered light

Designer perturbs the system according to these tolerances and makes sure the system still meets the specifications

Cost and schedule for delivery

1. Focal length
2. Weight,
3. Spectral range,
4. Image quality
5. Number of elements
6. Available space
7. cost

Detailed description of a lens

- Sequentially numbered set of spherical surfaces
 - Curvature
 - Thickness to the next surface
 - Index of refraction of the medium after the surface
 - Surface shape
 - Orientation
 - Dimension
- Operating condition

Merit functions used in design evaluation

Evaluation of a lens is done through sampling state of aberration of the lens by computing light distribution across the lens including diffraction effect

1. Ray intercept plots
2. Spot diagrams
3. Point Spread Function (PSF)
4. Optical transfer function (OTF)
5. Modulation transfer function (MTF)

Ray Intercept Diagrams

- A geometrical optics calculation

Spot Diagrams / Ray Scatter Diagrams

- A geometrical optics calculation
- They show symmetry of aberration

Point Spread Function (PSF)

- A physical optics calculation
- Light intensity distribution in the diffraction image if the lens were made perfectly (diffraction-limited).
- It is simply image of a point object.

Optical Transfer Function (OTF)

A physical optics calculation

- A functional representation of the spatial frequency response of the lens.
- OTF is a Fourier transform of the PSF
- Spatial frequency or wavenumber: number of wavelengths in a unit length (not time)
- OTF describes the contrast between images of different sinusoidal elements with specified spatial frequency
- Image analysis with OTF is equivalent to Frequency band analysis in communication

Modulation Transfer Function (MTF)

A physical optics calculation

- MTF which is modulus of the OTF is a more interesting function to the lens users.
- Shows the contrast between the different frequency components.

Designing a simple low-cost fixed-focus digital camera lens

- A simple objective lens for a fixed-focus digital camera
- Interpret general design specifications.
- Identify starting points based on design specs.
- Match the starting points to the requirements
- Perform basic analysis, compare results with specs.
- Determine guidelines for optimization.
- Optimize the lens
- Identify problems for potential refinements.

Fixed-focus VGA digital camera objective specs (Ref: CODE V user guide)

- Number of elements: 1-3
- Material: common glasses or plastics
- Image sensor: Agilent FDCCS-2020
 - Resolution: 640x480
 - Pixel size: 7.4x7.4 microns
 - Sensitive area: 3.55x4.74 mm (full diagonal 6 mm)
- Objective lens:
 - Focus: fixed, depth of field 750 mm (2.5 ft) to infinity
 - Focal length: fixed, 6.0 mm
 - Geometric Distortion: <4%
 - f/number: Fixed aperture, f/3.5
 - Sharpness: MTF through focus range (central area is inner 3 mm of CCD)

Low frequency 17 lp/mm	>90% (central)	>85% (outer)
High frequency, 51 lp/mm	>30% (central)	>25% (outer)

- Vignetting: Corner relative illumination > 60%
- Transmission: Lens alone >80% 400-700 nm
- IR filter: 1 mm thick Schott IR638 or Hoya CM500

Interpretation of the design parameters; FOV

The system is small:

Focal length of the system (effective focal length EFL) is $f = 6mm$

Sensor size = half of the diagonal size = $h' = 3mm$.

The sensor size and effective focal length (EFL) will establish the field of view (FOV)

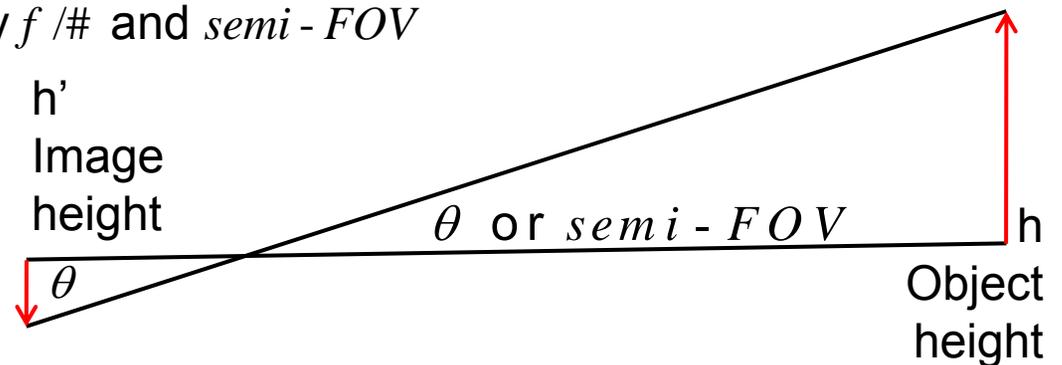
For object at ∞ the image height is: $h = f \tan \theta$

For the image height equal to the sensor size, $h' \approx 3mm$ (here):

$$h' = EFL \times \tan(\text{semi-FOV}); \quad EFL = 6mm$$

$$\text{semi-FOV} = \tan^{-1}\left(\frac{h'}{EFL}\right) = \tan^{-1}\left(\frac{3mm}{6mm}\right) = 26.57^\circ$$

Most patent databases are listed by $f/\#$ and semi-FOV



Interpretation of the design parameters; sharpness

Sharpness or MTF quantifies the systems ability on imaging as a function of spatial frequency.

$$MTF(f_x, f_y) = |OTF| = |H(f_x, f_y)|.$$

Maximum limits: $0.0 \leq MTF \leq 1.0$

Spatial frequency is measured in # of lines/mm and defines the level of details in the image.

Low frequency 17 lp/mm	>90% (central)	>85% (outer)
High frequency, 51 lp/mm	>30% (central)	>25% (outer)

Interpretation of the design parameters II

The sensor is a CCD array of fixed-size cells called pixels.

There are three colour pixels in each cell. For simplicity we assume each cell consists of one pixel.

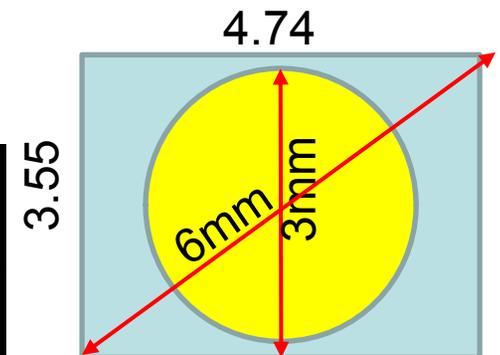
$$\text{Pixel size} = x_{pxl} \times y_{pxl} = 7.4 \times 7.4 \mu^2$$

$$\text{CCD cut off frequency: } (f_x)_{\max} = (f_y)_{\max} = \frac{1}{2} \frac{1}{x_{pxl}} = \frac{1}{2} \frac{1}{7.4 \times 10^{-3}} = 67.6 \text{ lines / mm}$$

Any image with spatial frequency components higher than 67 lp/mm will not be seen with our *CCD* array.

The optics should provide details beyond the cut-off frequency of the *CCD* so that the combined optics/detector *MTF* will produce usable contrast up to the *CCD*'s cut off frequency listed in the specs.

Low frequency 17 lp/mm	>90% (central)	>85% (outer)
High frequency, 51 lp/mm	>30% (central)	>25% (outer)



The design starting point

- Start your software
- File>new>
- New lens wizard (if exists)>Patent lens>filter
 - Can select from expired patent database of the software. Code V offers 2456 of them
 - You can also access patent search from tools>patent lens search
- Select the filter parameters according to the needs +/- a range so you don't miss the design options. You can always optimize for the perfect match later. $F=6mm$: a fast (small $f/\#$) lens and a wide FOV: 26.5° semi-FOV, cheap so small number of elements (1-3).
- Always choose the lens with a larger FOV than needed because it is hard to expand FOV
- Choose the lens with smallest F# among the ones offer the needed FOV since stepping down a lens to a larger F# improves the image quality.
- The next is entering the system data (information about the lens usage)
 - Image $f/\#$
 - Wavelength of simulations (you can increase weight of any wavelength to increase sensitivity for that wavelength. Make weight for the green 2.
 - Reference wavelength used for paraxial and reference ray-tracing (default is ok)
 - Fields lists the simulated fields angle. Usually minimum three is required 0, 0.7 and full field. For wide angle lenses more than three is good. We choose 4 at 0,11,19,26.5⁰
- The lens data will appear after clicking next and done

GUI of a typical lens design program (Code V)

CODE V - 258_digital camera FN3.5 CCD6

File Edit Lens Display Review Analysis Optimization Tools Window Help

* 1 - JAPAN PATENT 50 ; Z

Lens Data Manager

Surface #	Surface Name	Surface Type	Y Radius	Thickness	Glass	Refract Mode	Y Semi-Aperture
Object		Sphere	Infinity	Infinity		Refract	0
1		Sphere	0.35606	0.11000	786500.50	Refract	0.19495
2		Sphere	0.70116	0.07000		Refract	0.15575
3		Sphere	-0.65975	0.02000	717360.29	Refract	0.11822
Stop		Sphere	0.41684	0.03500		Refract	0.10884
5		Sphere	0.92080	0.06500	834810.42	Refract	0.12426
6		Sphere	-0.54079	0.77426 ^S		Refract	0.13413
Image		Sphere	Infinity	-0.00403		Refract	0.47189

End Of Data

Command Window

```

OBJECT SURFACE:
RDY 0.000000 ; THI 0.1000E+14 ; GLA AIR
CODE V> S 0 0
SURFACE
1:
RDY 0.000000 ; THI 0.000000 ; GLA AIR
CODE V> GO
CODE V> GO
CODE V>
    
```

CODE V

DIM: Millimeter; Apertures Used: User-Defined and Default; Use ZX Plane: No; Polarization Ray Tracing Active: No

Spring 2010 CODE V - 258... Microsoft Pow... PHYS 258 Eradat SJSU 99% 6:29 PM 18

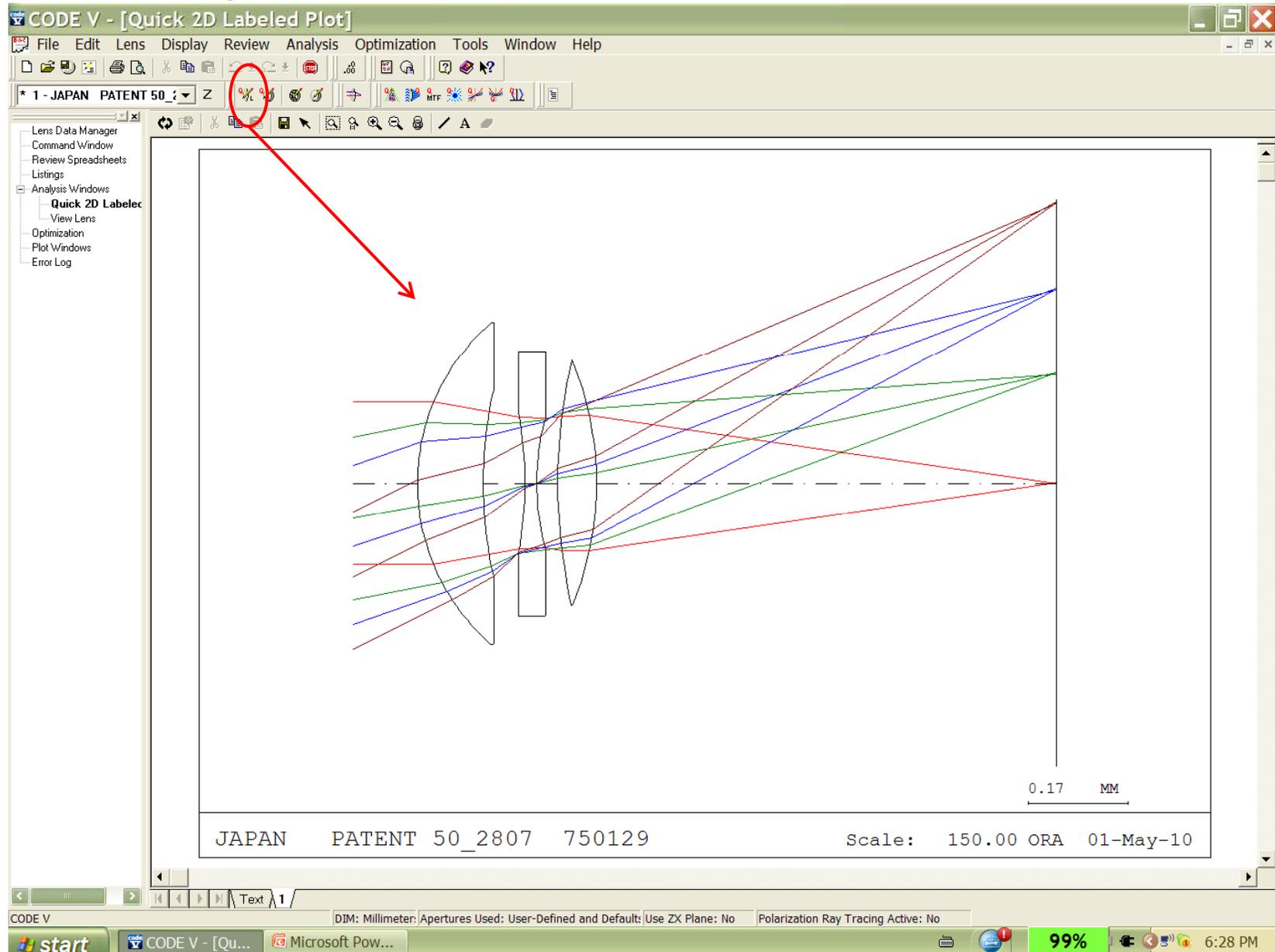
Lens Data manager

- The basic operation of the lens design software is ray-tracing. Everything else is a derivative of this operation.
- Ray tracing is done sequentially from surface to surface.
- The optical system is defined through a series of surfaces.
- The surface and system data are listed in the lens data manager (LDM) spreadsheet.
- The gray cells are result of calculations.
- All the data and operation related to a cell can be seen by right clicking on it.
- Format of the cells can be changed by tools>customize>format cells

Lens Data manager: Surface data

- All the systems will start with object surface and end with image surface.
- Stop: is the aperture surface
- The chief ray (principal ray) from every point passes through center ($x=0$ & $y=0$) of the stop surface.
- Surface number,
- surface name,
- surface type,
- Y radius (1/mm) or radius of curvature or reciprocal of curvature, for non-spherically symmetric surfaces we have X- & Y-curvatures,
- Thickness: distance to the next surface
- For the last surface (one before the image surface) the thickness is calculated by location of the paraxial image and is calculated by the paraxial image (PIM) solver. Location of the best focus is given from the PIM. Optimization determines the best focus
- Glass: material in the space following the surface (blank for air). You can use real glass from catalog or fictitious glass with variable index for optimization. Then convert the results to a real glass that one can buy. (glassfit.seg macro in Code V does that)
- Refract/reflect determines the basic property of the surface.
- Y Semi-Aperture represents the size of the optically useful portion of the lens. Usually it is a circular aperture centered on the optical axis and calculated by the system but possible to change it.
- It is possible to change the data by right click on the LDM or command prompt if one knows the commands.

Viewing a 2D picture of the lenses



Surface operations: Scale the lens

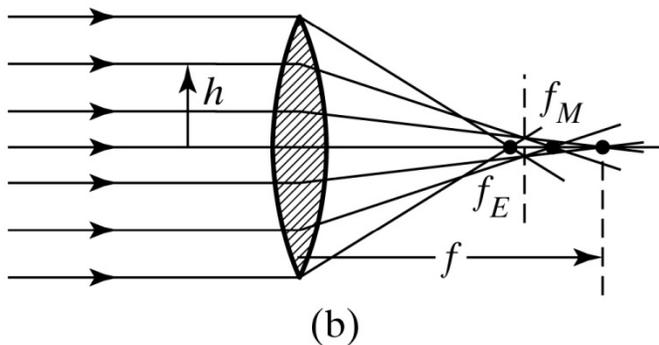
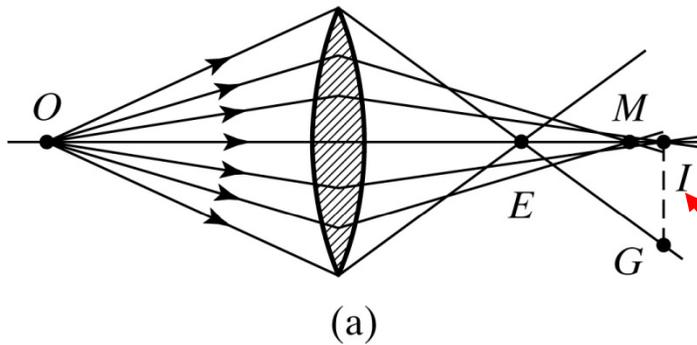
- We selected the $f\#$ and FOV but we need to make sure the lens has the desired physical dimensions EFL=6mm. This can be done through “window of the first order properties”:
 - Display>List Lens Data>First Order Data
- If the data is different than desired we can scale the lens to bring them to what we want.
- Highlight the surface number on the LDM and choose Edit>Scale and select scale the EFL and insert value 6.0
- Click the execute button on the “List first order data” and “Quick 2D plot” windows to refresh them.
- Now both EFL=6mm and image height=2.99 as desired
- Use the Lens>System data>System settings to select title and working condition and other system parameters.

Analyze the starting point

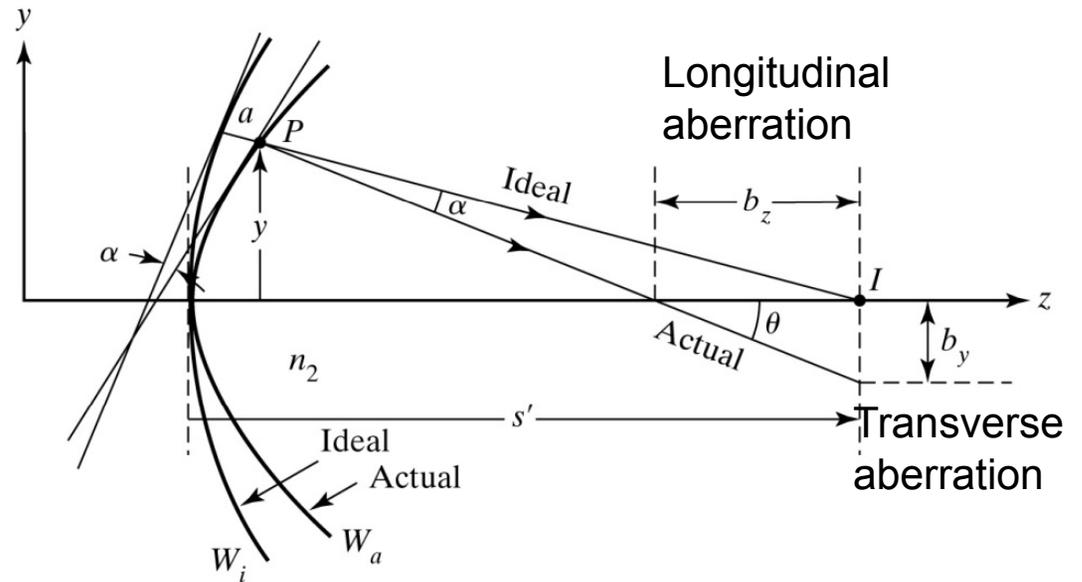
- Original check to see if we meet the specs
- First order requirements (focal length and Image height)
- Distortion (field curves and distortion grid)
- Sharpness (diffraction MTF)
- Depth of focus (MTF at different object distances)
- Vignetting / illumination (transmission analysis)
- Ray aberration curves
- Spot diagrams
- Establish feasibility (usually we use the existing experience to check):
 - Assume someone has to manufacture this lens or system, what practical issues will they face?
 - Size of the elements (too small /too big / too thin / too thick)
 - Ease of assembling and mounting
 - Price and availability of the required material (glass, etc.)
 - Tolerance analysis
 - Thermal analysis (usually system design software are good at this)

Aberrations: ray intercept curve

Ray intercept curve:
Plotted as a function of position on the stop (pupil) for a line of evenly spaced rays.



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distance from a particular ray to the chief ray for the same field point on the image surface

Off-axis object point

Consider off-axis pencil of rays from point P .

The aberration function for the point Q on the wavefront:

$$a'(Q) = (PQP' - PBP')_{opd} = c(BQ)^4 = c\rho'^4$$

The aberration function for the point O on the wavefront:

$$a'(O) = (POP' - PBP')_{opd} = c(BO)^4 = cb^4$$

The off-axis aberration function: $a(Q) = a'(Q) - a'(O) = c\rho'^4 - cb^4 = c(\rho'^4 - b^4)$

In $\triangle BOQ \rightarrow \rho'^2 = r^2 + b^2 + 2rb \cos \theta$

In $\triangle OBC$ and $SCP' \rightarrow OB = b \propto h' \rightarrow b = kh'$

Replace ρ'^2 and b in $a(Q)$ and regroup all the terms

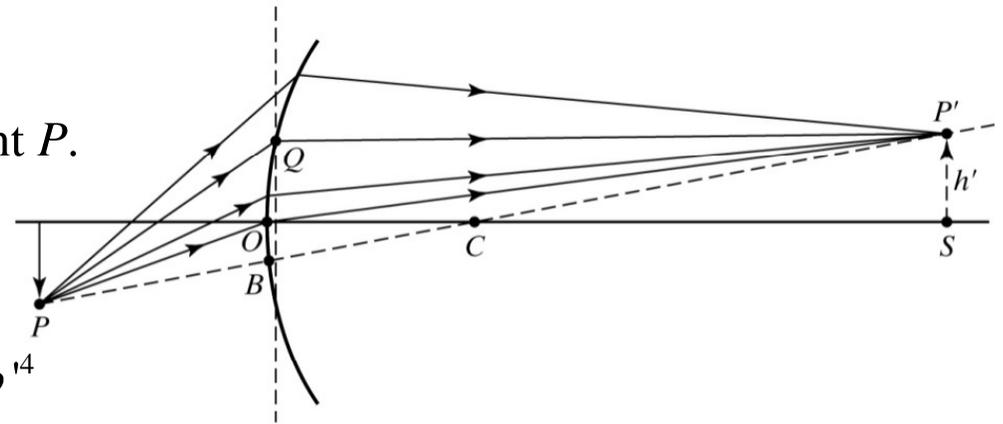
$$a(Q) = {}_0C_{40}r^4 + {}_1C_{31}h'r^3 \cos \theta + {}_2C_{22}h'^2 r^2 \cos^2 \theta + {}_2C_{20}h'^2 r^2 + {}_3C_{11}h'^3 r \cos \theta$$

The ${}_iC_{jk}$ coefficients have indices that are powers of the terms

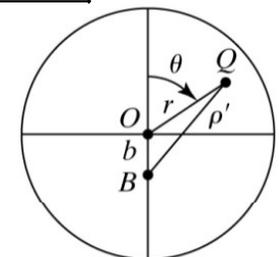
h' : departure from axial image, r : aperture of the refracting surface,

$\cos \theta$: azimuthal angle on the aperture.

i is power of h' , j is power of r , k is power of $\cos \theta$.



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Detail

Spherical aberration

Off-axis aberration function

$$a(Q) = {}_0C_{40}r^4 + {}_1C_{31}h'r^3 \cos \theta + {}_2C_{22}h'^2 r^2 \cos^2 \theta + {}_2C_{20}h'^2 r^2 + {}_3C_{11}h'^3 r \cos \theta$$

Each term comprises one kind of monochromatic aberration or Seidel aberration as follows:

${}_0C_{40}r^4$ ← Spherical aberration (r is the system aperture).

$h'r^3 \cos \theta$ ← coma

$h'^2 r^2 \cos^2 \theta$ ← astigmatism

$h'^2 r^2$ ← curvature of field

$h'^3 r \cos \theta$ ← distortion

The only term independent of the h' (departur from axial imaging) so it exists even for paraxial and axial points.

The rays refracted from the extremities of the lens generate two types of spherical aberrations:

$$b_y = \frac{s'}{n_2} \frac{da}{dy} = \frac{s'}{n_2} \frac{da}{dr}$$

$$a(Q) = {}_0C_{40}r^4 \rightarrow \frac{da}{dr} = 4{}_0C_{40}r^3$$

$$\left. \begin{array}{l} b_y = \frac{s'}{n_2} \frac{da}{dr} \\ \frac{da}{dr} = 4{}_0C_{40}r^3 \end{array} \right\} b_y = \frac{4{}_0C_{40}s'}{n_2} r^3$$

$$b_z = \frac{s'b_y}{y} = \frac{s'b_y}{r} \rightarrow b_z = \frac{4{}_0C_{40}s'^2}{n_2} r^2$$

Coma (resembles comet)

$$a(Q) = {}_0C_{40}r^4 + \underbrace{{}_1C_{31}h'r^3\cos\theta}_{\text{Coma, an off-axial aberration}} + {}_2C_{22}h'^2r^2\cos^2\theta + {}_2C_{20}h'^2r^2 + {}_3C_{11}h'^3r\cos\theta$$

Off-axis aberration

Coma, an off-axial aberration

$h' \neq 0$, and is not symmetrical about the optical axis or $\cos\theta \neq \text{constant}$

Coma rapidly increases with system aperture (r^3).

Zone: a thin annular region of a lens centered at optical axis.

Comatic circle: is created by all the rays arriving from a distant object and passing through a zone.

Radius of the comatic circles increase with radius of the generating zone (figure a).

Figure b: formation of different comatic circles.

Each zone produces a different magnification.

h_e : magnification due to extreme rays.

h_c : magnification due to central rays.

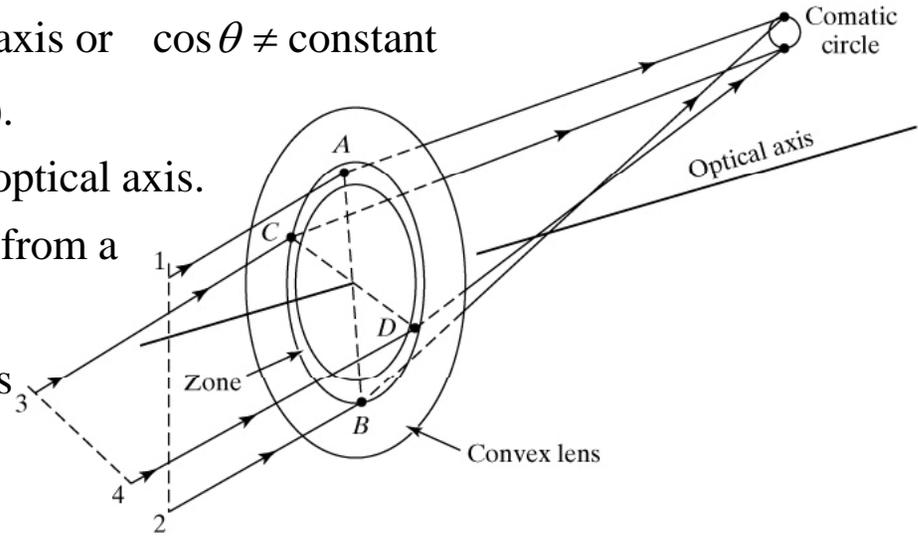
Coma may occur in two forms:

a positive quantity ($h_e > h_c$)

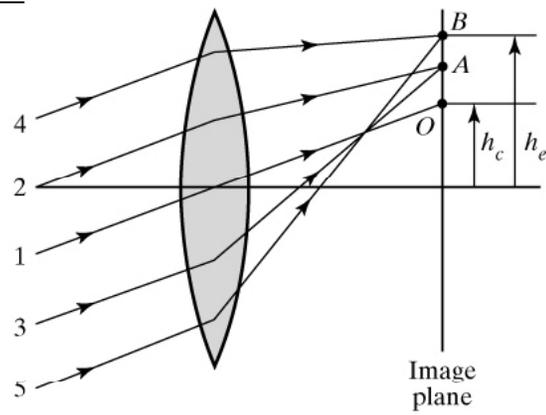
a negative quantity ($h_e < h_c$)

Maximum extent of a comatic image: $3R_e$

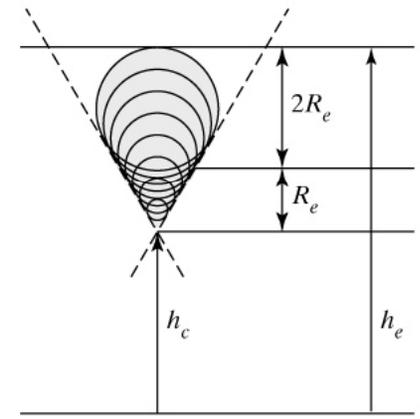
R_e is the radius of the extreme comatic circle.



(a)



(b)



(c)

Minimizing coma

Law of sines in $\triangle PCM$: $\frac{\sin \theta}{r} = \frac{\sin(\pi - \phi)}{PC} = \frac{\sin \phi}{PC}$

Law of sines in $\triangle P'CM$: $\frac{\sin \theta'}{r} = \frac{\sin \phi'}{P'C}$ $n \sin \phi = n' \sin \phi'$

$\frac{\sin \theta'}{r} = \frac{\sin \phi'}{P'C}$ using Snell's law $\rightarrow \frac{\sin \theta'}{r} = \frac{n \sin \phi}{n' P'C}$
 $n \sin \phi = n' \sin \phi'$

$$\left. \begin{aligned} \frac{\sin \theta'}{r} &= \frac{n}{n' P'C} \frac{PC \sin \theta}{r} \\ -\frac{h}{h'} &= \frac{PC}{P'C} \end{aligned} \right\} \frac{\sin \theta'}{1} = -\frac{nh \sin \theta}{n' h'}$$

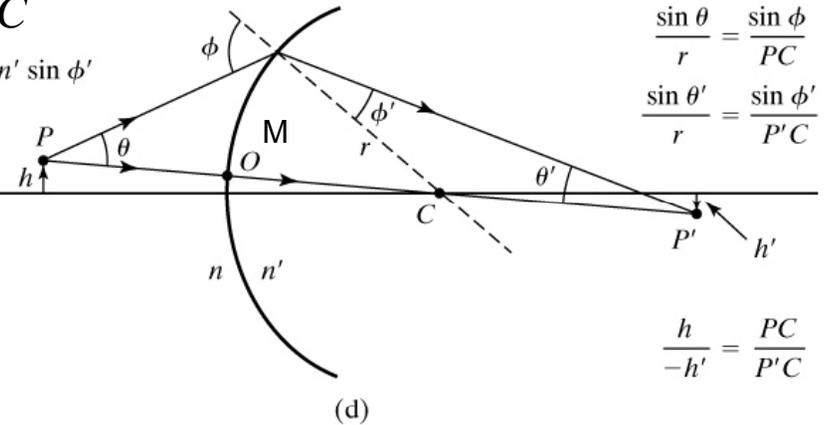
$nh \sin \theta + n' h' \sin \theta' = 0$ *Abbe's sine condition*

We can rewrite the condition: $m = \frac{h'}{h} = -\frac{n \sin \theta}{n' \sin \theta'}$

For small objects near axis, any ray refracted at a spherical surface must satisfy the *Abbe* sine condition.

To prevent coma all magnifications must be independent of θ and that is only possible if:

$\frac{\sin \theta}{\sin \theta'} = \text{constant}$



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Example for minimizing coma

Abbe's sine condition:
$$m = \frac{h'}{h} = -\frac{n \sin \theta}{n' \sin \theta'}$$

The proper Coddington shape factor for absence of coma:
$$\sigma = \left(\frac{2n^2 - n - 1}{n + 1} \right) \left(\frac{s - s'}{s + s'} \right)$$

Example: $n = 1.50$, object at infinity, we get $\sigma = 0.8$ very close to the value of minimum spherical aberration 0.7.

Thus we can minimize both spherical and coma aberration simultaneously in one design called aplanatic optics.

Astigmatism and curvature of field

Off-axis aberration function:

$$a(Q) = \underbrace{{}_0C_{40}r^4 + {}_1C_{31}h'r^3 \cos\theta}_{\text{Aplanatic optics corrects spherical and coma}} +$$

$$+h'^2 r^2 \left(\underbrace{{}_2C_{22} \cos^2\theta}_{\text{Astigmatism}} + \underbrace{{}_2C_{20}}_{\text{Curvature of field}} \right) + {}_3C_{11}h'^3 r \cos\theta$$

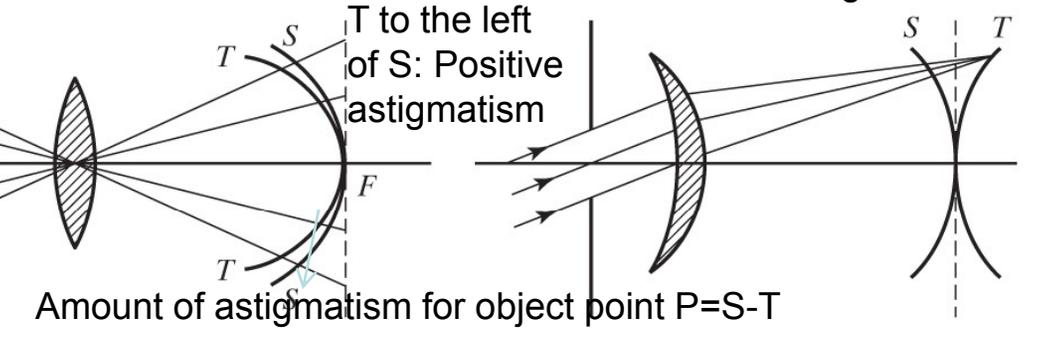
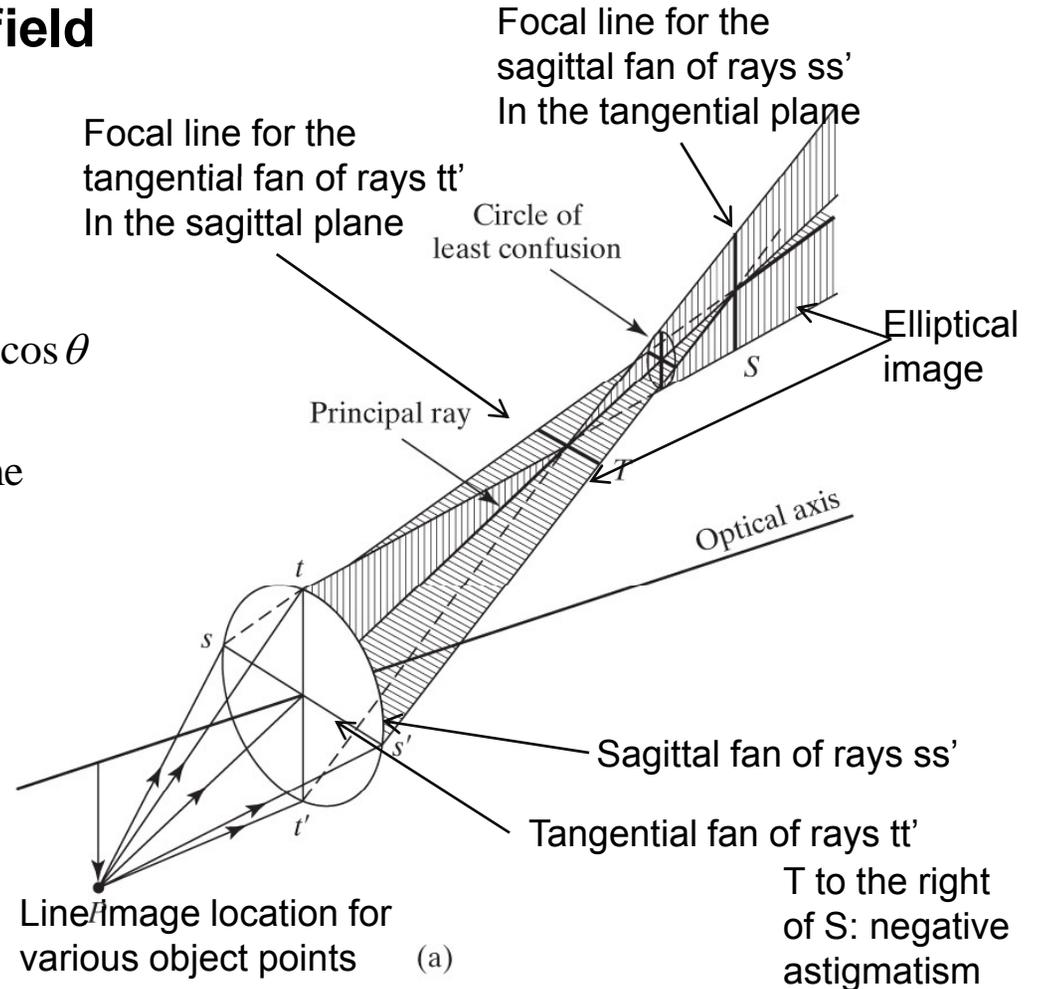
Astigmatism and curvature of field have same dependencies to:

- a) off-axis distance of the object
- b) aperture of the system

If we use combination of lenses so that S and T image planes coincide on a single plane so called **Petzval surface**, we no longer have astigmatism but the image plane is now curved.

This kind of aberration is called **curvature of (image) field**.

The sharp image forms on a curved surface.



Astigmatism and curvature of field

For two thin lenses the Petzval surface is flat if

$$n_1 f_1 + n_2 f_2 = 0$$

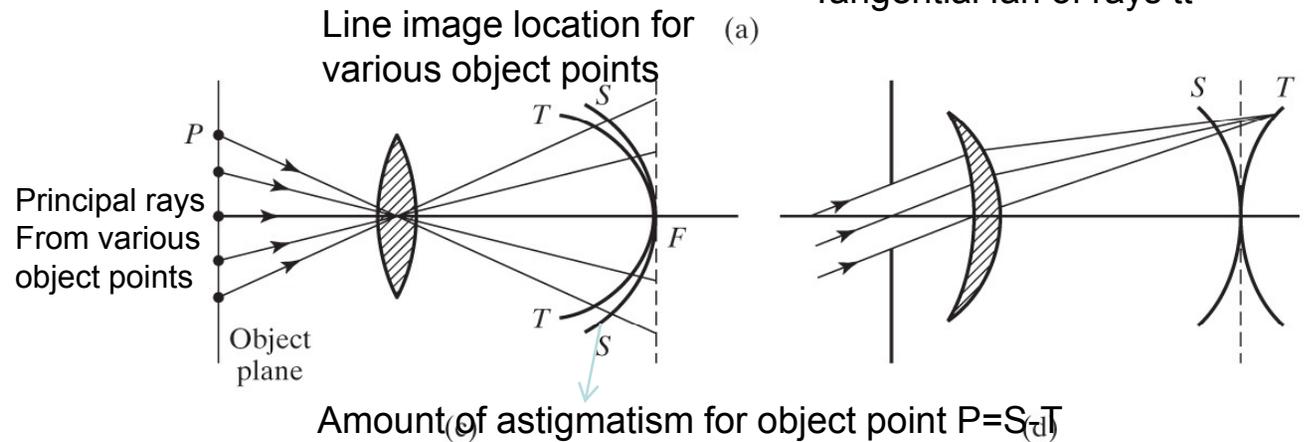
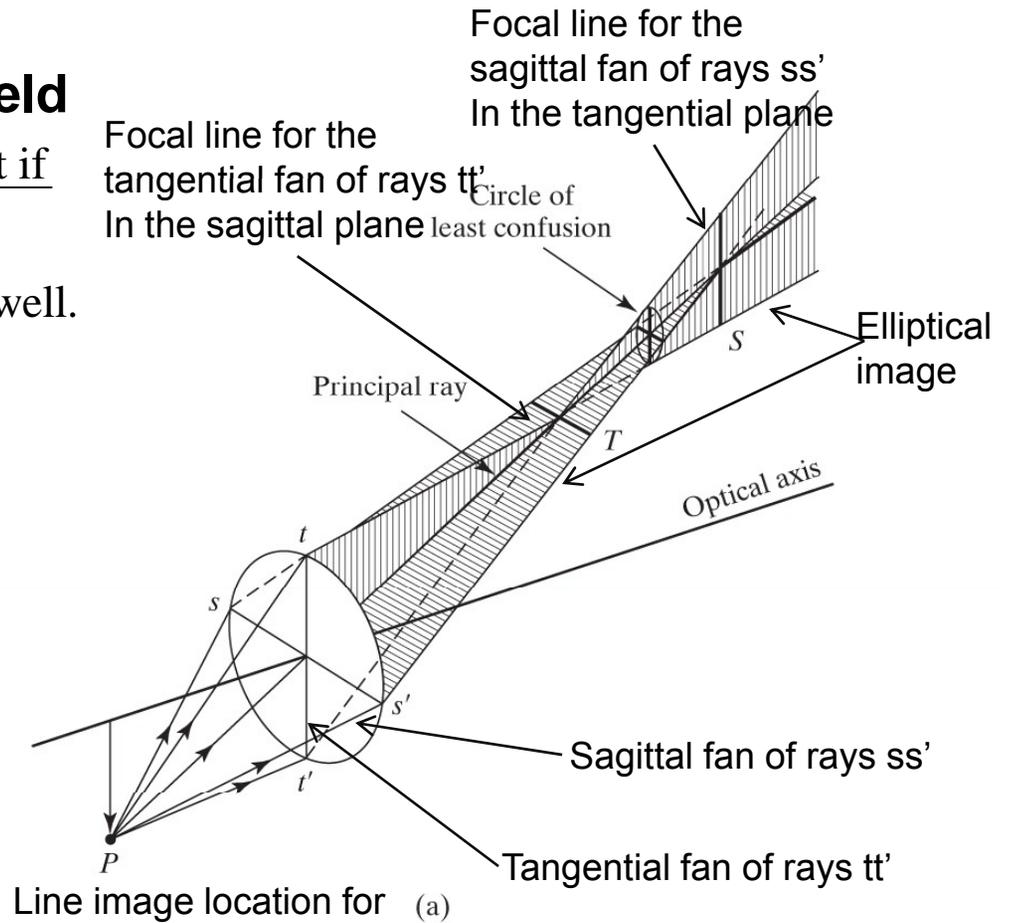
This eliminates the curvature of the field as well.

For k, thin lenses:
$$\sum_{i=1}^k \frac{1}{n_i f_i} = \frac{1}{R_p}$$

where R_p is the radius of Petzval surface.

We can also use apertures to flatten fields like in a simple box camera.

For actual flattening the curvature of field we need 5th order analysis.

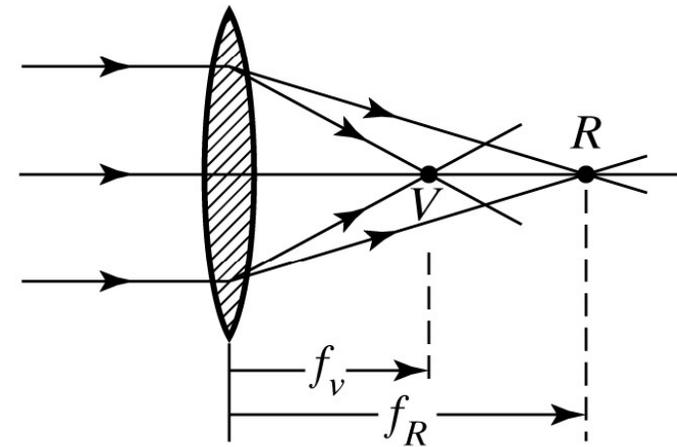


Chromatic aberration

Chromatic aberration is not a Seidel aberration.
It is caused by variation of refractive index with wavelength or dispersion.

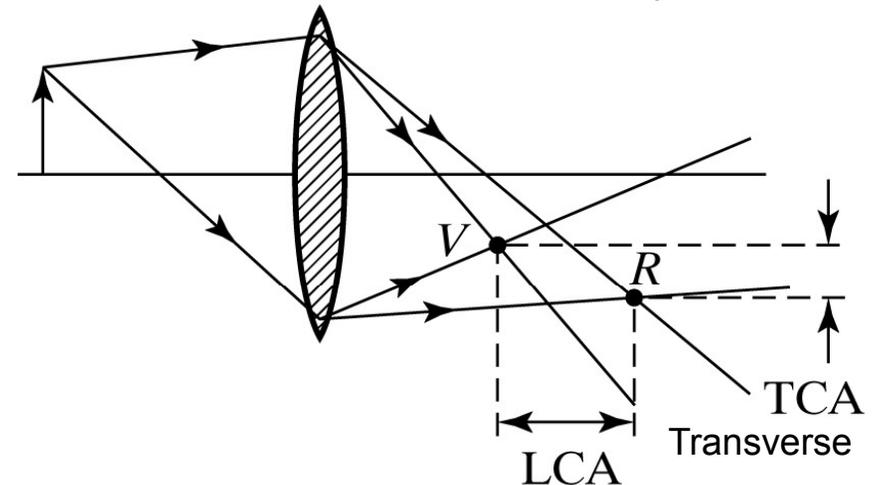
f , focal length of a lens depends on n and
 n depends on wavelength so $f \rightarrow f(\lambda)$

Chromatic aberration for axial object points



(a)

Chromatic aberration for off-axial object points



(b) Longitudinal

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Eliminating chromatic aberration

We can eliminate chromatic aberration by using refractive elements of opposite power.

Goal: finding the proper radii of curvature for an achromatic doublet.

Fraunhofer spectral lines:

$$\lambda_F = 486.1nm \text{ (hydrogen)}; \quad \lambda_D = 587.6nm \text{ (sodium)}; \quad \lambda_C = 656.3nm;$$

Dispersive constant of a glass defined as: $V \equiv \frac{1}{\Delta} = \frac{n_D - 1}{n_F - n_C}$ where Δ is dispersive power.

Assume variations of n with λ is: $\frac{\partial n}{\partial \lambda} \cong \frac{n_F - n_C}{\lambda_F - \lambda_C}$

Power of the two lenses for the sodium yellow line:

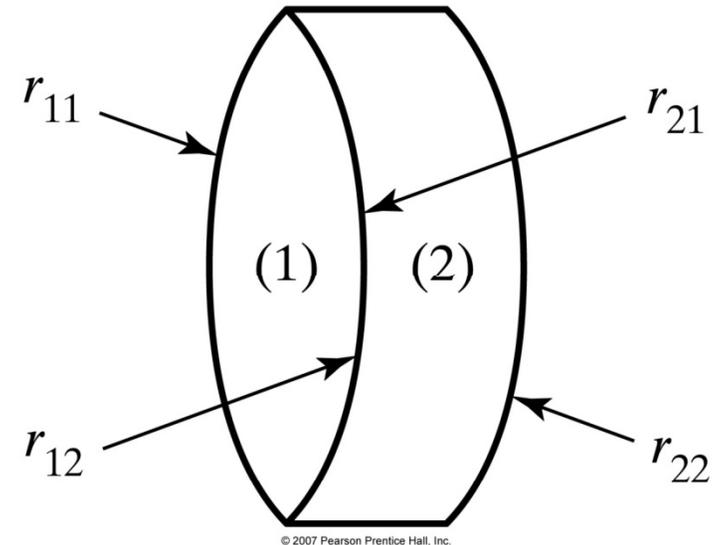
$$P_{1D} = \frac{1}{f_{1D}} = (n_{1D} - 1) \left(\frac{1}{r_{11}} - \frac{1}{r_{12}} \right) = (n_{1D} - 1) K_1$$

$$P_{2D} = \frac{1}{f_{2D}} = (n_{2D} - 1) \left(\frac{1}{r_{21}} - \frac{1}{r_{22}} \right) = (n_{2D} - 1) K_2$$

Total power of two thin lenses with distance L between them:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2} \rightarrow P = P_1 + P_2 - LP_1 P_2$$

Total power of two thin lenses cemented: $P = P_1 + P_2 = (n_1 - 1) K_1 + (n_2 - 1) K_2$



Eliminating chromatic aberration

Total power of two thin lenses cemented: $P = (n_1 - 1)K_1 + (n_2 - 1)K_2$

If the power of the combination is independent of wavelength, λ , to achieve that $(\partial P / \partial \lambda)_D = 0$

$$\frac{\partial P}{\partial \lambda} = K_1 \frac{\partial n_1}{\partial \lambda} + K_2 \frac{\partial n_2}{\partial \lambda} = 0 \quad \text{with} \quad \frac{\partial n}{\partial \lambda} \cong \frac{n_F - n_C}{\lambda_F - \lambda_C}$$

$$K_1 \frac{\partial n_{1D}}{\partial \lambda} = K_1 \left(\frac{n_{1F} - n_{1C}}{\lambda_F - \lambda_C} \right) \left(\frac{n_{1D} - 1}{n_{1D} - 1} \right) = \frac{P_{1D}}{(\lambda_F - \lambda_C)V_1}; \quad K_2 \frac{\partial n_{2D}}{\partial \lambda} = K_2 \left(\frac{n_{2F} - n_{2C}}{\lambda_F - \lambda_C} \right) \left(\frac{n_{2D} - 1}{n_{2D} - 1} \right) = \frac{P_{2D}}{(\lambda_F - \lambda_C)V_2}$$

$$\frac{\partial P}{\partial \lambda} = \frac{P_{1D}}{(\lambda_F - \lambda_C)V_1} + \frac{P_{2D}}{(\lambda_F - \lambda_C)V_2} = 0 \rightarrow V_2 P_{1D} + V_1 P_{2D} = 0$$

The powers of individual elements are:

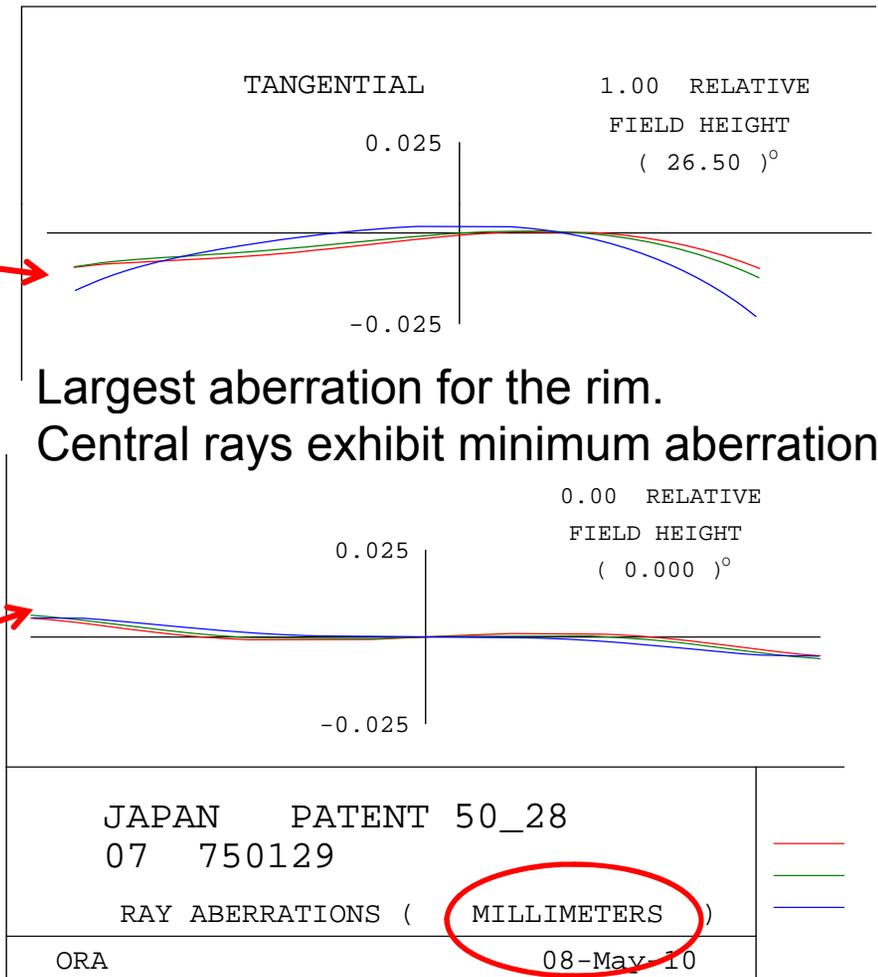
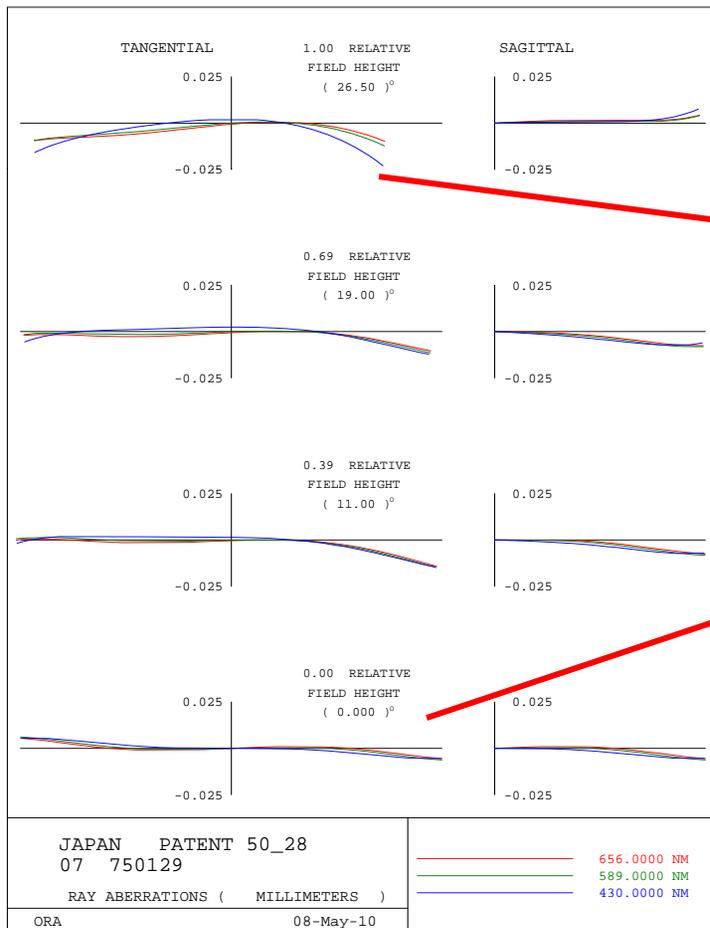
$$\begin{cases} V_2 P_{1D} + V_1 P_{2D} = 0 \\ P = P_{1D} + P_{2D} \end{cases} \rightarrow \begin{cases} P_{1D} = P_D \frac{-V_1}{V_2 - V_1} \\ P_{2D} = P_D \frac{V_2}{V_2 - V_1} \end{cases} \rightarrow \begin{cases} K_1 = \frac{P_{1D}}{n_{1D} - 1} = \left(\frac{1}{r_{11}} - \frac{1}{r_{12}} \right) \\ K_2 = \frac{P_{2D}}{n_{2D} - 1} = \left(\frac{1}{r_{21}} - \frac{1}{r_{22}} \right) \end{cases}$$

For simplicity we choose the crown glass to be equiconvex: $r_{12} = -r_{11}$

The curvature of the cemented surfaces has to match: $r_{21} = r_{12}$ and $r_{22} = \frac{r_{12}}{1 - K_2 r_{12}}$

Ray aberration curve

- Based on ray trace data we look for patterns that may cause problem
 - Transverse ray aberration (zero for perfect lens). Plotted as a function of position on the stop (pupil) for a line of evenly spaced rays.
 - For quick ray aberration plot choose RIM option in: Analysis>Diagnosis>Ray Aberration Curves for "rim rays" only



Ray aberration curve

- Based on ray trace data we look for patterns that may cause problem
 - Transverse ray aberration (zero for perfect lens). Plotted as a function of position on the stop (pupil) for a line of evenly spaced rays.
 - For quick ray aberration plot choose RIM option in: Analysis>Diagnosis>Ray Aberration Curves for "rim rays" only
 - The rim ray is usually the most problematic one so if this is ok we don't expect any problem.
 - What is big and what is small on this graph(scale is in mm)?
 - We compare the aberrations to the size of the airy disc on the diffraction pattern:

$$D_{Airy} = \frac{(2)1.22\lambda}{D_L / f} = 2.44\lambda f \#$$

$$D_{Airy} = 2.44 \times 3.5 \times 500nm = 4270 \times 10^{-9}$$

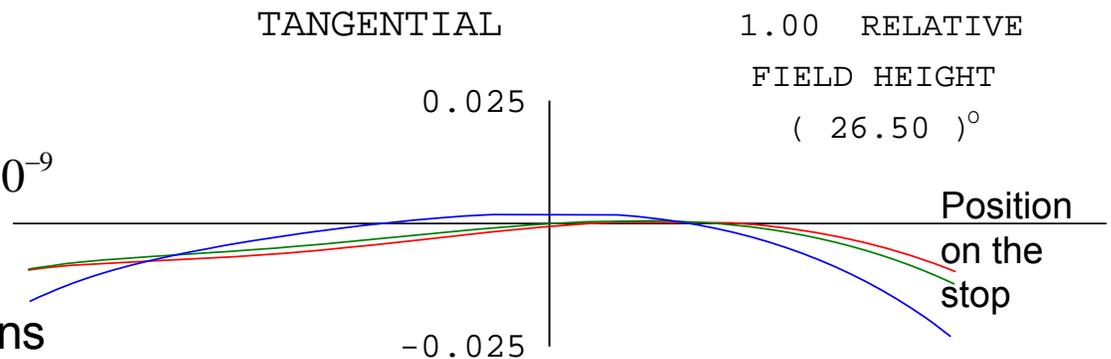
$$D_{Airy} \approx 4\mu$$

The largest aberration for this lens

is 23μ , about $6 \times D_{Airy}$.

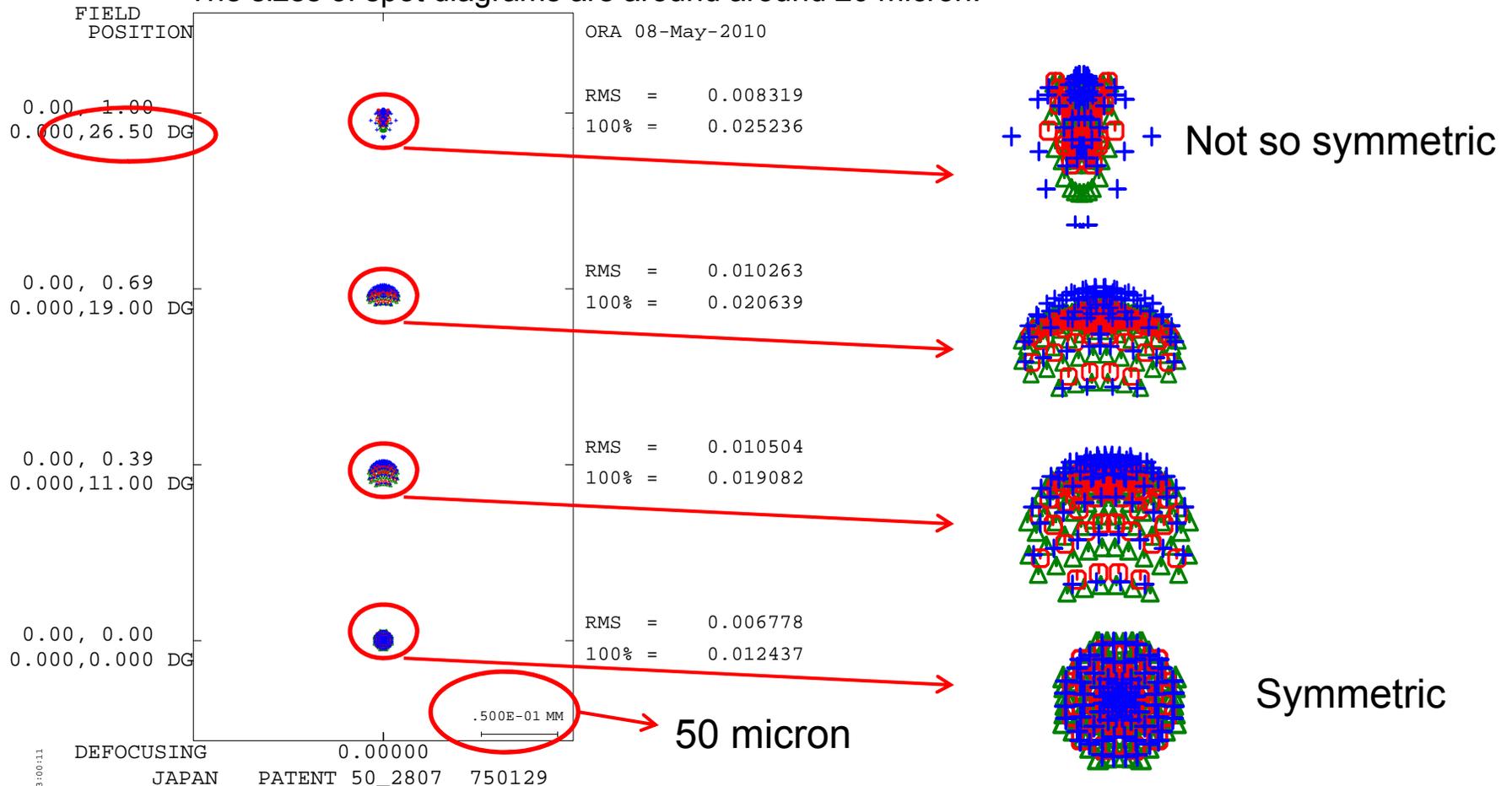
If the aberrations were less or equal to the D_{Airy} then our lens was a diffraction-limited lens but it is not and we don't expect it to be.

Note: for positive-dispersion glasses ($dn/df > 0$) the largest chromatic aberration belongs to the highest frequency (shortest λ) in the spectrum.



Quick spot diagram

- A quick and easy graphical measure of the image quality
 - Many rays from each field point (forming a rectangular grid on the entrance pupil) are traced.
 - A scatter plot of the rays position on the image plane is generated.
 - This is done for each field angle and color coded for wavelength.
 - Analysis>Geometrical>Spot Diagram (note the scale bar is 50 micron).
 - The sizes of spot diagrams are around around 20 micron.

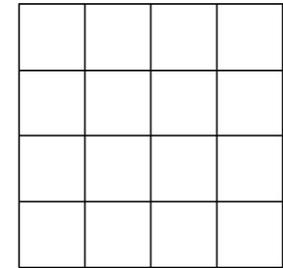


Distortion

$$a(Q) = \underbrace{{}_0C_{40}r^4 + {}_1C_{31}h'r^3 \cos \theta}_{\text{Aplanatic optics corrects spherical and coma}} + h'^2 r^2 \left(\underbrace{{}_2C_{22} \cos^2 \theta}_{\text{Astigmatism}} + \underbrace{{}_2C_{20}}_{\text{Curvature of field}} \right) + \underbrace{{}_3C_{11}h'^3 r \cos \theta}_{\text{Distortion}}$$

Distortion exists even if all the other chromatic & Seidel aberrations (third order) have been eliminated.

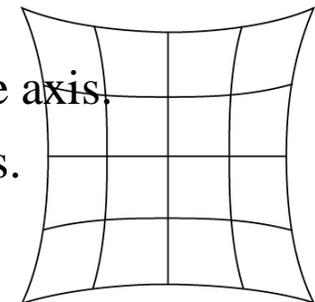
It is caused by variations of the lateral magnifications for the object points at different distance from the optical axis.



(a)

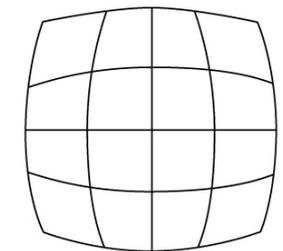
Pincushion distortion: if magnification increases with distance from the axis.

Barrel distortion: if magnification decreases with distance from the axis.



(b)

The image is sharp but distorted. Can be treated by using stops and apertures at appropriate locations between the lens and object or lens and image.



(c)

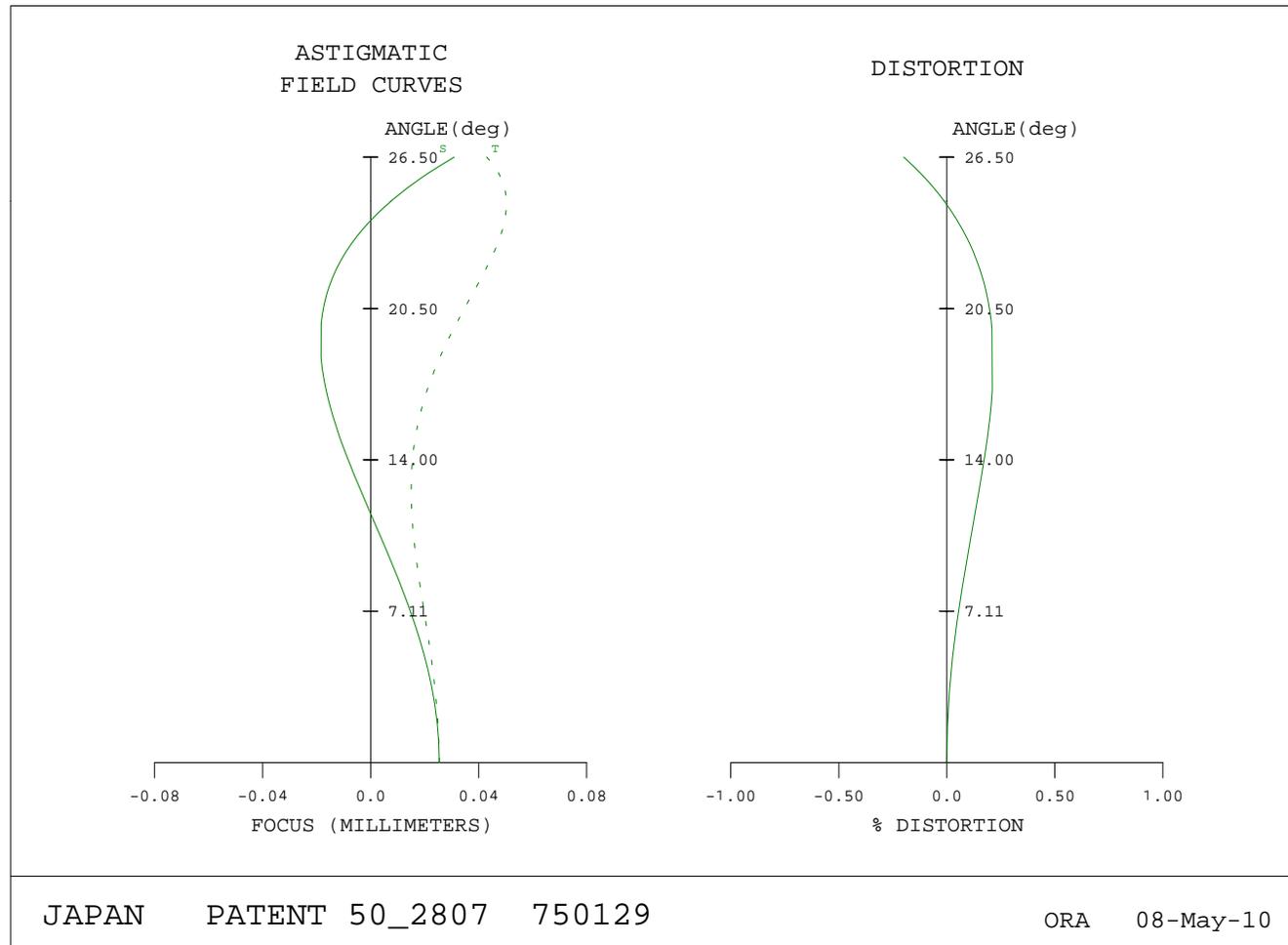
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Distortion

If the real image height differs from the paraxial image height, we have distortion.

Paraxial image height: $h = f \tan \theta$, Real image height: h' , $distortion = \frac{h' - h}{h} \times 100$

Distortion is a field-related aberration. It is usually plotted with another field aberration astigmatism. Use: Analysis>Diagnostics>Field curves or "Quick field plot"



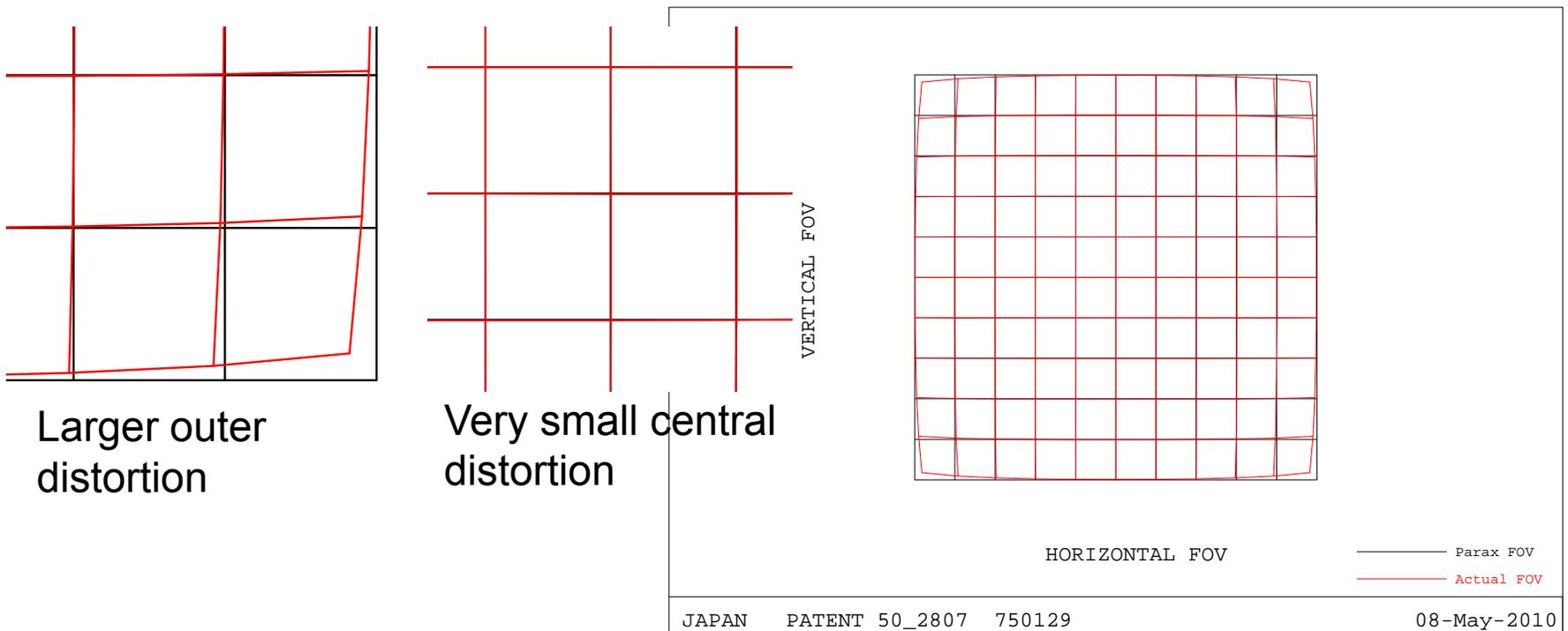
Distortion grid

Distortion grid displays the distortion on a grid that displays the ideal paraxial image superimposed on a distorted grid. We need to convert the CCD horizontal (X) and vertical (Y) dimensions to the angles. $h = f \tan \theta$

$$\theta_{horizontal} = \theta_x = \tan^{-1}\left(\frac{4.74}{2} \frac{1}{6}\right) = 21.55^\circ \quad \& \quad \theta_{vertical} = \theta_y = \tan^{-1}\left(\frac{3.55}{2} \frac{1}{6}\right) = 16.48^\circ$$

Choose: Analysis>Diagnostics>Distortion Grid:

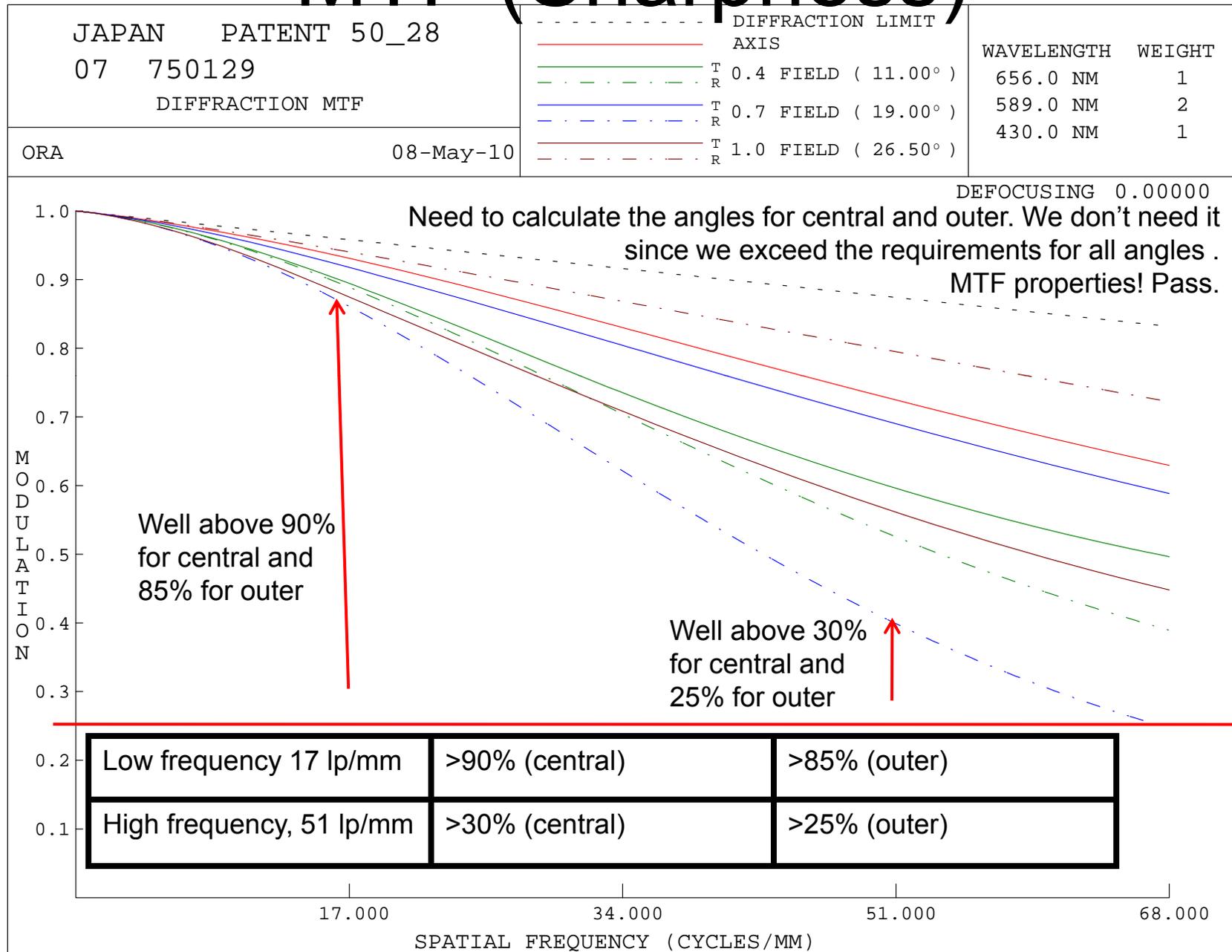
X - FOV semi - field = 21.55 & y - FOV semi - field = 16.48



MTF (Sharpness)

- User wants sharpness of resolution, designer has to relate these to Modulation Transfer Function (MTF).
- MTF is the relative contrast. 1 for ideal and 0 for no contrast at all.
- For low frequencies (large features) any lens will have good contrast
- For high frequencies (small features we need good lenses)
- Use Analysis>Diffraction>MTF
 - Maximum frequency 68 (max spatial frequency of our CCD array is 67, low frequency of the specs is 17 lines/mm so $4 \times 17 = 68$ will cover all the information we need)
 - Increment frequency 17
 - Number of rays across diameter 60
- Usually the MTF is used in its one-dimensional form, calculated for one azimuthal section through the image plane. The azimuth (section plane) of the object pattern is called **radial (sagittal) azimuth** when the **prolongation of the slit or object passes through the reference axis**. When the prolongation of the slit pattern is perpendicular to the reference axis, the azimuth is called tangential azimuth.
- The depth of field issue (MTF for object distances between 750 mm and infinity) will be considered in optimization section)

MTF (Sharpness)



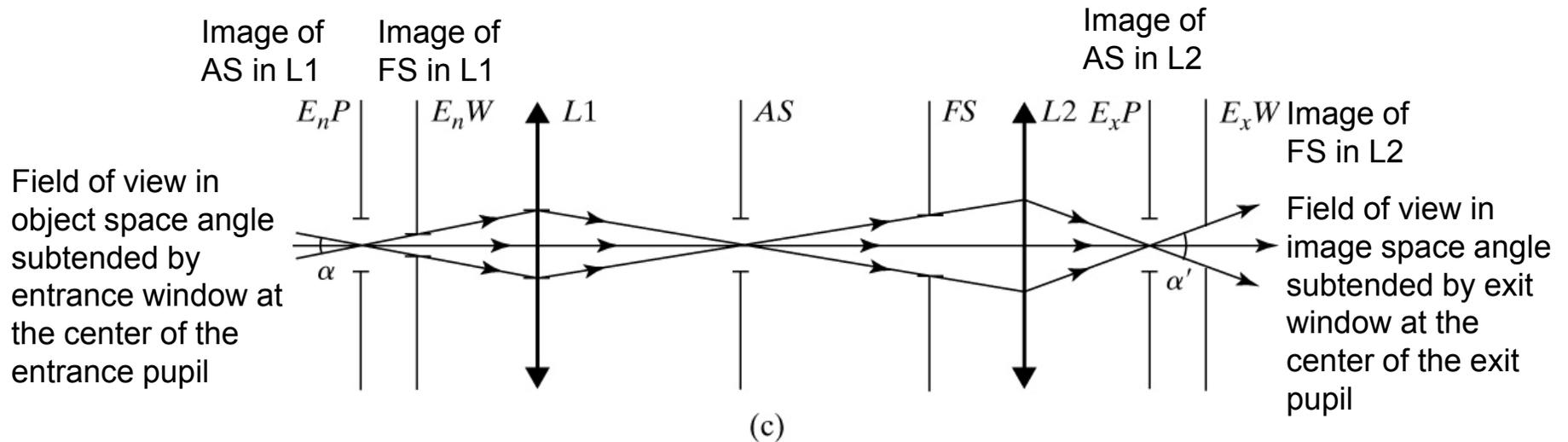
Field stops, Entrance Window, Exit window

Field stop (FS): the aperture that controls the field of view to **eliminate poor quality image points** due to aberration or vignetting.

Practical criteria to determine field stop: as seen from the center of the entrance pupil, the field stop or its image subtends the smallest angle.

Entrance window (E_nW): is the image of the field stop by all optical elements preceding it (to the left of it). It outlines the lateral dimensions of the object being imaged. Conjugate of FS.

Exit window (E_xW): is the image of the field stop by all optical elements following it (to the right of it). This is like a window limiting outside view as seen from inside of a room.



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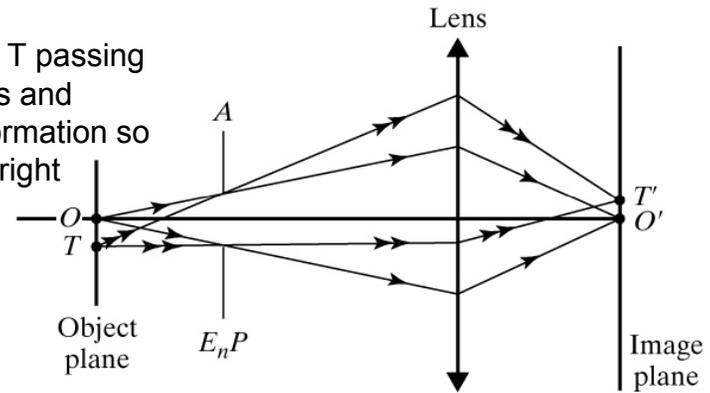
Field of view

Apertures limit the image brightness and field of view like a window does.

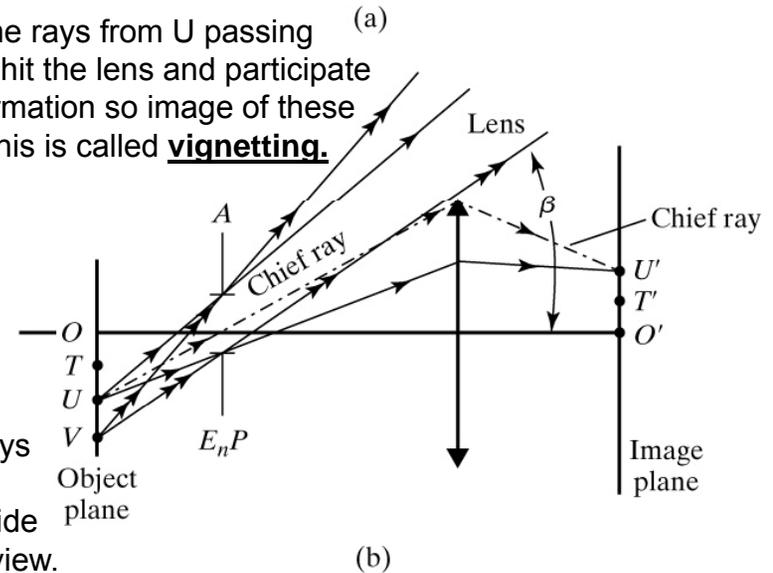
If we consider image points with at least half of brightness (irradiance W/m^2) of the axial image points acceptable then we define **field of view** a circle with radius of OU where U is a point on the object plane that only half of its rays passing through the aperture reaches the image plane (or chief ray from U touches edges of the lens).

Angular field of view: twice the angle β between the chief ray from the last object point with half brightness and the optical axis.

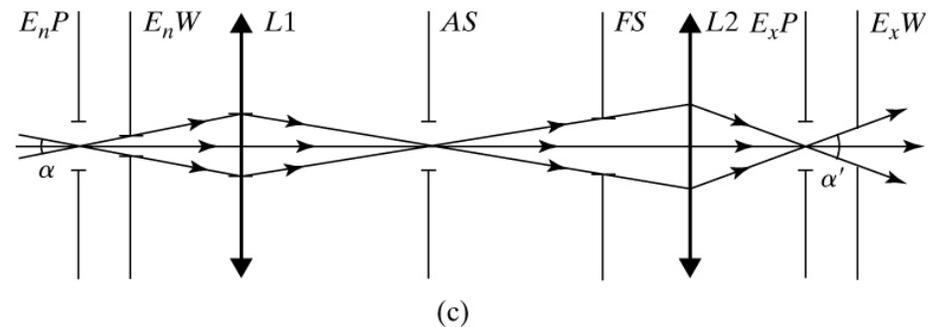
All of the rays from O and T passing through the AS hit the lens and participate in the image formation so image of these points is bright



Only some of the rays from U passing through the AS hit the lens and participate in the image formation so image of these points is dim. This is called **vignetting**.



None of the rays from V hit the lens. V is outside of the field of view.

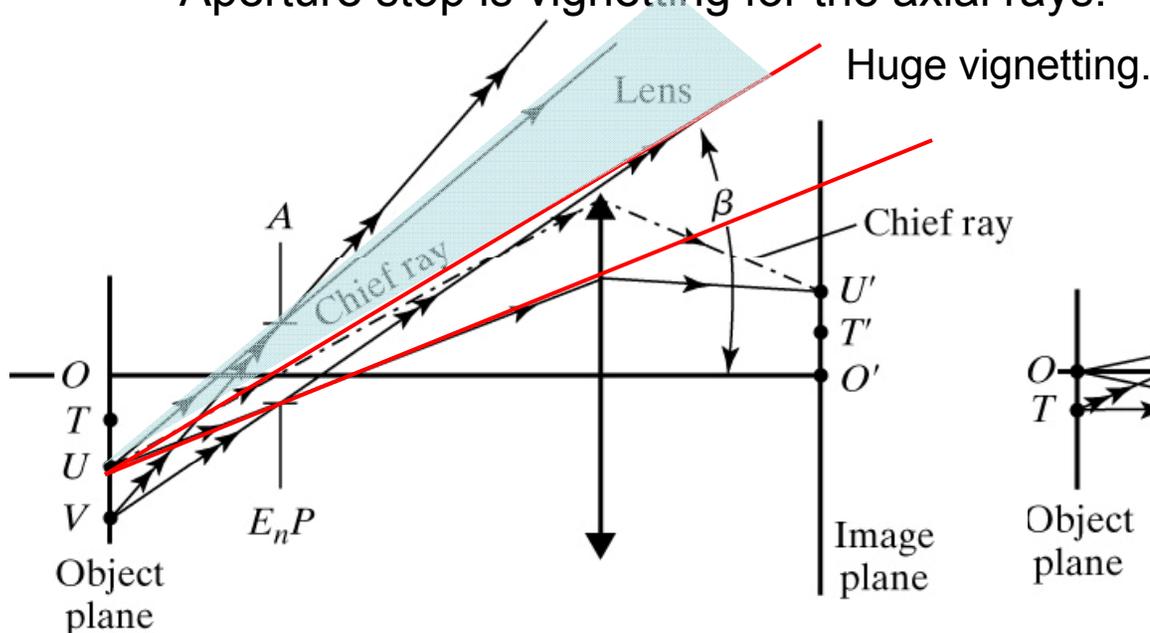


Reference rays

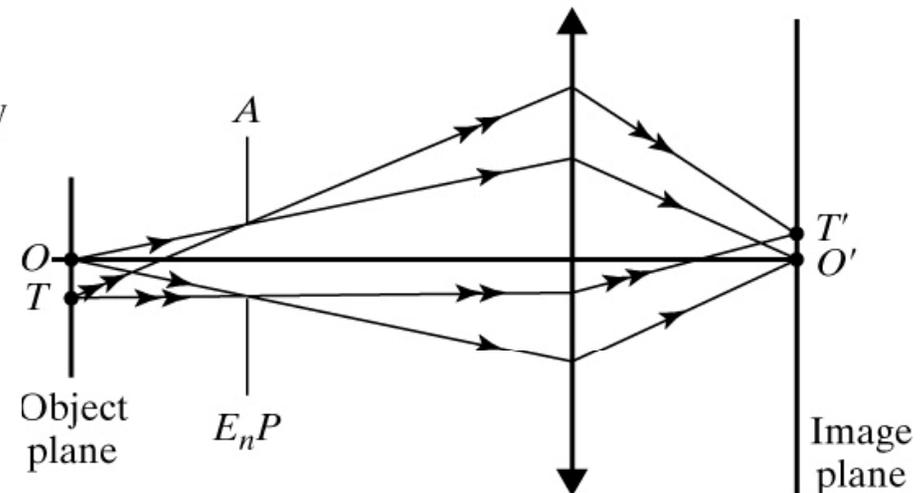
- Reference rays: a special set of rays traced from each field position (angle) of the lens with no symmetry. Usually five rays are traced from each field. The coordinates of the five rays are retained for every surface and recalculated any time a surface is changed.
 - Chief ray (one ray)
 - Outmost rays in x-z and y-z planes of the bundle from each field point (4 rays)
- Reference rays are used to determine:
 - Default clear apertures
 - Edge thickness
 - Spatial frequencies for MTF calculations
 - Any value that depends on the sizes of elements or the extent of the light bundles.
- Reference rays are determined based on the system data such as:
 - Pupil size $f\#$, or numerical aperture, (EPD?)
 - The field position
 - Vignetting factors
- The chief ray always goes through the center of the aperture stop and it is considered the representative of its field
- Other 4 rays pass through the edge of the paraxial entrance pupil

Vignetting

- For non-axial field points, the aperture stop may not be the limiting aperture and portions of off-axis bundles may be clipped by apertures on other surfaces.
- If vignetting is present then the reference rays can not reach the edges of the paraxial entrance pupil. Instead they are pushed inward (towards the optical axis) by a factor called “vignetting factor”.
- Aperture stop is vignetting for the axial rays.



There is vignetting, and vignetting factor is less than one, the reference rays can not hit the edges of the entrance pupil. They have to come closer to the optical axis



There is no vignetting so vignetting factor is one, the reference rays hit the edges of the entrance pupil

Vignetting: problem and solution

- Solution: we may use vignetting to remove the bad rays (rays with severe aberration)
- Problem: vignetting reduces the illumination of the off-axis points (the rays do not reach the system).
- Vignetting as result of limitation in size of the lenses is also a problem.
- Vignetting limits the FOV

Vignetting/Illumination?

- Vignetting: clipping of off-axis rays due to apertures on surfaces other than the aperture stop surface.
- Angular effects:
 - $\text{relative illumination}(\text{angle}) = \cos^4(\text{field angle})/4$
- Requirements: at least 60% relative illumination at the corner of the field (full field).
- Vignetting factors determine the reference rays
- Reference rays determine default apertures
- Default apertures determine the rays that are included in calculation of MTF
- Designer can change the vignetting factors to expand or contract the cone of light entering the system from off-axis fields.
- You can see the vignetting-related data from the MTF screen click on the text.

MTF (Sharpness)-text

GEO NO
MFR 68
IFR 17
PLO FRE Y
GO

APERTURE STOP

SEMI-DIAMETER = 0.685366

(Based on the maximum reference ray height at the stop.)

X and Y focal lengths for each field angle

X	0.595785E+01	0.607819E+01	0.632056E+01	0.665978E+01
Y	0.595785E+01	0.632659E+01	0.708043E+01	0.798371E+01

X and Y F-numbers for each field angle

X	3.475412	3.545608	3.714552	3.975541
Y	3.475412	3.690510	4.230523	5.524230

Reference sphere radius for each field angle

	0.535692E+01	0.548322E+01	0.574632E+01	0.614527E+01
--	--------------	--------------	--------------	--------------

Relative execution time = 1.21 UNITS

POSITION 1 DIFFRACTION MTF ORA 09-May-10
JAPAN PATENT 50_2807 750129

WAVELENGTH WEIGHT NO. OF RAYS

FIELD (X,Y)=(0.00, 0.00)MAX, (0.00, 0.00)DEG 656.0 NM 1 524
RELATIVE ILLUMINATION = 100.0 PER CENT 589.0 NM 2 648
ILLUMINATION (UNIT BRIGHTNESS) = 0.064999 430.0 NM 1 1224
DISTORTION = 0.00 PER CENT

DIFFRACTION LIMIT FOCUS POSITION

Formula Actual 0.00000

L/MM f/3.500 RAD TAN RAD TAN

0 .999 .999 .999
17 .957 .958 .931
34 .914 .916 .830
51 .872 .874 .725
68 .829 .832 .629

POSITION 1 DIFFRACTION MTF ORA 09-May-10
JAPAN PATENT 50_2807 750129

WAVELENGTH WEIGHT NO. OF RAYS

FIELD (X,Y)=(0.00, 0.39)MAX, (0.00, 11.00)DEG 656.0 NM 1 486
RELATIVE ILLUMINATION = 93.0 PER CENT 589.0 NM 2 604
ILLUMINATION (UNIT BRIGHTNESS) = 0.060457 430.0 NM 1 1134
DISTORTION = 0.12 PER CENT

DIFFRACTION LIMIT FOCUS POSITION

Formula Actual 0.00000

L/MM f/3.500 RAD TAN RAD TAN

0 .999 .999 .999 .999 .999
17 .957 .957 .955 .889 .894
34 .914 .915 .910 .704 .735
51 .872 .872 .865 .525 .596
68 .829 .829 .821 .390 .496

POSITION 1 DIFFRACTION MTF ORA 09-May-10
JAPAN PATENT 50_2807 750129

WAVELENGTH WEIGHT NO. OF RAYS

FIELD (X,Y)=(0.00, 0.69)MAX, (0.00, 19.00)DEG 656.0 NM 1 406
RELATIVE ILLUMINATION = 78.6 PER CENT 589.0 NM 2 510
ILLUMINATION (UNIT BRIGHTNESS) = 0.051069 430.0 NM 1 964
DISTORTION = 0.21 PER CENT

DIFFRACTION LIMIT FOCUS POSITION

Formula Actual 0.00000

L/MM f/3.500 RAD TAN RAD TAN

0 .999 .999 .999 .999 .999
17 .957 .955 .948 .861 .918
34 .914 .911 .896 .621 .804
51 .872 .866 .844 .399 .690
68 .829 .821 .794 .247 .588

POSITION 1 DIFFRACTION MTF ORA 09-May-10
JAPAN PATENT 50_2807 750129

WAVELENGTH WEIGHT NO. OF RAYS

FIELD (X,Y)=(0.00, 1.00)MAX, (0.00, **26.50**)DEG 656.0 NM 1 310
RELATIVE ILLUMINATION = 58.8 PER CENT 589.0 NM 2 380
ILLUMINATION (UNIT BRIGHTNESS) = 0.038205 430.0 NM 1 720
DISTORTION = -0.20 PER CENT

DIFFRACTION LIMIT FOCUS POSITION

Formula Actual 0.00000

L/MM f/3.500 RAD TAN RAD TAN

0 .999 .999 .999 .999 .999
17 .957 .954 .935 .940 .875
34 .914 .908 .869 .868 .708
51 .872 .863 .803 .795 .561
68 .829 .818 .739 .722 .448

We need 60%
This is close enough
that we leave it to see if
it will be fixed during the
optimization
 $\text{Cos}^4(26.5)=0.64_{49}$

Vignetting/Illumination

- Analysis>system>Transmission data provides more detailed illumination data including coatings. Default is a quarter wavelength anti-reflection coating for each glass surface.

FIELD (X,Y)=(0.00, 26.50) DEG

```
-----  
PRODUCT REF 0.9412  0.9696  0.8985  0.9447  
ABS 1.0000  1.0000  1.0000  1.0000  
Ave Transmittance: 0.9412  0.9696  0.8985  0.9447  
ILLUMINATION:      0.03571  0.03682  0.03430  0.03591  
RELATIVE ILLUM:   56.5    57.2    61.1    57.9  
PROJECTED SOLID  
ANGLE (IMAGE SPACE): 0.03794  0.03798  0.03818  0.03802  
USED AREA OF  
ENTRANCE PUPIL:     2.02455  2.02488  2.02510  2.02485
```

Analysis>Fabrication support>Cost Analysis

15-May-10

CODE V

POSITION 1

BLOCKING COST FACTORS FOR LENS PRODUCTION

JAPAN PATENT 50_2807 750129

SURFACE NUMBER	RADIUS OF CURVATURE	THICKNESS	OUTSIDE DIAMETER	GLASS NAME	BLOCKING FACTOR	APPROXIMATE NUMBER	SPINDLE LOADS PER 1000 ELTS
1	2.2451	0.693	3.126	LAKN17	1.000	1	1000
2	4.4211		3.126		0.467	4	250
3	-4.1545	0.126	2.011	SF1	0.362	6	167
4	2.6249		2.011		0.574	2	500
5	5.7998	0.409	2.245	NLASF41	0.327	7	143
6	-3.4062		2.245		0.543	2	500

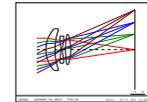
Spindle loads for 1000 assemblies = 2560

NOTE:

1. Linear dimensions are in millimeters unless otherwise noted.
2. The "OUTSIDE DIAMETER" is larger than the clear aperture for beveling/mounting purposes
3. "APPROXIMATE NUMBER" is an estimate of the number of elements on a block, based on the following assumptions:
 - A. Maximum blocker diameter is 300.0 MM.
 - B. Maximum blocker "STEEPNESS" is 160.0 degrees.

Establish feasibility: Issues

- Size of the elements:
 - At 6 mm EFL this lens system is tiny. 126 micorn thickness of the central part of the middle lens.
 - Reasonable values for the thickness at the center is about 0.8 mm and at the edges about 0.9 mm
 - We need optimization to fix this.
- Index of the glasses:
 - They are too high (1.786, 1.717, 1.835)
 - High ($n > 1.65$) and low index ($n < 1.45$) glasses are less common and more expensive
- Tools>Macro Manager>glassfit.seg
 - Use the dialog box and answer the questions



Catalog Code Definition

```
-----
All      3  Discontinued
         4  Discontinued with recommended replacement glass
         5  Preliminary
Schott   0  Preferred glass
         1  Standard Glass
         2  Inquiry Glass
```

Surf	Catalog	Glass	Delta Nd	Delta Vd	Avail	Price	DPF	Bubl	Stain
1	SCHOTT	LAKN17	-0.00103	0.1666	3	0.00	-73	0	0
3	SCHOTT	SF1	0.00000	-0.0129	0	25.00	0	1	1
5	SCHOTT	NLASF41	-0.00020	-0.2291	0	0.00	-79	1	1

Conclusion

- Interpret the specs for the design
- Locate the suitable starting design using patent search
- Select the lens
- Analyze the starting points
- Determine the guidelines for optimization
 - Constrain the glass choices
 - Constrain the size of the lenses

Optimization

- Generate “best” possible system that can be achieved within a given set of constraints
- “best” is measured by an error function.
- The error function for a software is usually predetermined but it might be possible to change the weighting factors. Code V uses accelerated Damped Least Square (DLS) method.
- We need to
 - Setting up variables
 - Determine and define the constraints
 - Analyze the results